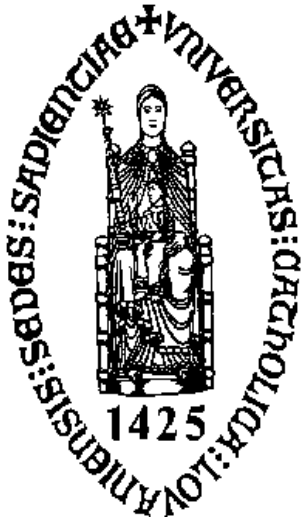

Noise performance of elementary transistor stages



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SNR and SNDR

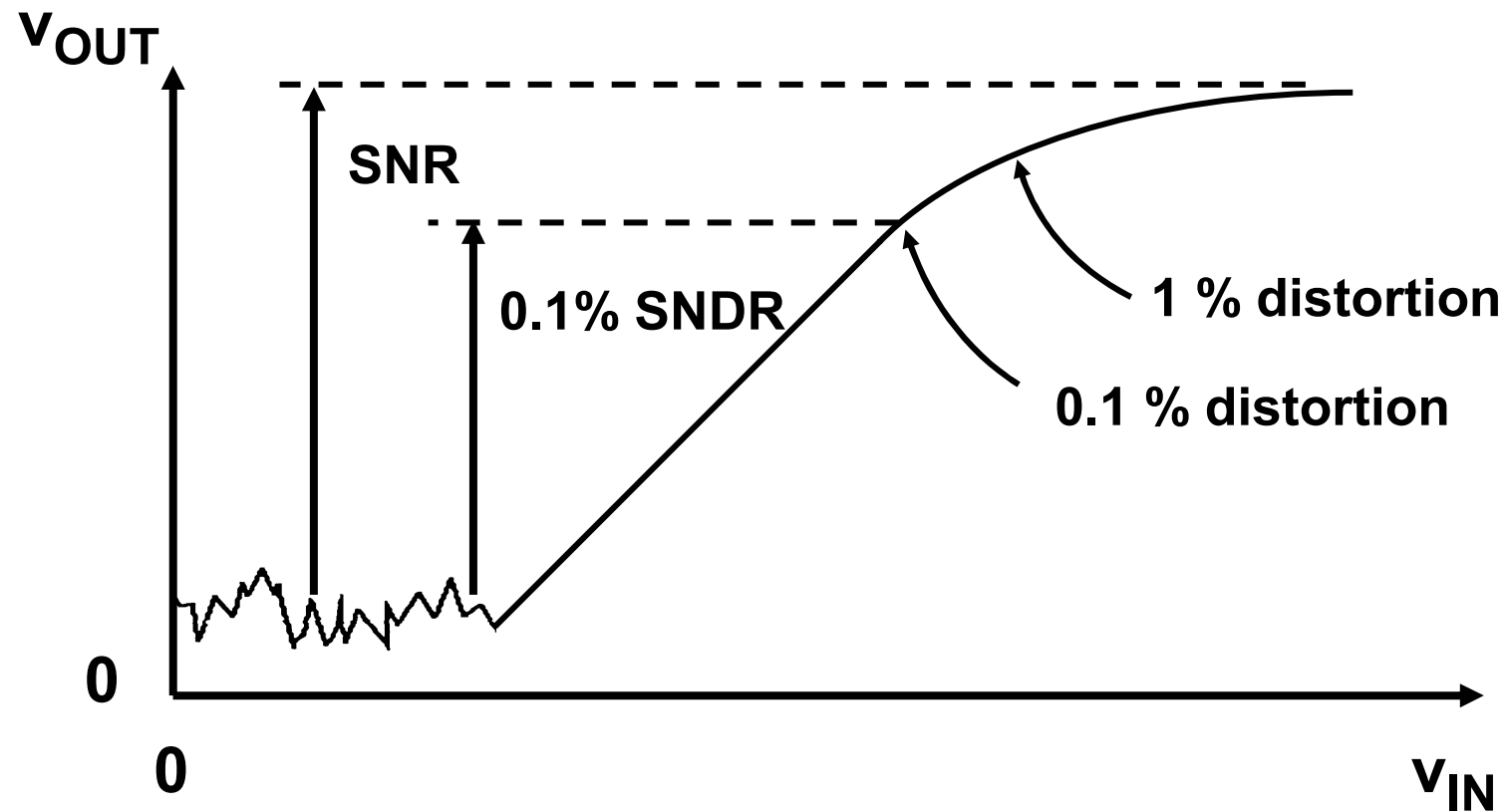
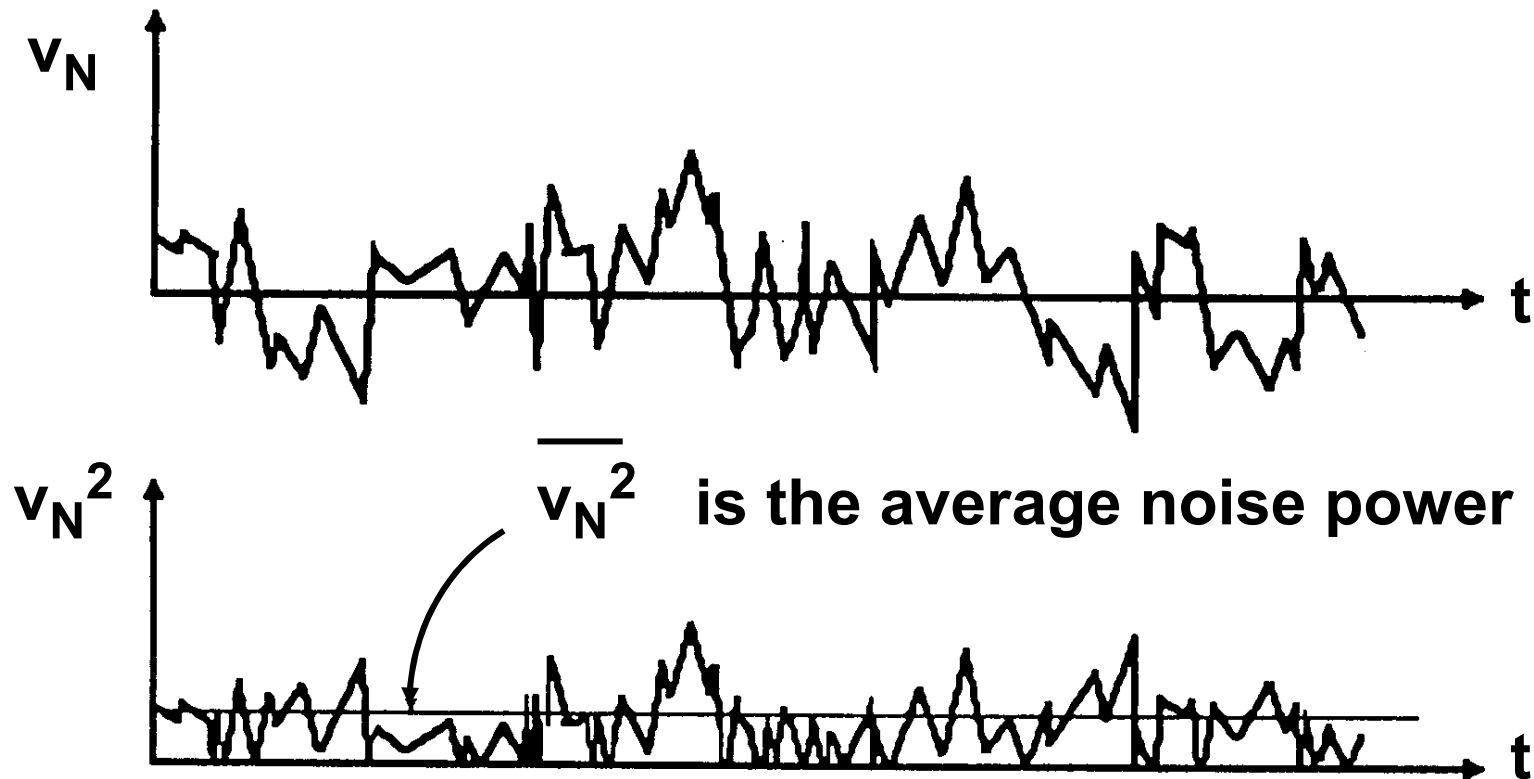


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- ◆ **Definitions of noise**

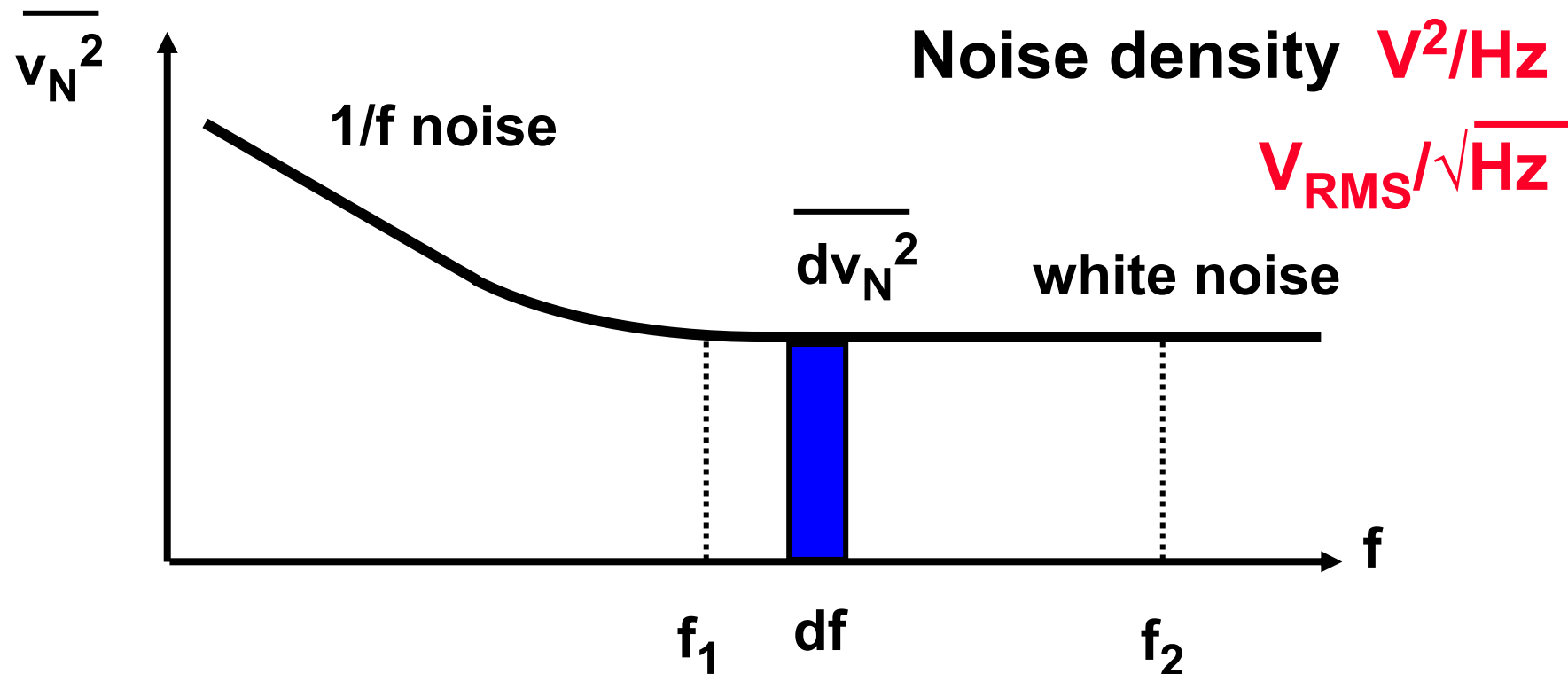
- ◆ **Noise of an amplifier**
- ◆ **Noise of a follower**
- ◆ **Noise of a cascode**
- ◆ **Noise of a current mirror**
- ◆ **Noise of a differential pair**
- ◆ **Capacitive noise matching**

Noise versus time



Ref. Van der Ziel (Prentice Hall 1954, Wiley 1986), Ott (Wiley 1988)

Noise versus frequency

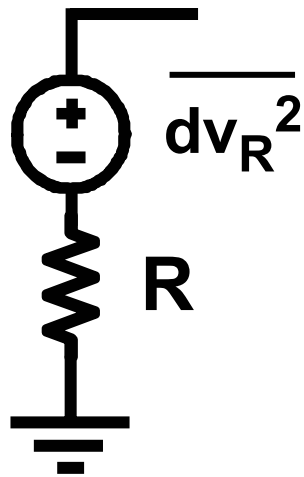


Integrated noise

V_{RMS}

$$\overline{v_{12}} = \sqrt{\overline{v_N^2}} = \sqrt{\int_{f_1}^{f_2} d\overline{v_N^2} df} = \sqrt{(f_2 - f_1) d\overline{v_N^2}}$$

Noise of a resistor is thermal noise

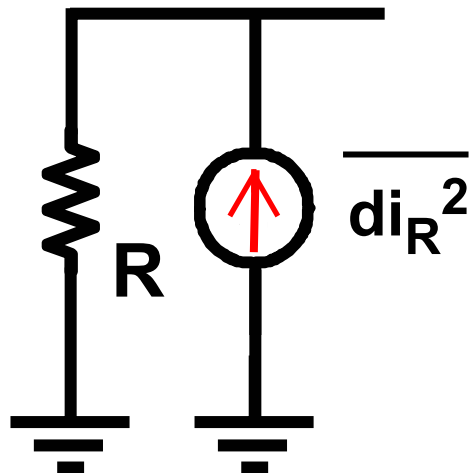


$$\overline{dv_R^2} = 4kT R df \quad \text{is white}$$

depends on T , not on I_R

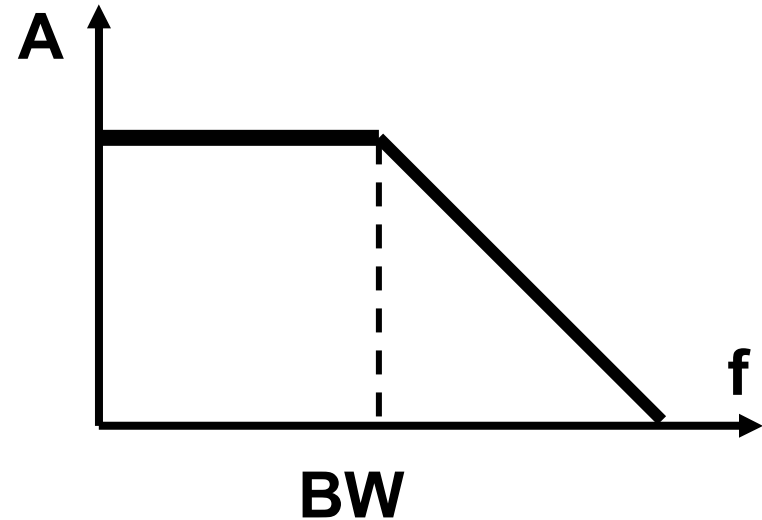
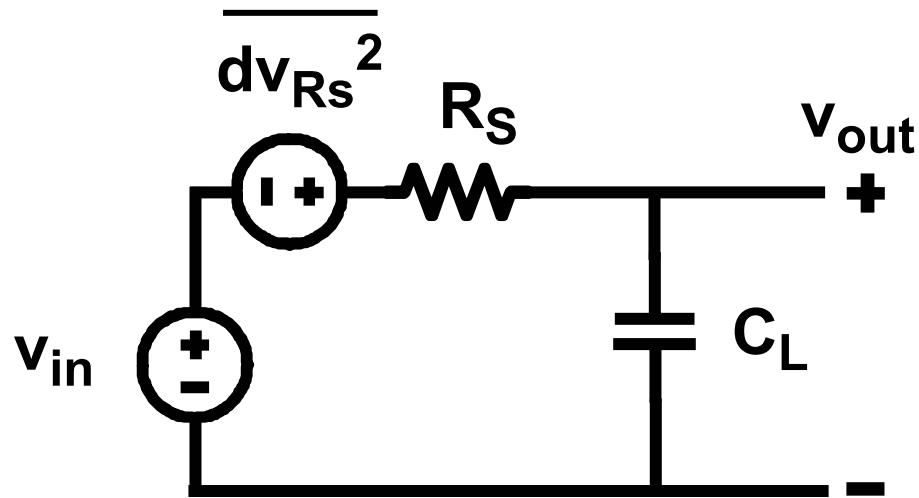
$$\text{for } R = 1 \text{ k}\Omega \quad \sqrt{\overline{dv_R^2}} = 4 \text{ nV}_{\text{RMS}} / \sqrt{\text{Hz}}$$

at $T = 300 \text{ K}$ or 27°C



$$\overline{di_R^2} = \frac{\overline{dv_R^2}}{R^2} = \frac{4kT}{R} df \quad \text{is white}$$

Integrated Noise of Resistor - 1

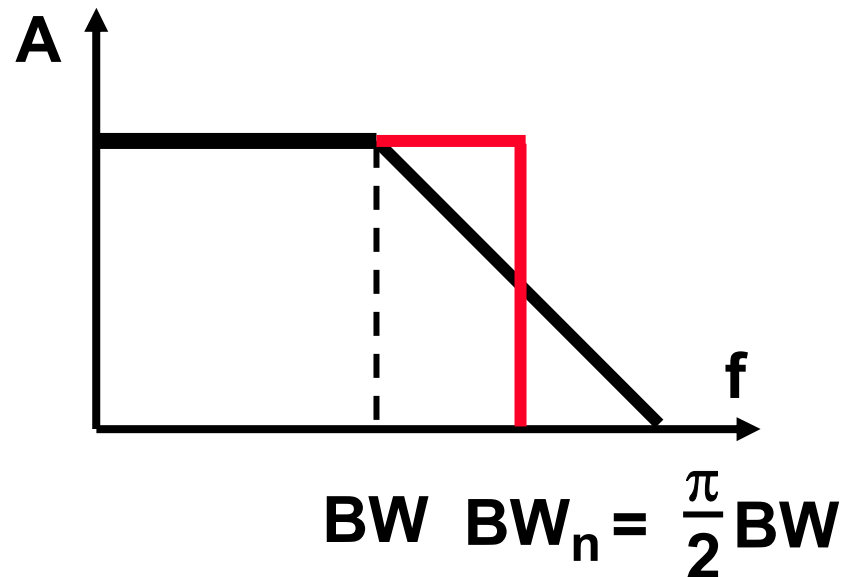


$$\overline{dv_{Rs}^2} = 4kT R_S df$$

$$\overline{v_{Rs}^2} = \int_0^{\infty} \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2}$$

$$BW = \frac{1}{2\pi R_S C_L}$$

Integrated Noise of Resistor - 2



$$\overline{v_{Rs}^2} = \int_0^{\infty} \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2}$$

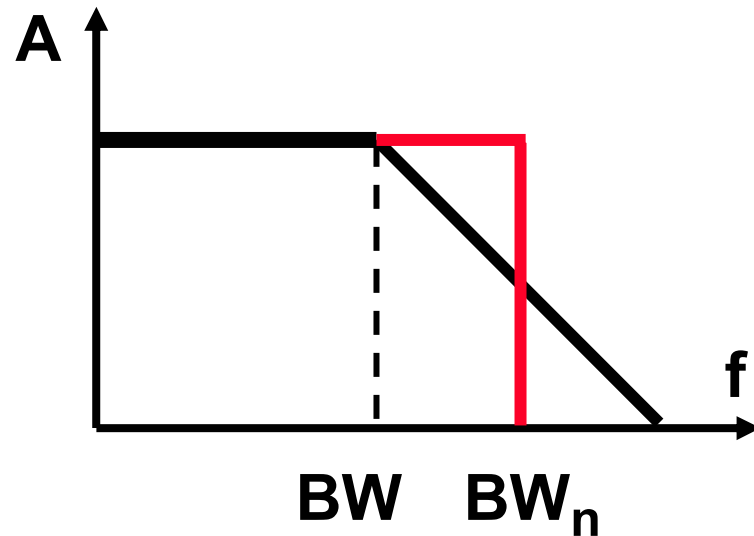
$$\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$$

$$\overline{v_{Rs}^2} = 4kT R_S BW \frac{\pi}{2} df$$

$$\overline{v_{Rs}^2} = \frac{kT}{C_L}$$

$$C_L = 1\text{pF} \quad v_{Rs} = 65 \mu V_{RMS}$$

Noise density vs integrated noise



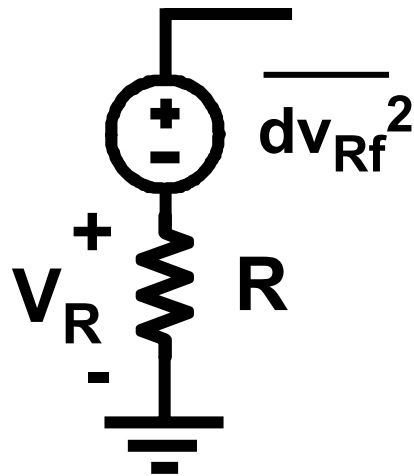
$$\overline{dv_{Rs}^2} = 4kT R_S df$$

$$\overline{v_{Rs}^2} = \int_0^\infty \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2} = \frac{kT}{C_L}$$

Noise density (V^2/Hz) $\sim R_S$ (or $1/g_m$)

Integrated noise (V_{RMS}) $\sim 1/C_L$

A resistor also has 1/f noise



$$\overline{dv_{Rf}^2} = V_R^2 \frac{KF_R R \square}{A_R} \frac{df}{f} \quad \text{is } 1/f$$

$$KF_{RSi} \approx 2 \cdot 10^{-21} \text{ Scm}^2$$

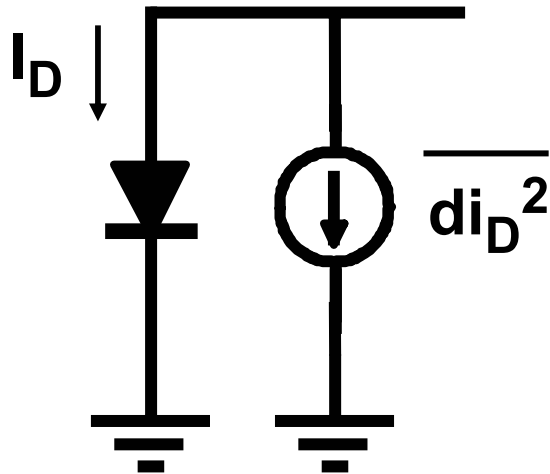
$$KF_{Rpoly} \approx 10 KF_{RSi}$$

for $R = 1 \text{ k}\Omega$ with 20 $\square$'s of $50 \text{ }\Omega/\square$ and $1 \text{ }\mu\text{m}$ wide and $V_R = 0.1 \text{ V}$

$$\sqrt{\overline{dv_{Rf}^2}} = 16 \text{ nV}_{\text{RMS}} / \sqrt{\text{Hz}} \text{ at } 1 \text{ Hz}$$

Ref. Vandamme, ESSDERC '04

Noise of a diode is shot noise



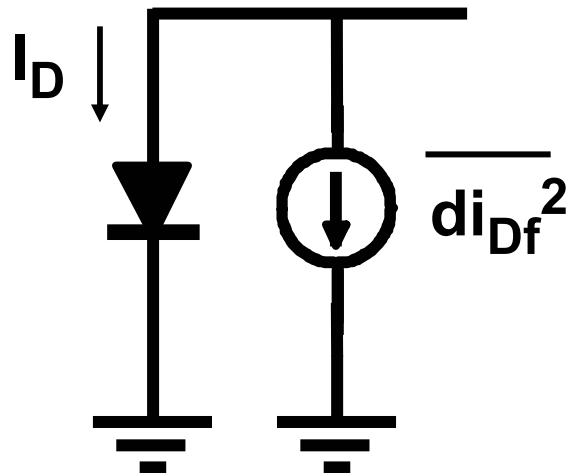
$$\overline{di_D^2} = 2q I_D df \quad \text{is white}$$

$$q = 1.6 \cdot 10^{-19} \text{ C}$$

depends on I_D , not on T

$$\text{for } I_D = 50 \mu\text{A} \quad \sqrt{\overline{di_D^2}} = 4 \text{ pA}_{\text{RMS}} / \sqrt{\text{Hz}}$$

A diode also has 1/f noise



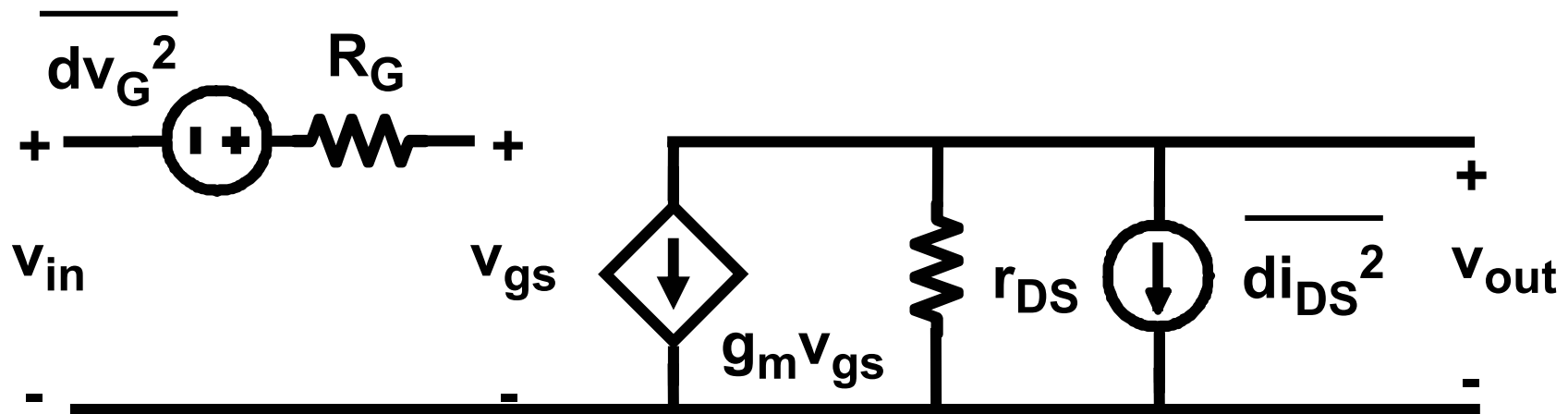
$$\overline{di_{Df}^2} = I_D \frac{KF_D}{A_D} \frac{df}{f} \quad \text{is } 1/f$$

$$KF_D \approx 10^{-21} \text{ Acm}^2$$

For a diode of $A_D = 5 \times 2 \text{ } \mu\text{m} = 10 \text{ } \mu\text{m}^2$ and $I_D = 0.1 \text{ mA}$

$$\sqrt{\overline{di_{Df}^2}} = 1 \text{ nA}_{\text{RMS}} / \sqrt{\text{Hz}} \text{ at } 1 \text{ Hz}$$

Noise of a MOST

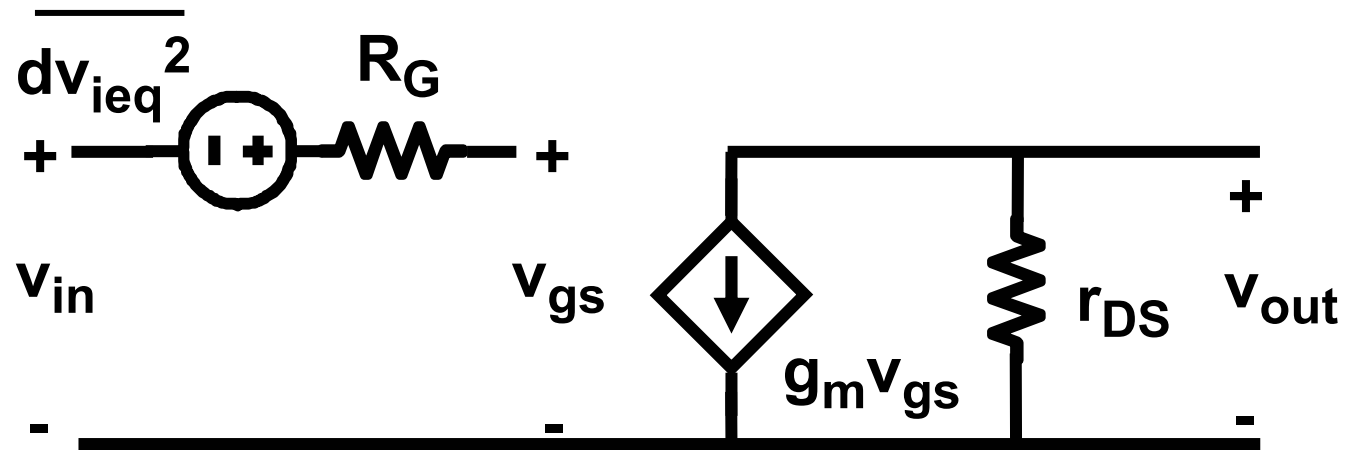


$$\overline{dv_G^2} = 4kT R_G df$$

$$\overline{di_{DS}^2} = \frac{4kT}{R_{CH}} df = 4kT \frac{2}{3} g_m df$$

Ref. Van der Ziel, Prentice Hall 1954, Wiley 1986.

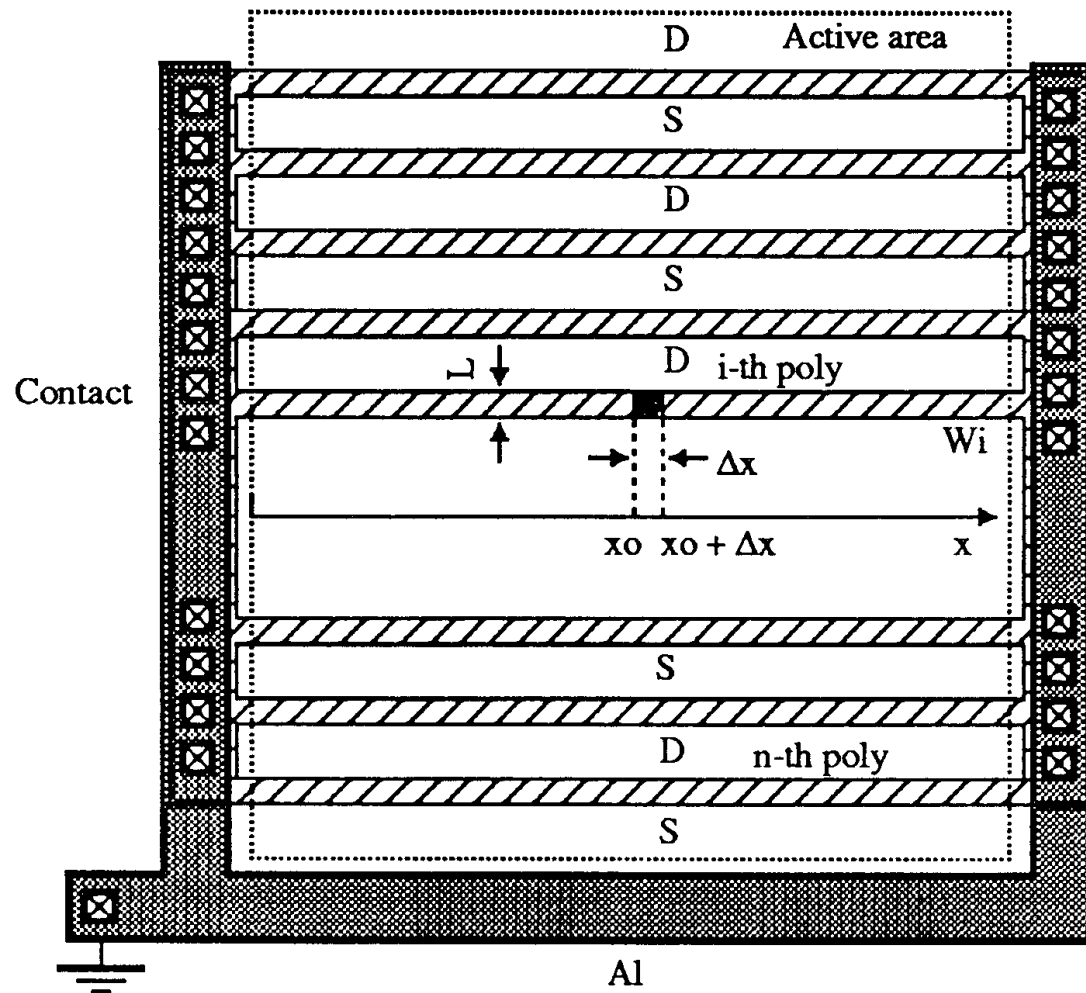
MOST: equivalent input noise : white



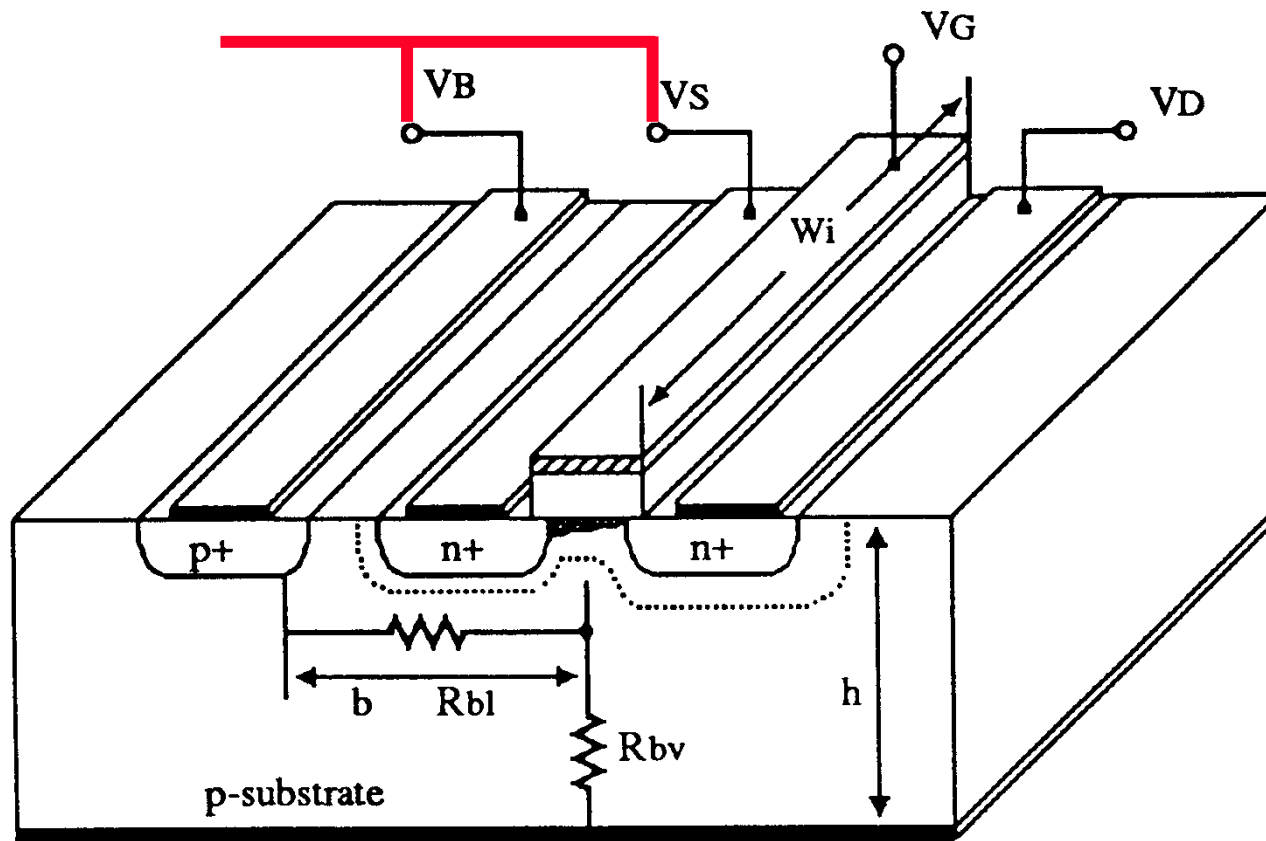
$$\overline{dv_{ieq}^2} = 4kT (R_{eff}) df \qquad R_{eff} = \frac{2/3}{g_m} + R_G$$

Hi Freq.: $\overline{di_{ieq}^2} = (C_{GS} \omega)^2 \overline{dv_{ieq}^2}$ is correlated

Poly Gate resistance r_G in a MOST

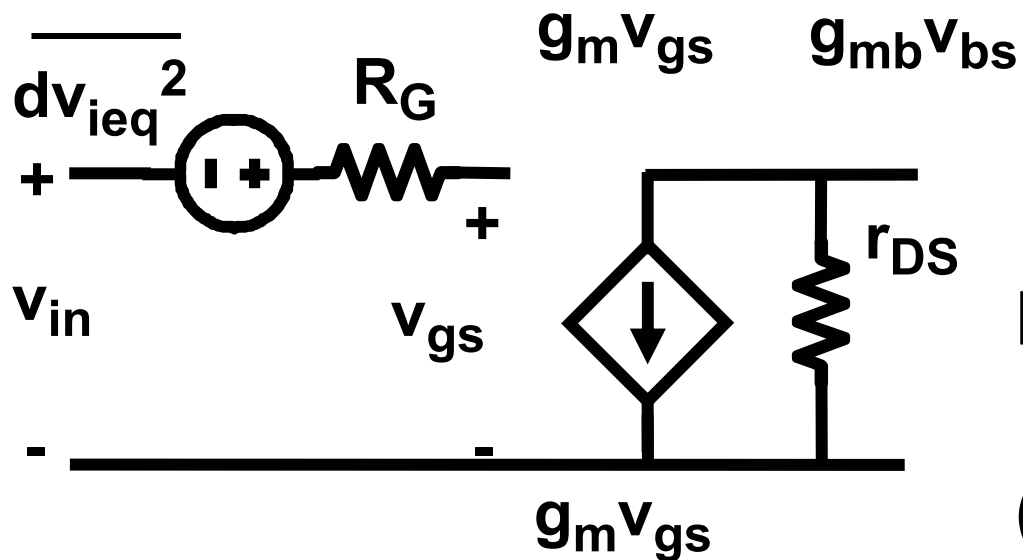
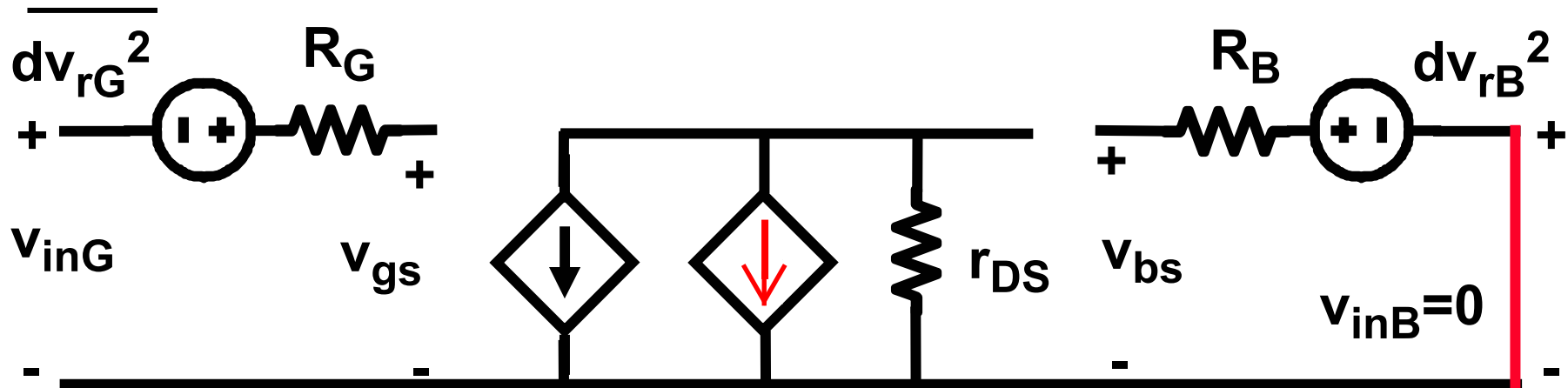


Substrate resistances r_B in a MOST



Ref. Chang, Kluwer 1991

Noise by the Bulk resistance

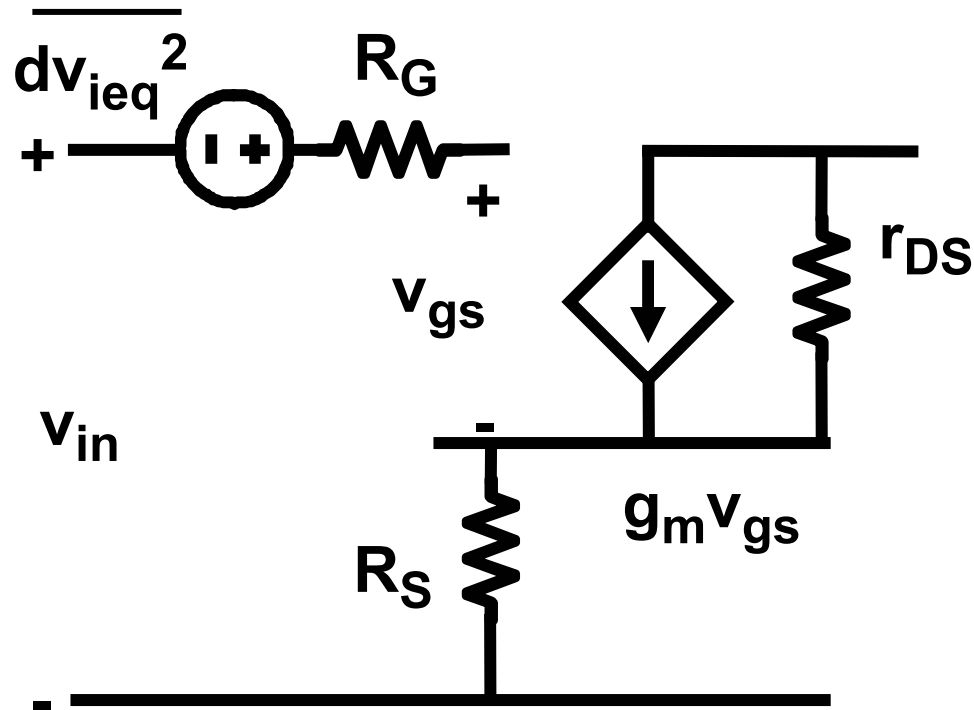


$$\overline{dv_{ieq}^2} = 4kT (R_{eff}) df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_B (n-1)^2$$

$$(n-1) = C_D/C_{ox} = g_{mb}/g_m$$

Noise by the Source resistance

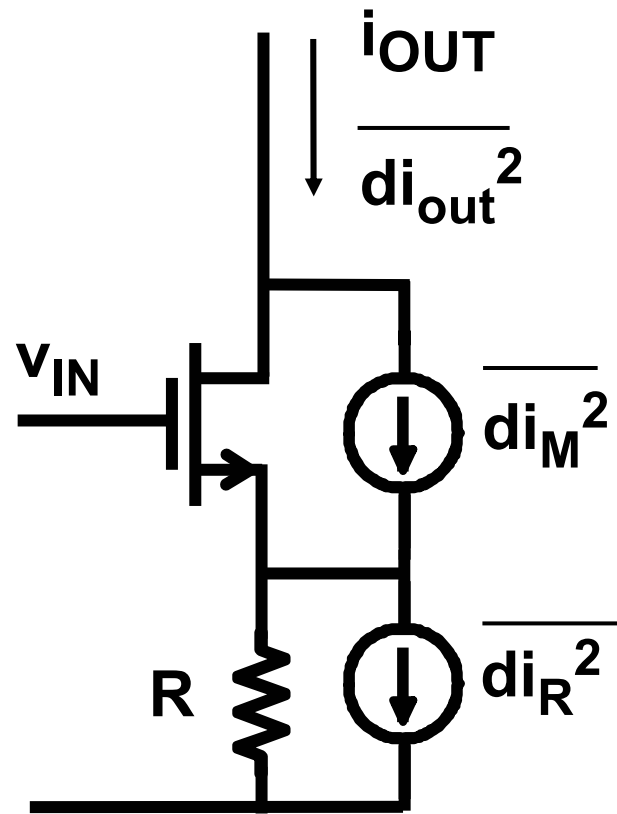


Noise of R_S
= noise R_G

$$\overline{dv_{ieq}^2} = 4kT (R_{eff}) df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_S + R_B (n-1)^2$$

Noise by Source resistor R



$$g_m R \gg 1$$

$$i_{out} = \frac{v_{in}}{R}$$

$$\overline{di_M^2} = 4kT \frac{2}{3} g_m df \quad \overline{di_{outM}^2} = \frac{\overline{di_M^2}}{(g_m R)^2}$$

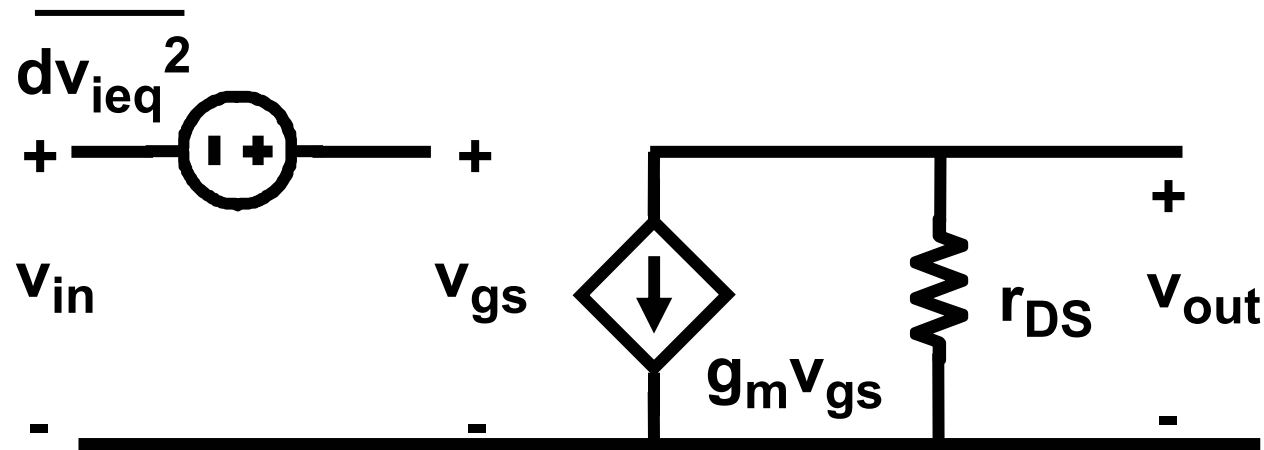
$$\overline{di_R^2} = \frac{4kT}{R} df$$

$$\overline{di_{outR}^2} = \overline{di_R^2}$$

$$\overline{di_{out}^2} = \frac{4kT}{R} \left(\frac{2}{3} \frac{1}{g_m R} + 1 \right) df \approx \frac{4kT}{R} df$$

$$\overline{dv_{in}^2} = 4kT R df$$

MOST: equivalent input noise : Exercise

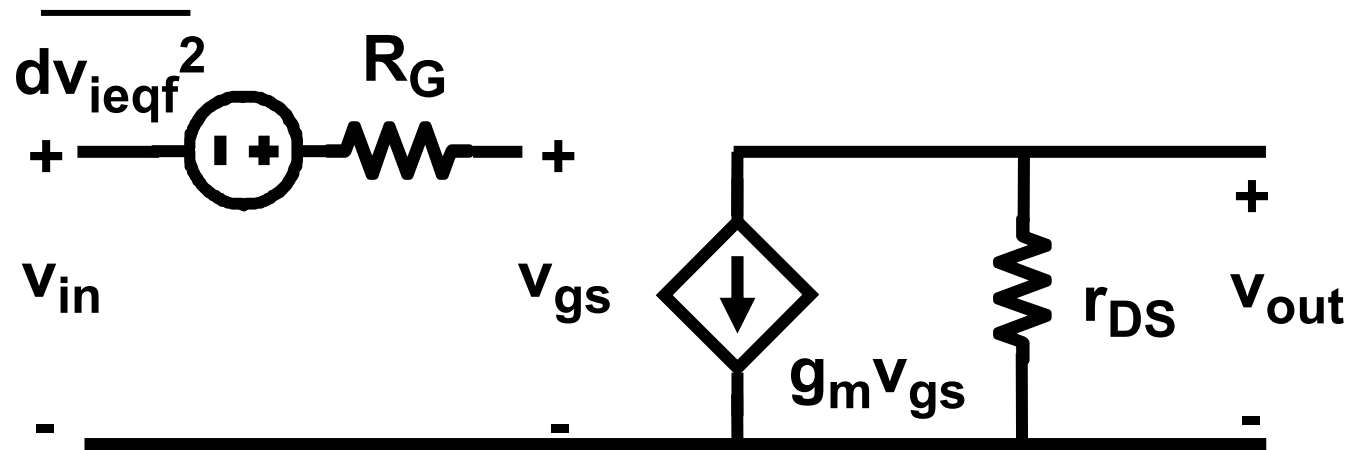


$$\overline{dv_{ieq}^2} \approx 4kT \left(\frac{2/3}{g_m} \right) df$$

$$\overline{dv_{ieq}^2} \approx ?$$

$$\text{for } I_{DS} = 65 \mu A$$

MOST: equivalent input noise : 1/f noise



$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL C_{ox}^2} \frac{df}{f}$$

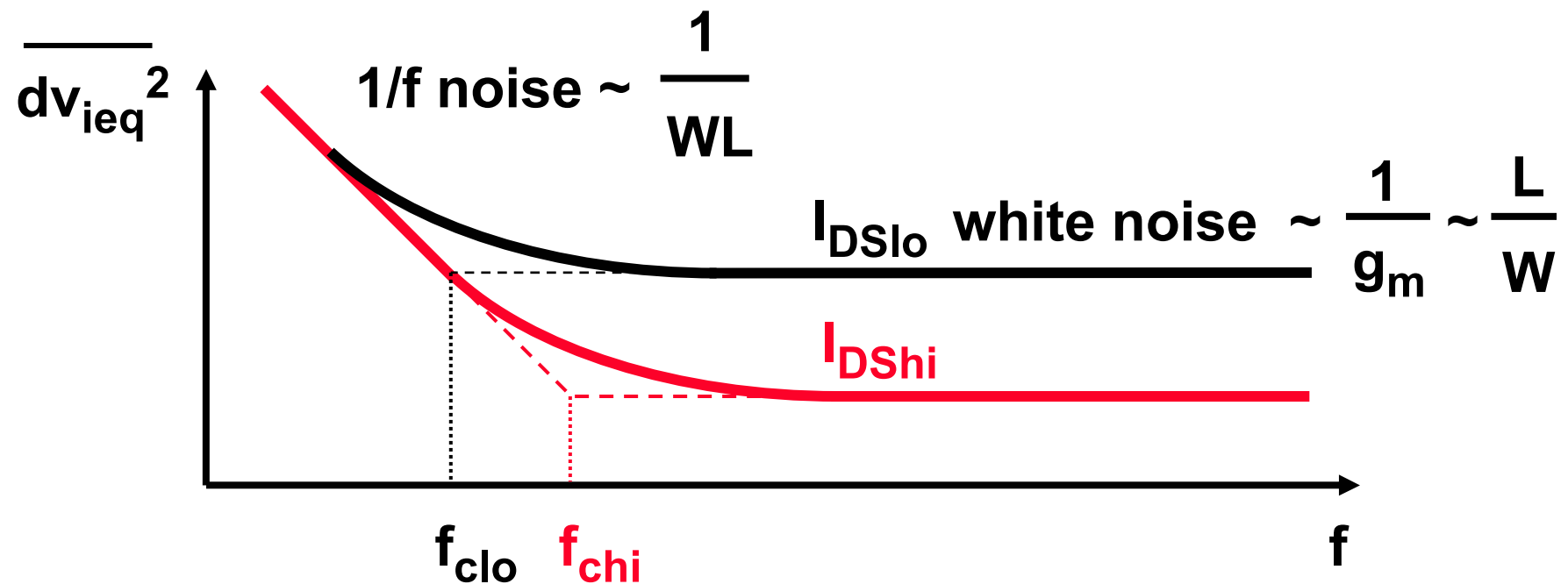
$$\text{pMOST } KF_F \approx 10^{-32} \text{ C}^2/\text{cm}^2$$

$$\text{nMOST } KF_F \approx 4 \cdot 10^{-31} \text{ C}^2/\text{cm}^2$$

$$\text{pJFET } KF_F \approx 10^{-33} \text{ C}^2/\text{cm}^2$$

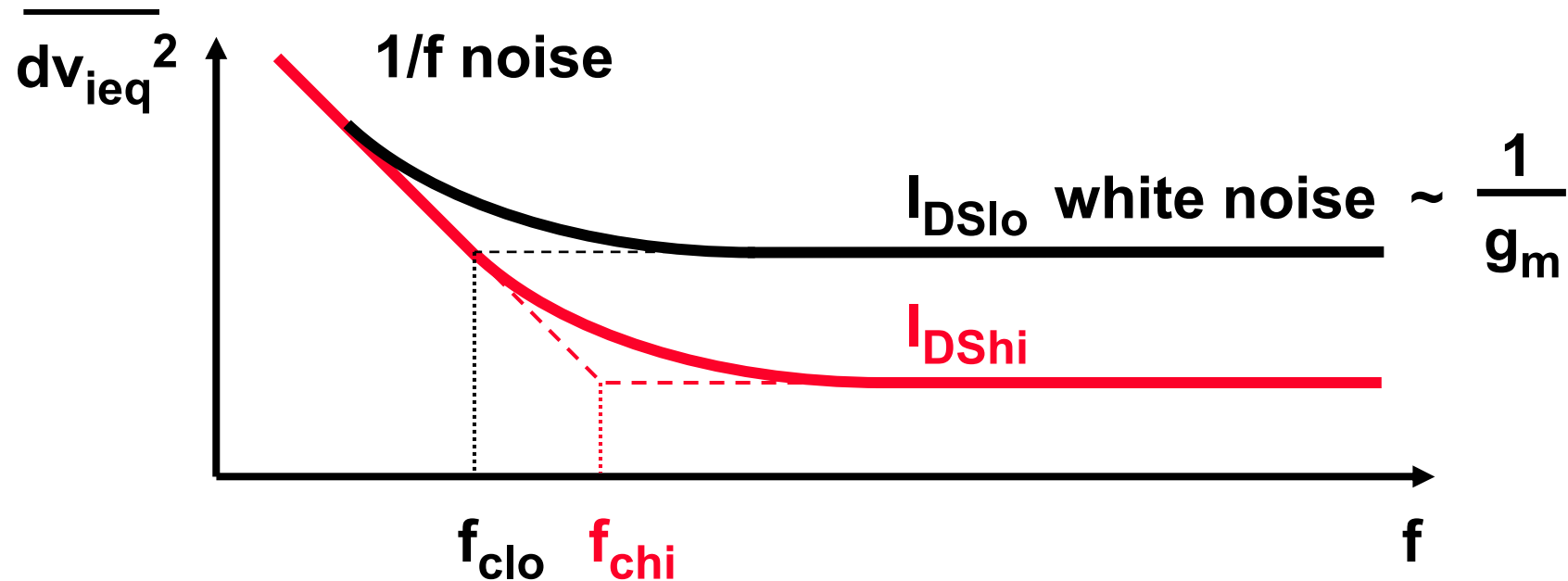
$$W \text{ \& L in cm; } C_{ox} \text{ in F/cm}^2$$

Noise vs current : corner frequency



Corner frequency $\sim g_m$

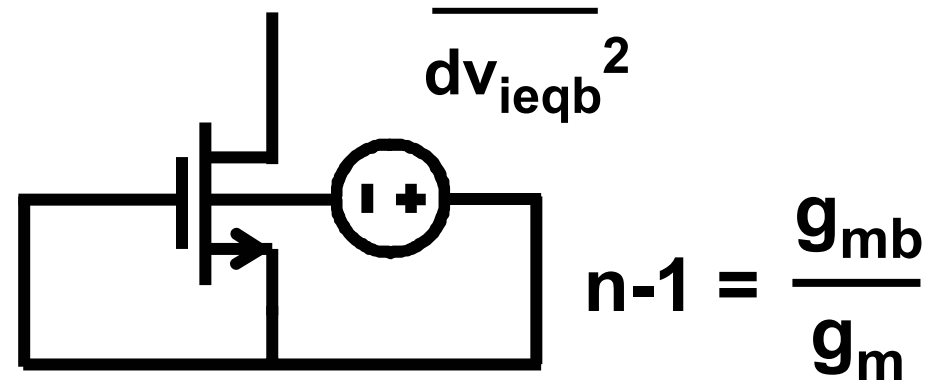
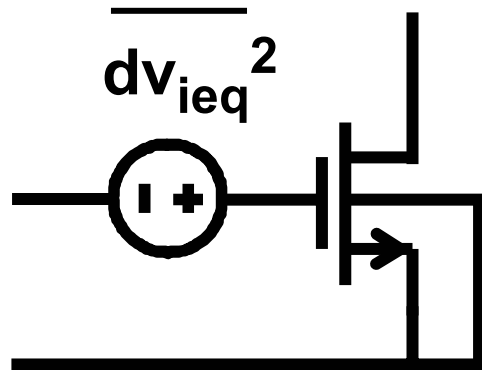
Noise vs current : exercise f_c



Ex. : f_c ? For $I_{DS} = 65 \mu A$;
 $K'_n = 60 \mu A/V^2$ and $L = 1 \mu m$ (0.35 μm process)

$f_c \approx 370 \text{ kHz}$

Noise seen at the Bulk



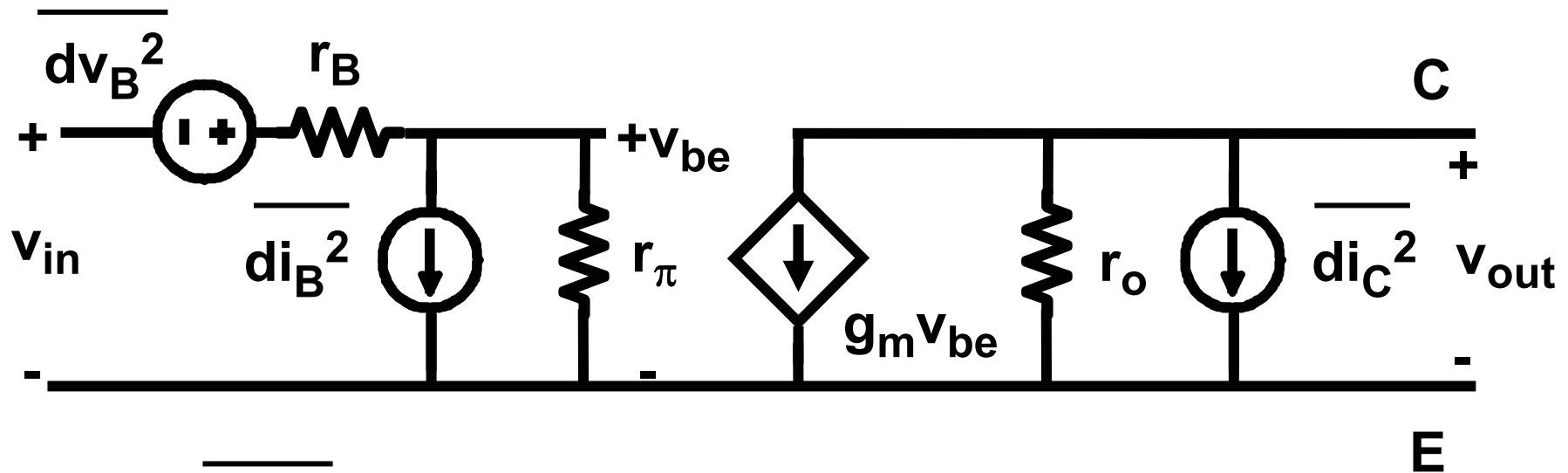
$$\overline{dv_{ieq}^2} = 4kT \left(\frac{2/3}{g_m} \right) df$$

$$\overline{dv_{ieqb}^2} = 4kT \left(\frac{2/3 g_m}{g_{mb}^2} \right) df$$

$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL C_{ox}^2} \frac{df}{f}$$

$$\overline{dv_{ieqfb}^2} = \frac{KF_F}{WL C_{ox}^2} \frac{g_m^2}{g_{mb}^2} \frac{df}{f}$$

Noise of a Bipolar transistor



$$\overline{dv_B^2} = 4kT r_B df$$

$$\overline{di_B^2} = 2q I_B df$$

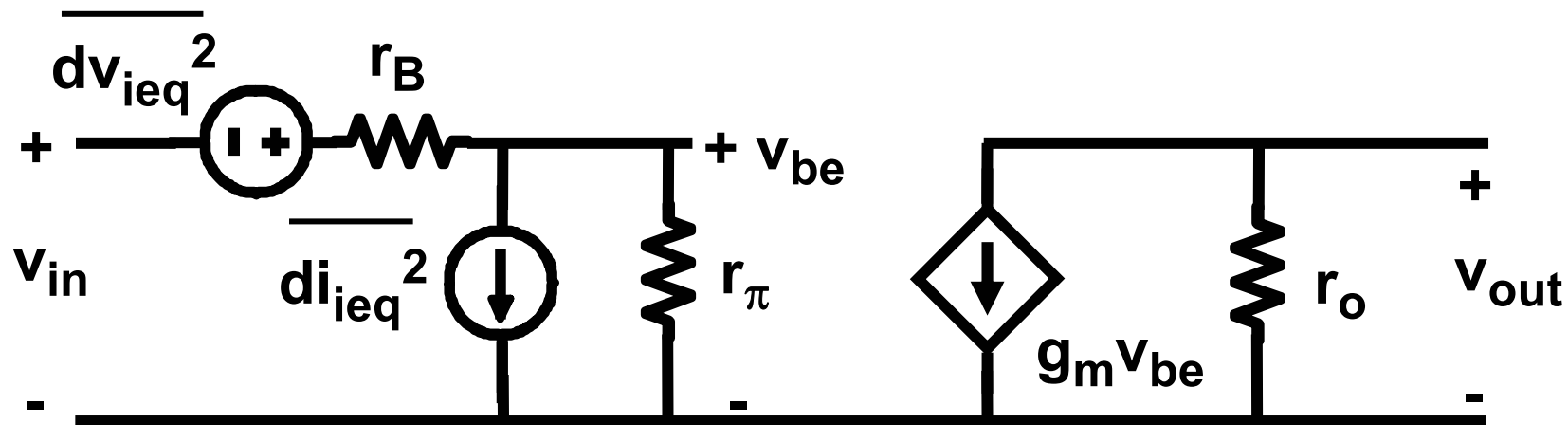
$$\overline{di_C^2} = 2q I_C df$$

$$\overline{di_{Bf}^2} = \frac{KF_B I_B}{A_{EB}} \frac{df}{f}$$

$$KF_B \approx 10^{-21} \text{ Acm}^2$$

Ref. Van der Ziel (Prentice Hall 1954)

Bipolar trans.: equivalent input noise



$$\overline{dv_{ieq}^2} = 4kT (R_{eff}) df$$

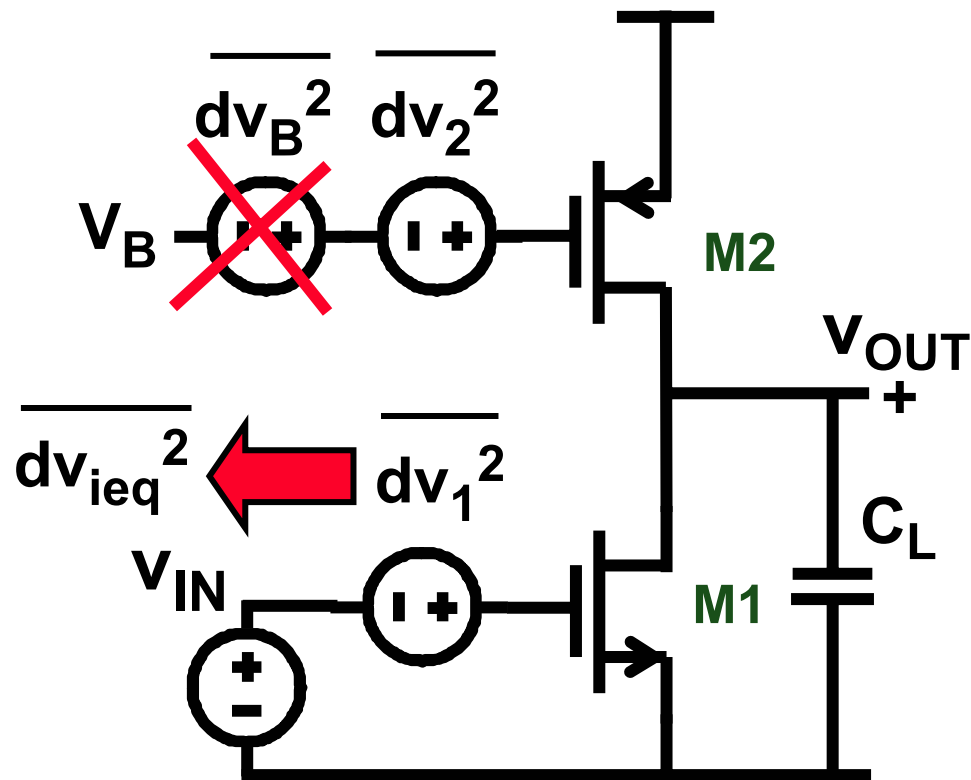
$$R_{eff} = \frac{1/2}{g_m} + r_B + r_E$$

$$\overline{di_{ieq}^2} = \overline{di_B^2} = 2q I_B df$$

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Noise of an amplifier with active load



If $\overline{dv_B^2}$ is negligible :

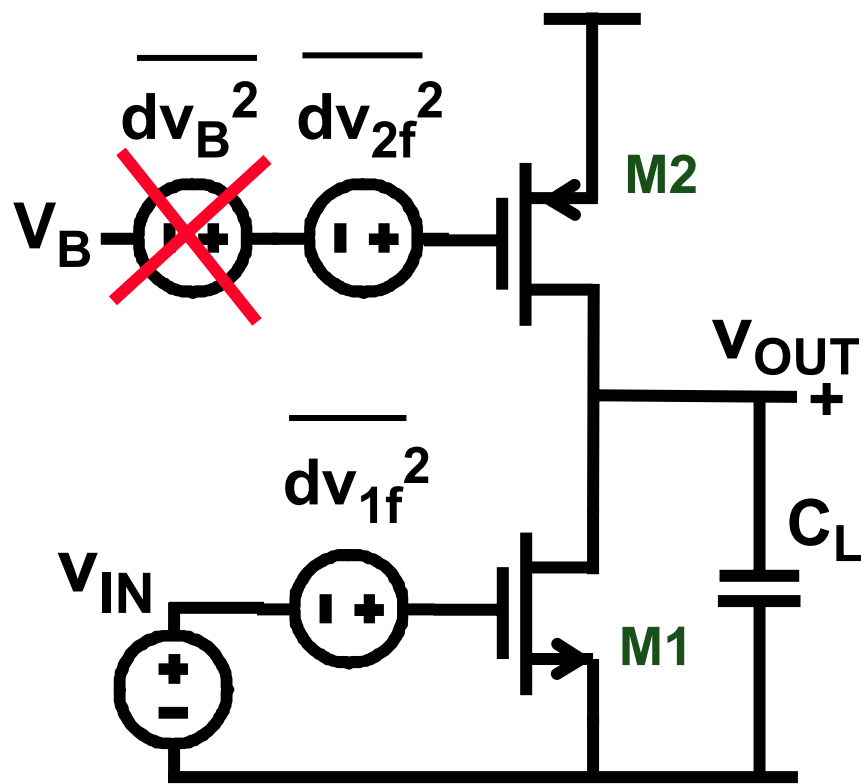
$$\overline{di_{out}^2} = g_{m1}^2 \overline{dv_1^2} + g_{m2}^2 \overline{dv_2^2}$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \left(\frac{g_{m2}}{g_{m1}} \right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} \left(1 + \frac{g_{m2}}{g_{m1}} \right)$$

Small g_{m2} : small $(W/L)_2$ or large $(V_{GS} - V_T)_2$

1/f Noise of amplifier with active load



If $\overline{dv_B^2}$ is negligible :

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} + \overline{dv_{2f}^2} \left(\frac{g_{m2}}{g_{m1}} \right)^2$$

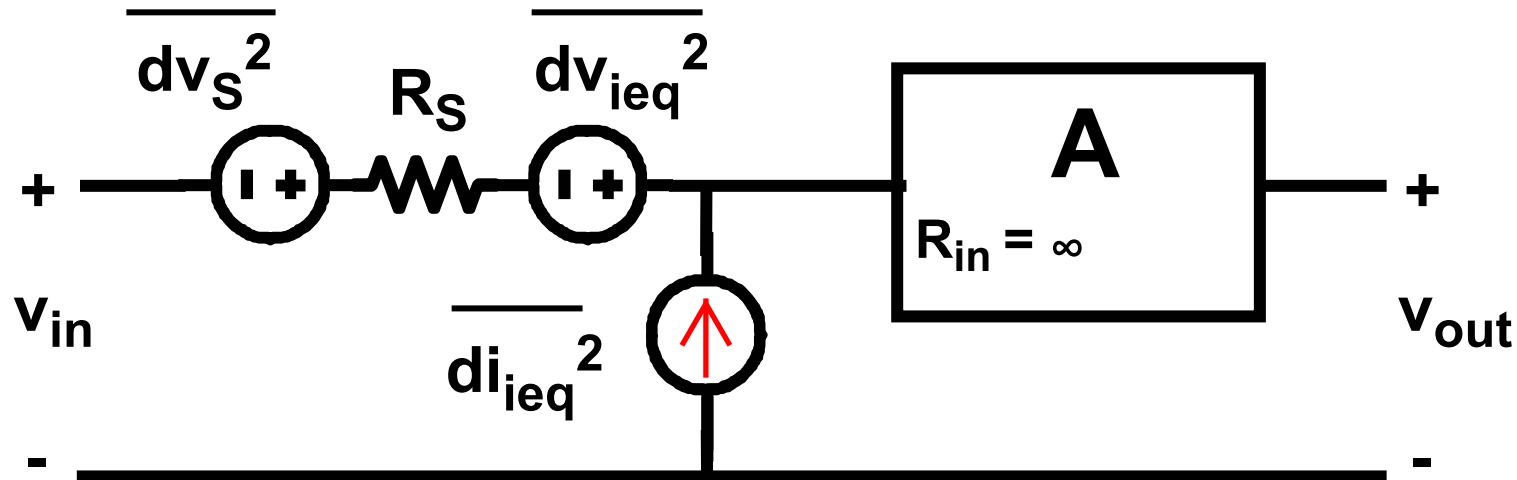
$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} \left[1 + \left(\frac{g_{m2}}{g_{m1}} \right)^2 \left(\frac{\overline{dv_{2f}}}{\overline{dv_{1f}}} \right)^2 \right]$$

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} \left[1 + \frac{KF_2}{KF_1} \frac{K'_2}{K'_1} \left(\frac{L_1}{L_2} \right)^2 \right]$$

$\overline{dv_{if}^2}$ has minimum at

$$L_{1opt} = L_2 \sqrt{\frac{KF_1}{KF_2} \frac{K'_1}{K'_2}} \approx 10 L_2 \quad \text{then} \quad \overline{dv_{if}^2} = 2 \overline{dv_{1f}^2}$$

Noise figure of an amplifier



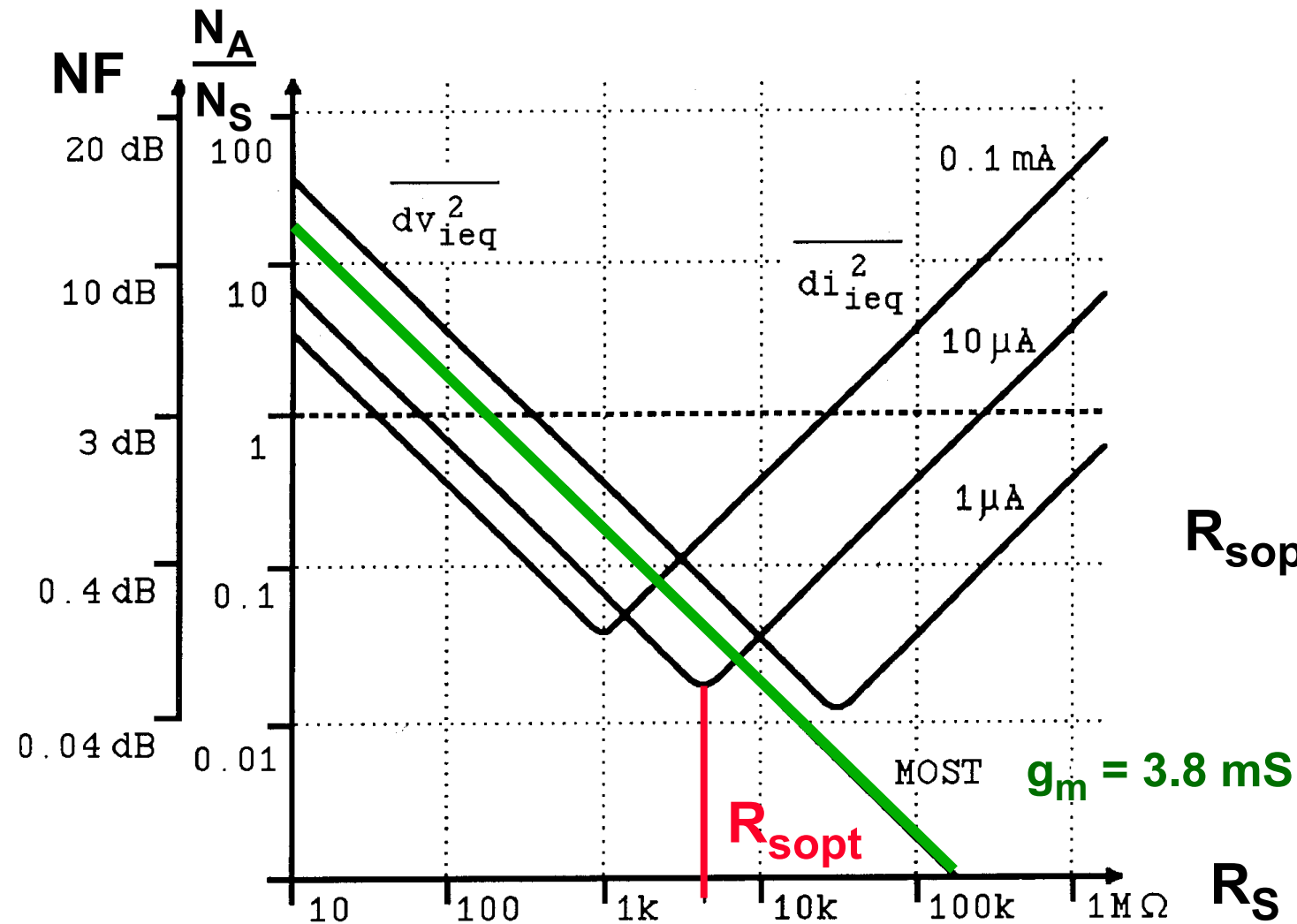
$$NF = \frac{N_S + N_A}{N_S} = 1 + \frac{N_A}{N_S}$$

$$NF = 1 + \frac{\overline{dv_{ieq}^2} + R_S^2 \overline{di_{ieq}^2}}{4kT R_S df}$$

Voltage drive $NF \sim \frac{1}{R_S}$

Current drive $NF \sim R_S$

Resistive noise matching



$$\beta = 100$$

$$r_B = 100 \Omega$$

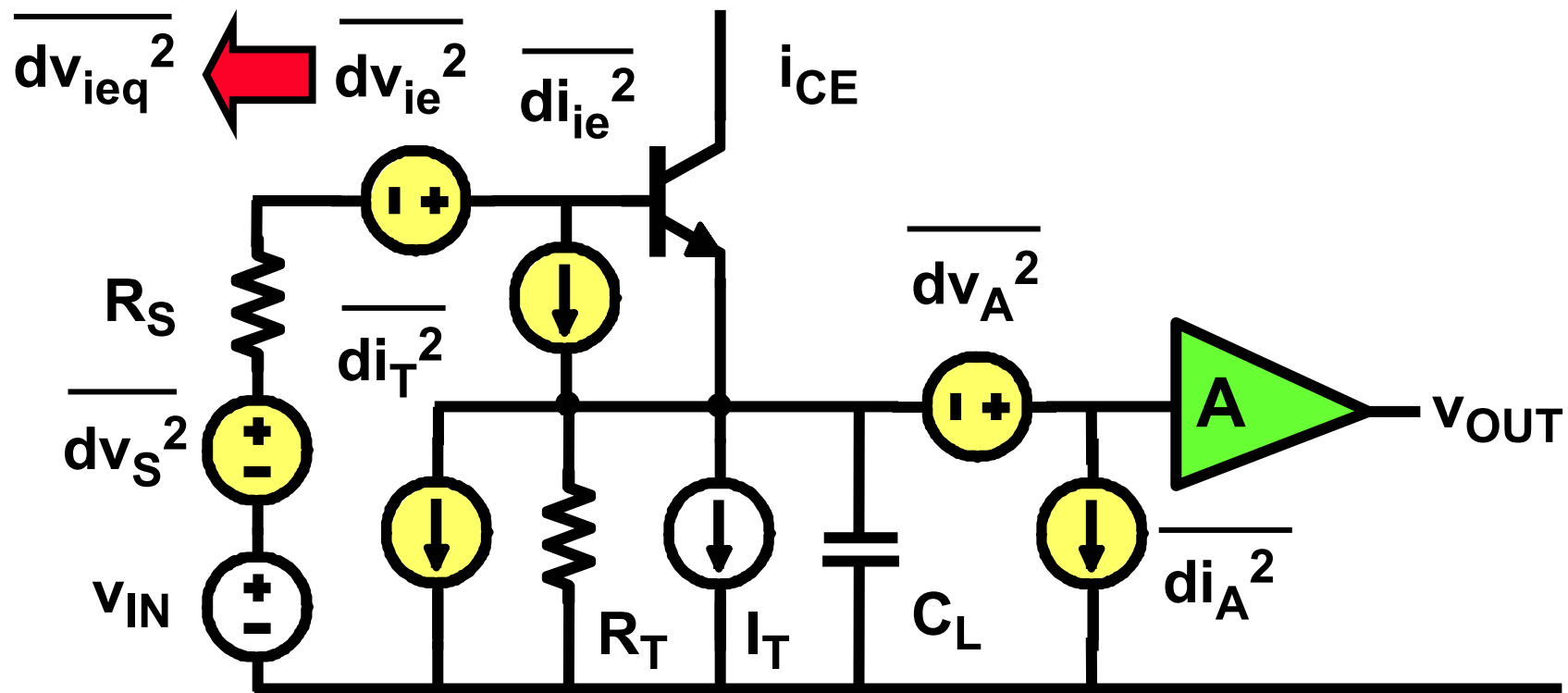
$$R_{sopt} = \sqrt{\frac{\overline{dv_{ieq}^2}}{\overline{di_{ieq}^2}}}$$

$$g_m = 3.8 \text{ mS}$$

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Noise of an emitter follower

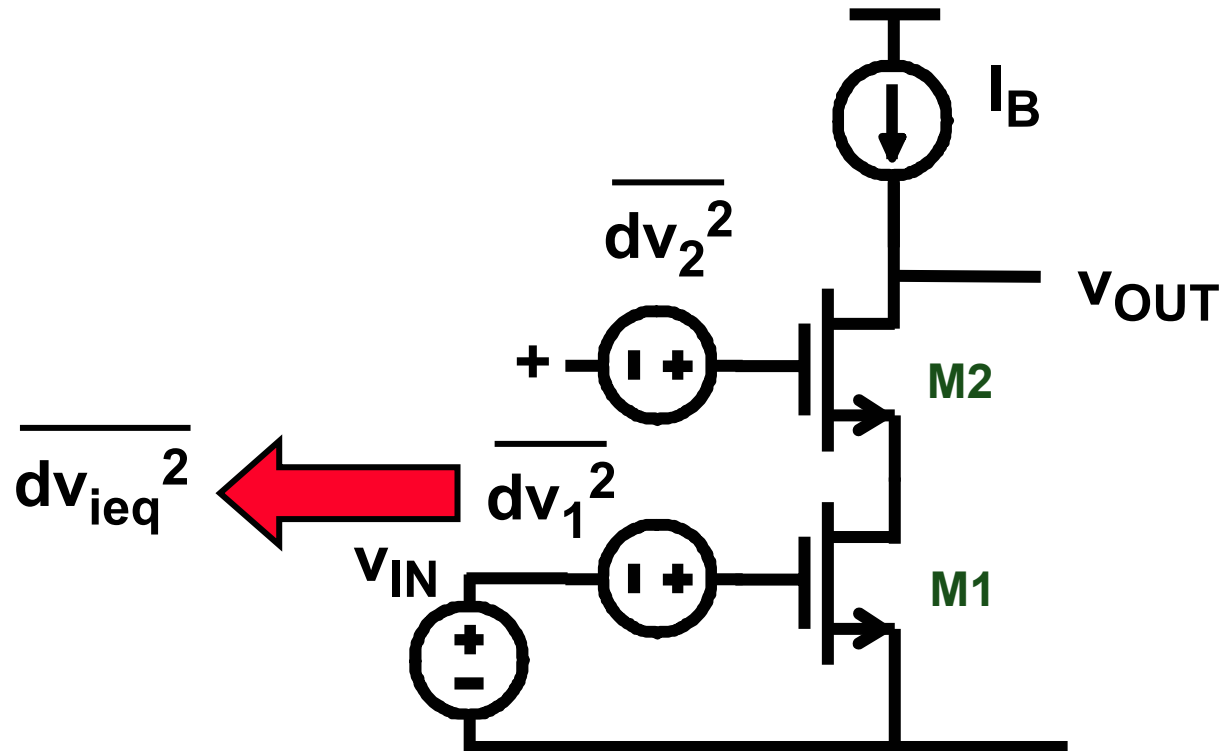


$$\overline{dv_{ieq}^2} = \overline{dv_{ie}^2} + \overline{dv_A^2} + \left(R_S - \frac{1}{g_m}\right)^2 \overline{di_{ie}^2} + \frac{\overline{di_T^2} + \overline{di_A^2}}{g_m^2}$$

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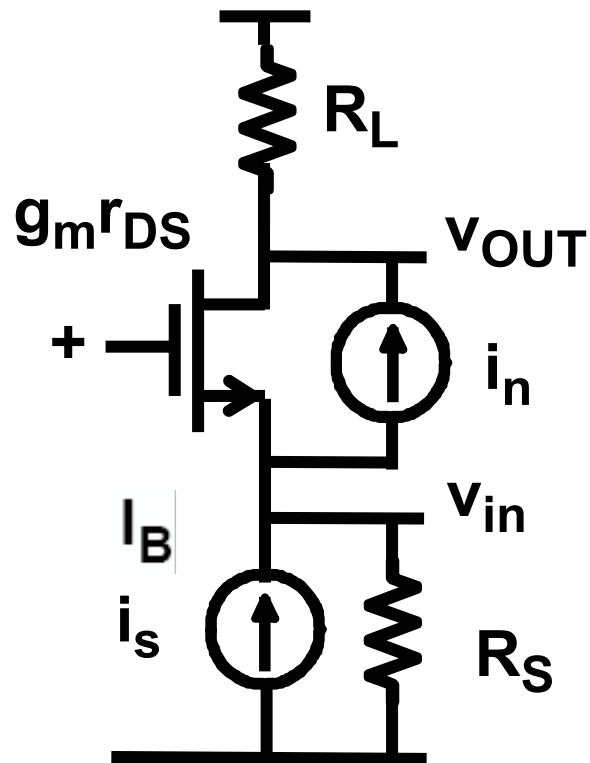
- ♦ **Definitions of noise**
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Noise of a cascode amplifier

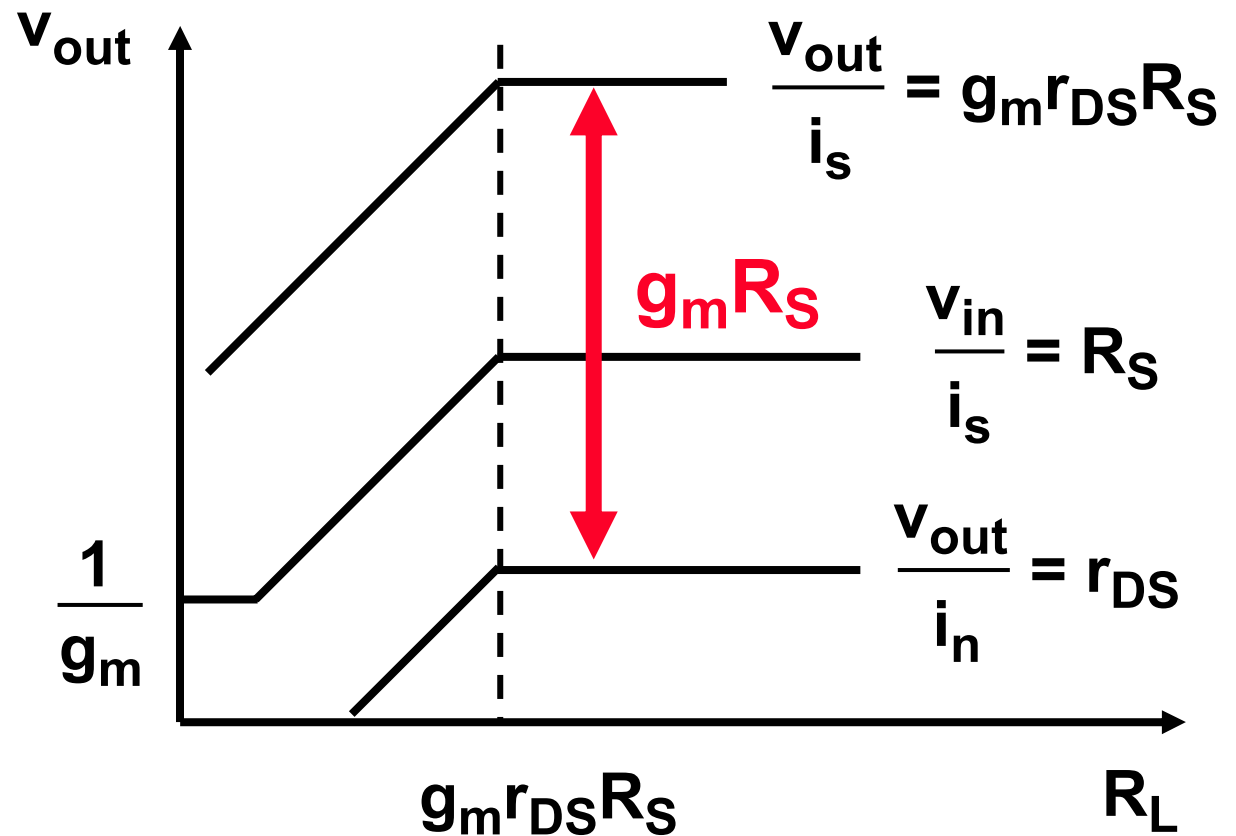


$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \frac{1}{(g_{m1} r_{o1})^2} \approx \overline{dv_1^2}$$

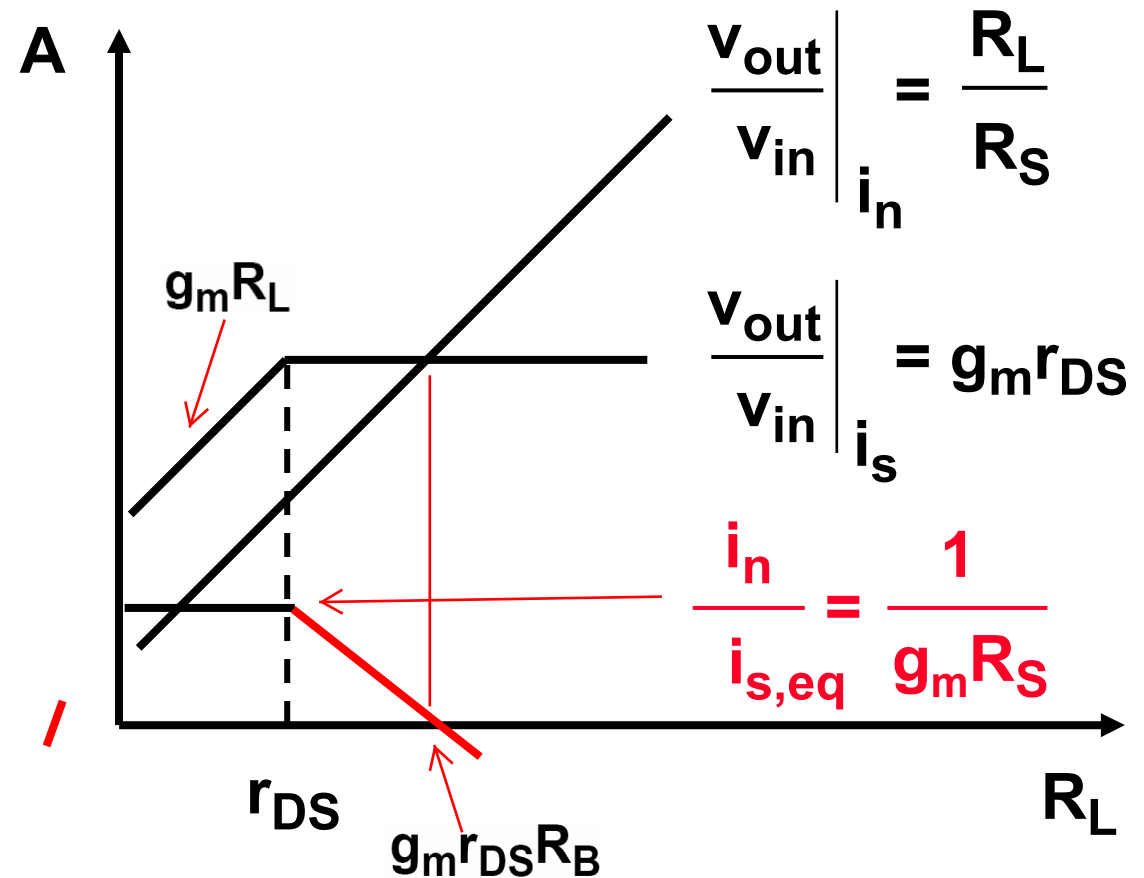
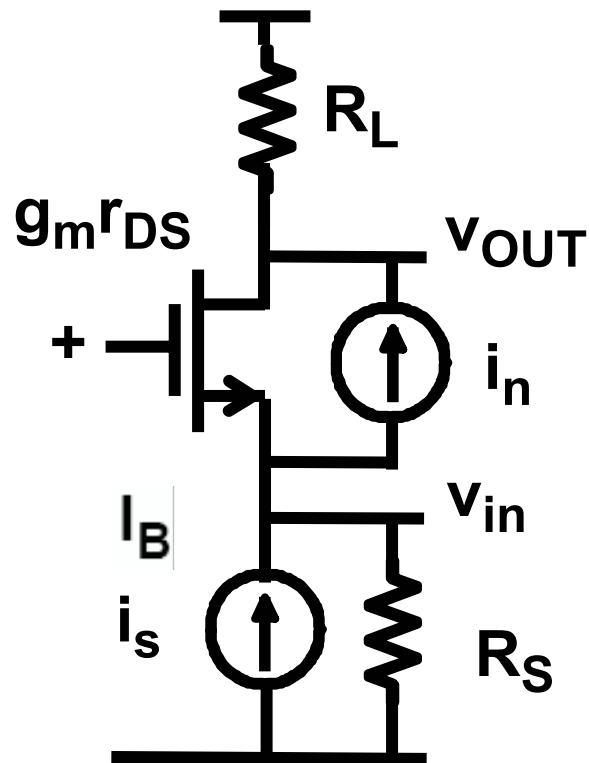
Input referred noise of a cascode



$$g_m r_{DS} \gg 1$$

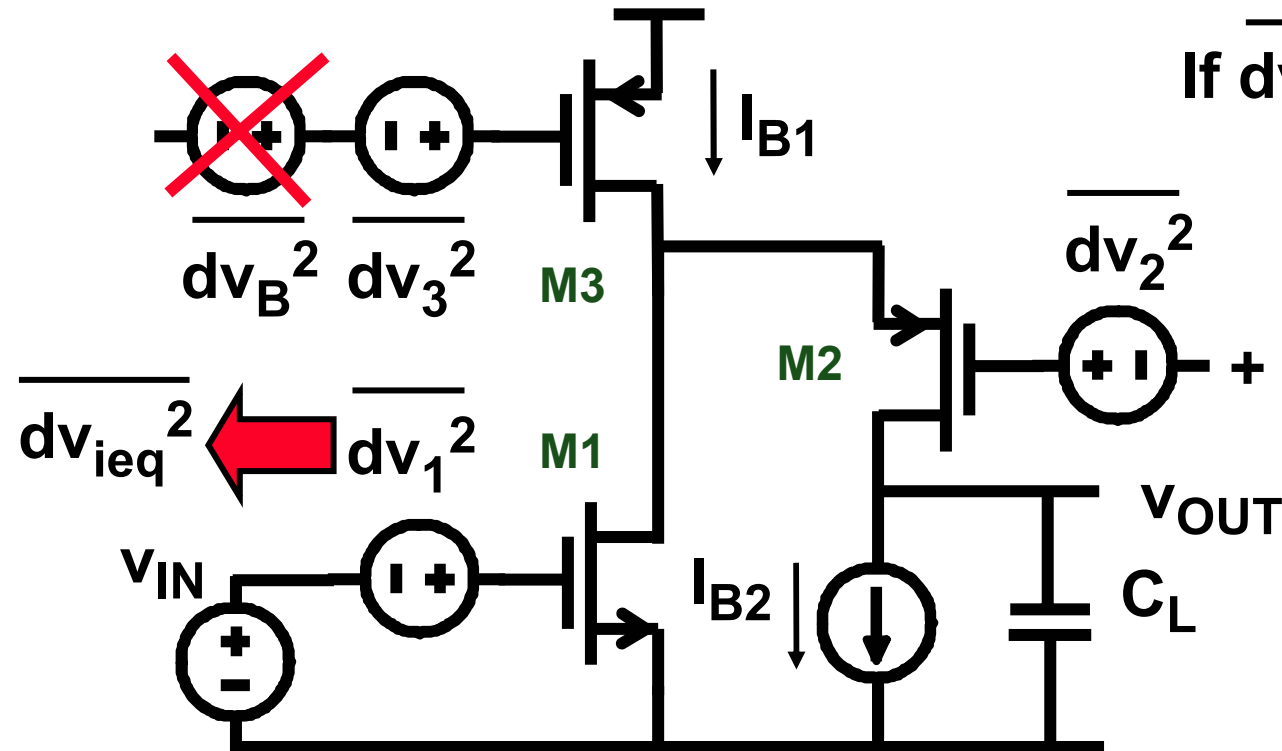


Noise gains in a cascode



Cascode noise i_n is only negligible **if R_S is large !!!**

Noise of a folded cascode

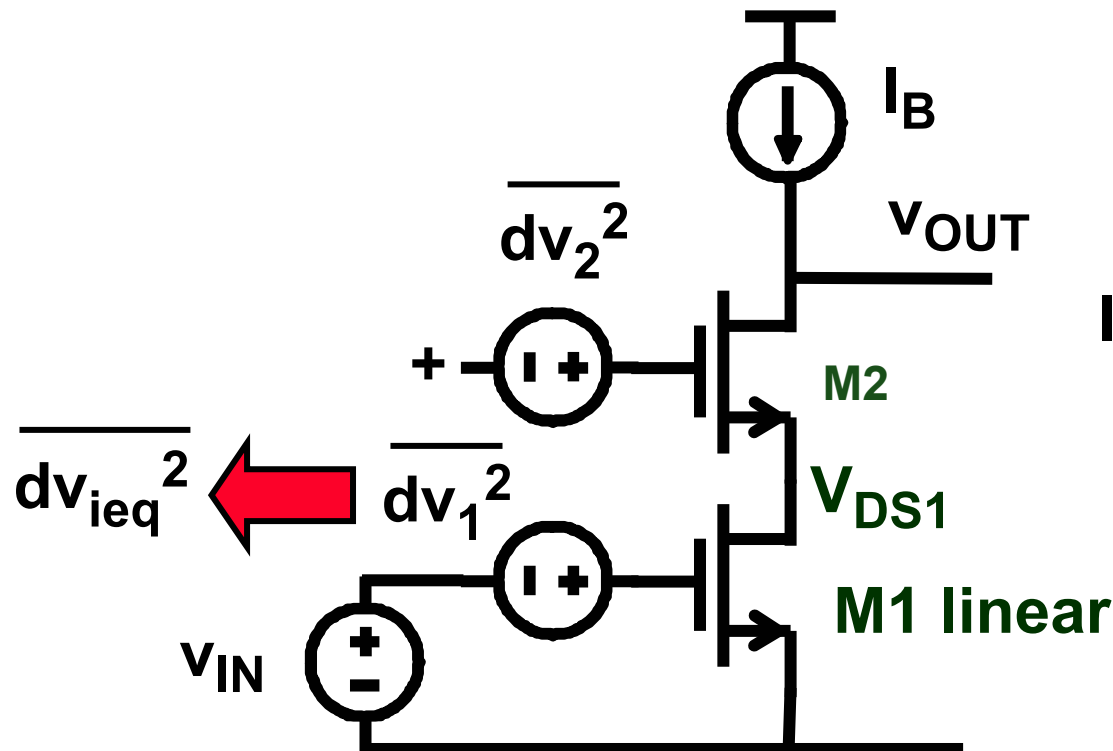


If $\overline{dv_B^2}$ is negligible :

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \frac{1}{(g_{m1} r_{o1} // r_{o3})^2} + \overline{dv_3^2} \frac{(g_{m3})^2}{(g_{m1})^2}$$

Small g_{m3} :
 $(W/L)_3 \downarrow$
 $(V_{GS}-V_T)_3 \uparrow$

Noise of a cascode with linear M1



$$\alpha_1 = \frac{V_{DS1}}{V_{GS1} - V_T} \quad \alpha_1 < 0.5$$

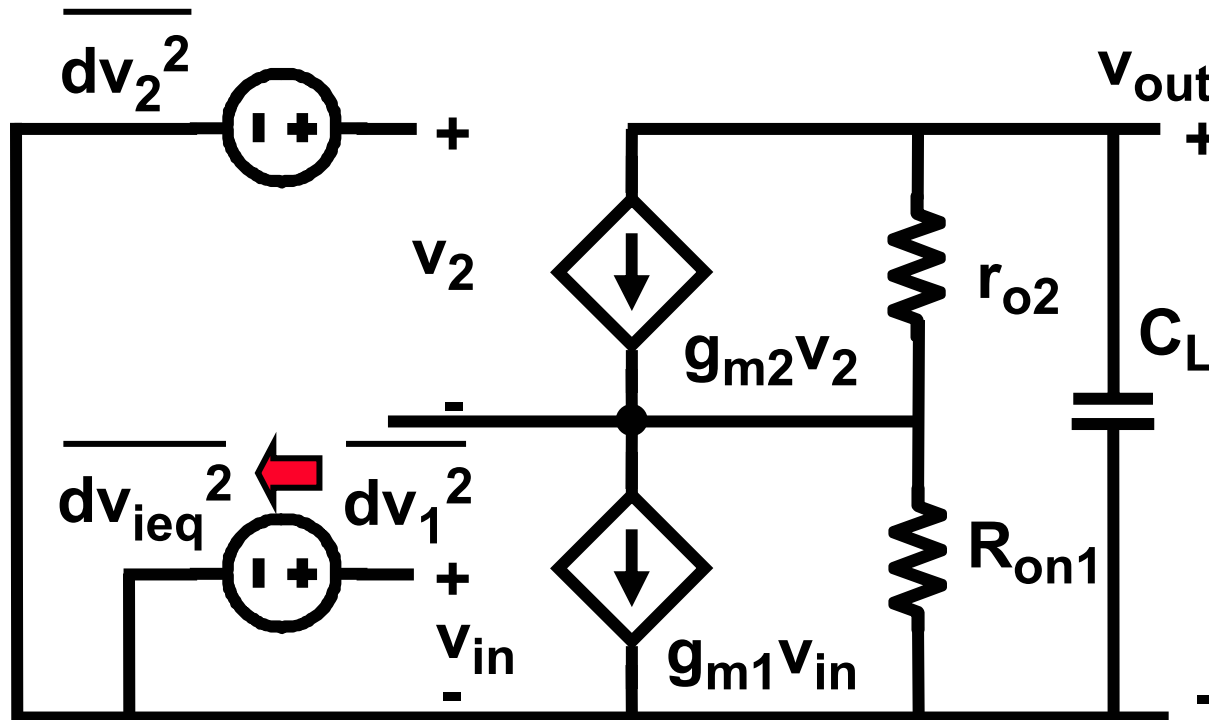
$$I_{DS1} = \beta_1 V_{DS1} (V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$A_v = \alpha_1 g_{m2} r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2} \left(R_{on1} + \frac{2/3}{g_{m2}} \right) df$$

Small-signal model of a cascode with linear M1



$$I_{DS1} = \beta_1 V_{DS1} (V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$\overline{dv_1^2} = \frac{4kT R_{on1}}{\alpha_1^2}$$

$$\overline{dv_2^2} = \frac{4kT \cdot 2/3}{g_{m2}}$$

$$v_{out} / dv_1 = \alpha_1 g_{m2} r_{o2}$$

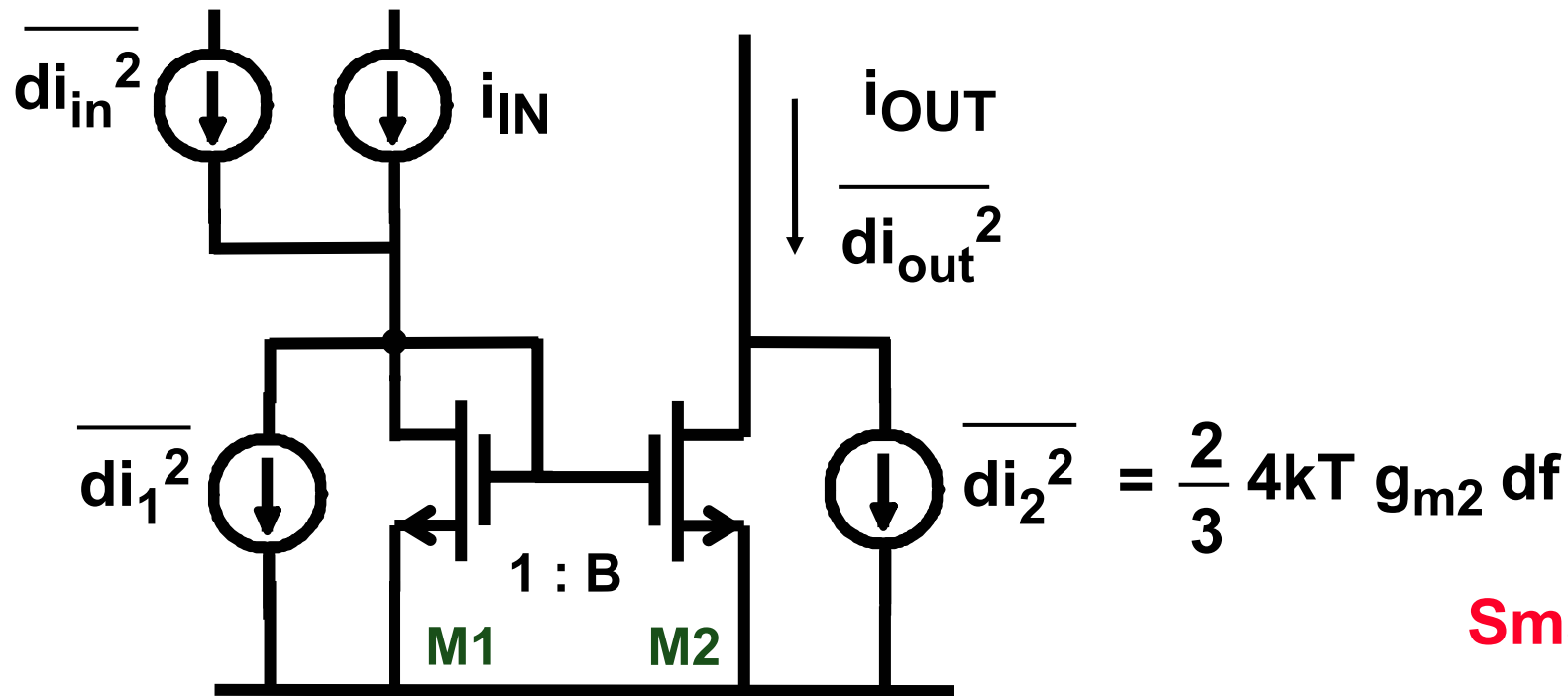
$$v_{out} / dv_2 = g_{m2} r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2} \left(R_{on1} + \frac{2/3}{g_{m2}} \right) df$$

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Noise of a current mirror



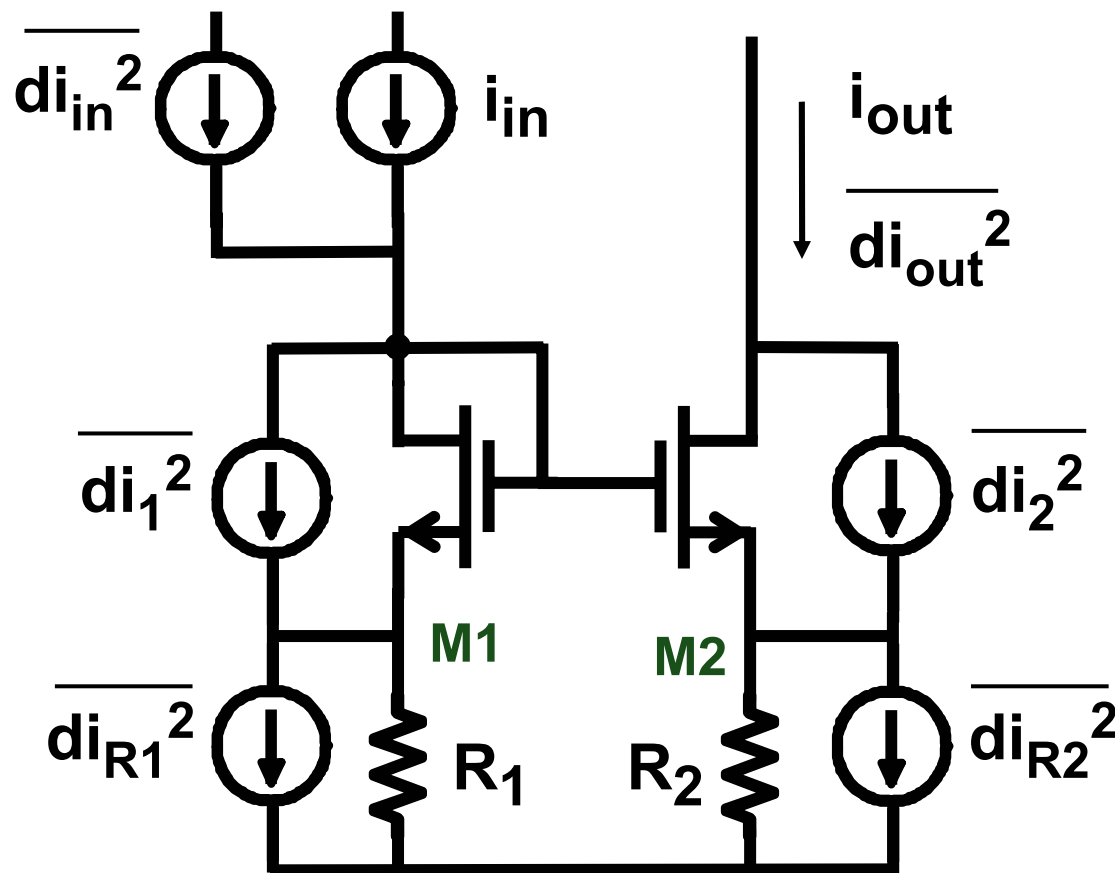
$$\overline{di_2^2} = \frac{2}{3} 4kT g_{m2} df$$

Small g_m :

$(W/L) \downarrow$

$(V_{GS} - V_T) \uparrow$

Noise of a current mirror with series R



$$\overline{di_{out}^2} : \boxed{\overline{di_2^2}} \boxed{\overline{di_{R2}^2}}$$

$$\frac{(R_1)^2}{(R_2)^2} (\boxed{\overline{di_1^2}} \boxed{\overline{di_{R1}^2}})$$

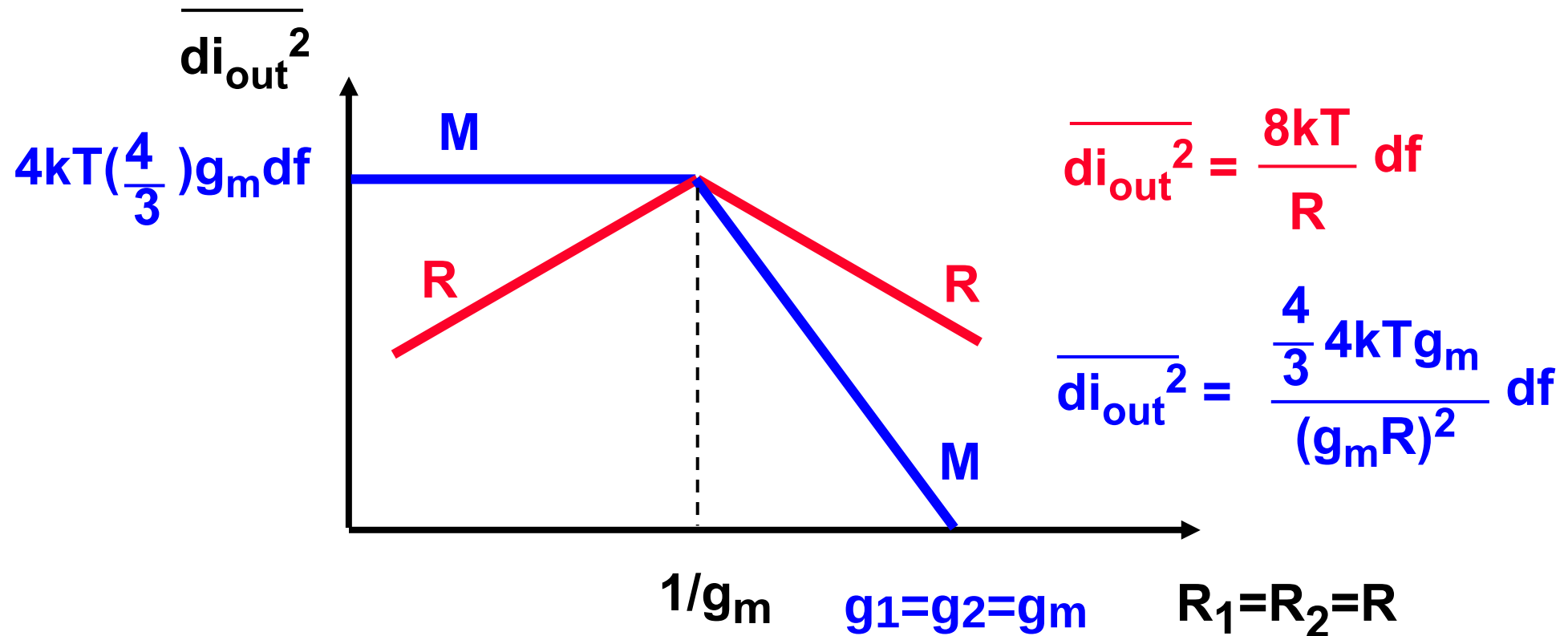
Small g_m :

$(W/L) \downarrow$

$(V_{GS} - V_T) \uparrow$

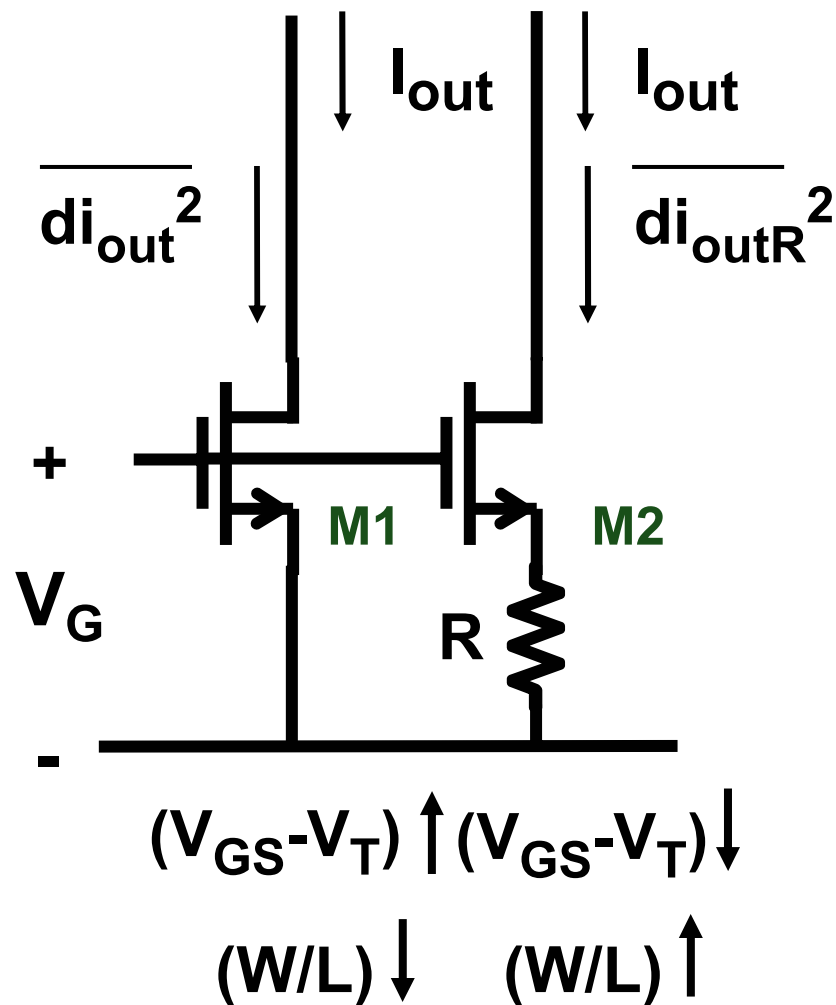
$R \uparrow$

Noise of a current mirror with series R



Bilotti, JSSC Dec 75, 516-524

Current mirror with series R



Same I_{out} & same V_G :

$$\overline{di_{outR}^2} = \overline{di_{out}^2}$$

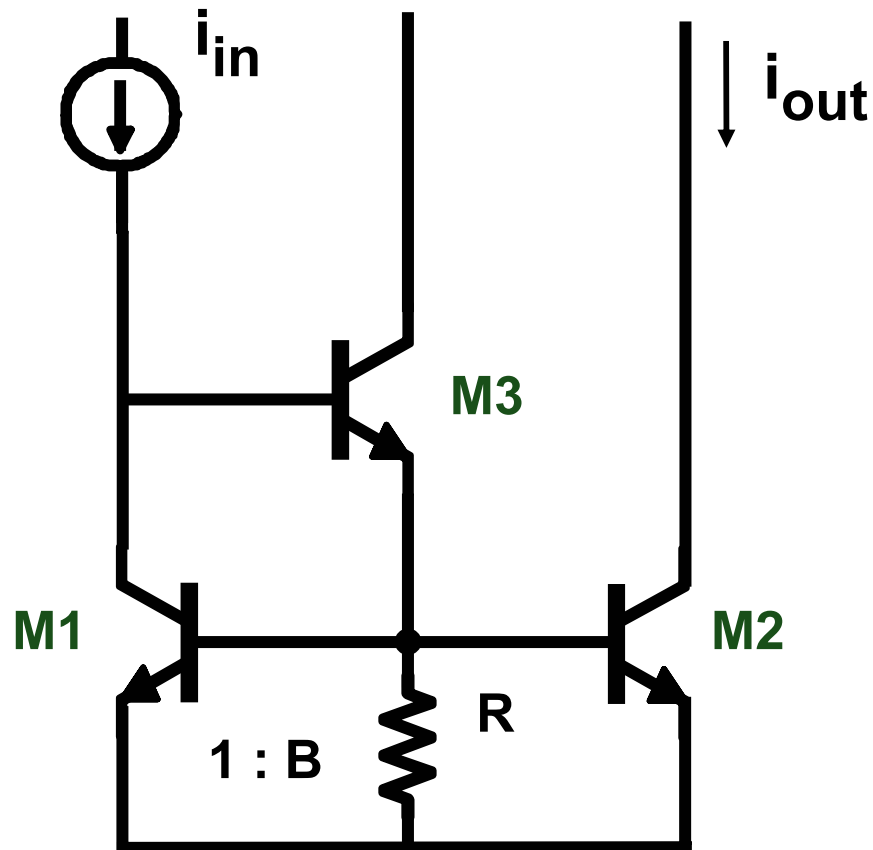
Small g_m :

$(W/L) \downarrow$

$(V_{GS} - V_T) \uparrow$

$V_G \uparrow$

Noise in bipolar current mirror



Noise added by M3 :

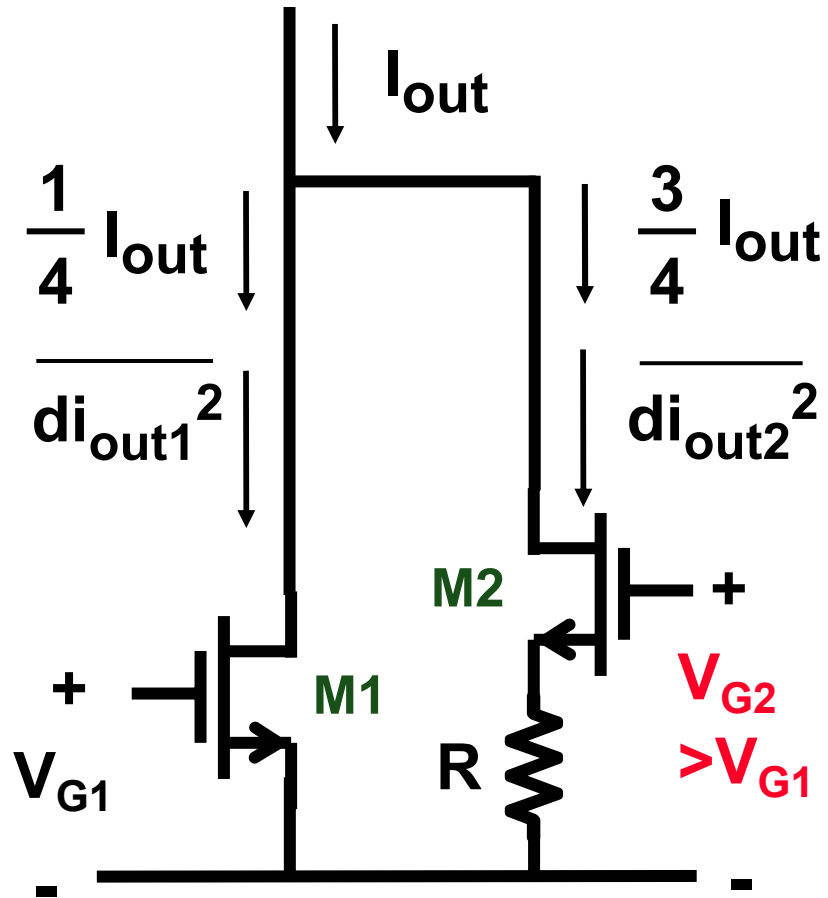
$$\overline{di_{outM3}^2} = 2qI_{C3} df$$

Noise added by R :

$$\overline{di_{outR}^2} = 4kT/R df$$

Both are divided by β_3^2
to be added to the output
and are thus negligible !

Low-noise current mirror with series R



Large R or V_{G2}

Same I_{out} & different V_G :

1 MOST: $\overline{di_{out}^2} = \frac{8kT}{3} \frac{2I_{out}}{V_{GS}-V_T} df$

2 MOSTs: $V_{G2} > V_{G1}$

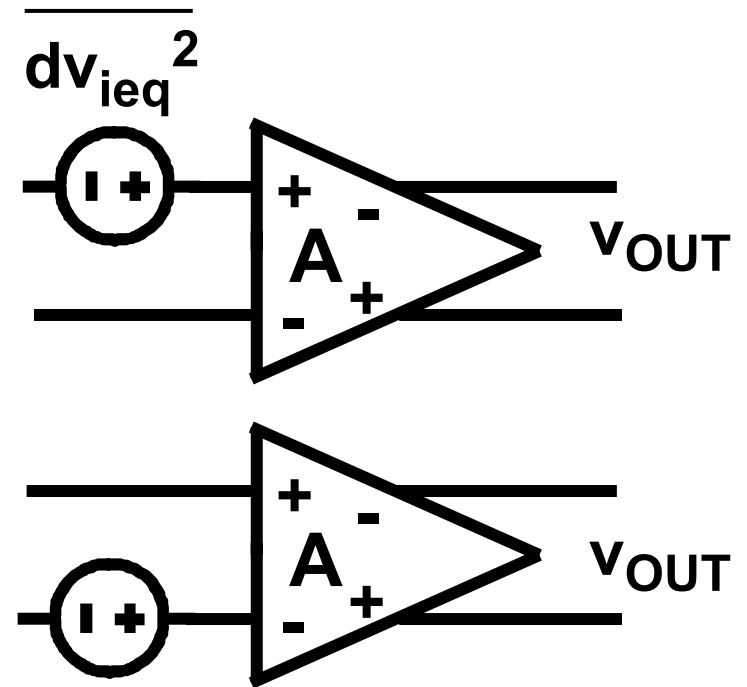
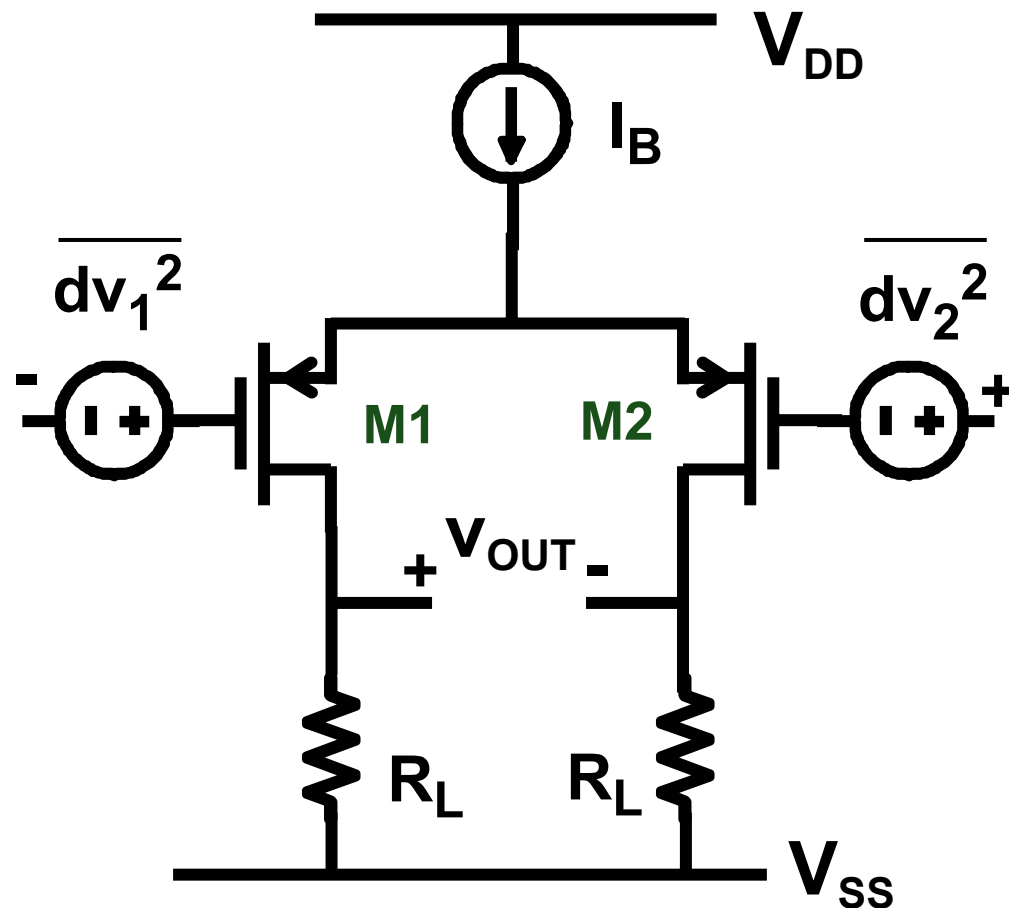
$$\overline{di_{out}^2} = \overline{di_{out1}^2} + \overline{di_{out2}^2} =$$

$$\frac{8kT}{3} \frac{2I_{out}}{V_{G1}-V_T} \left(\frac{1}{4} + \frac{9}{16} \frac{V_{G1}-V_T}{V_{G2}-V_{G1}} \right)$$

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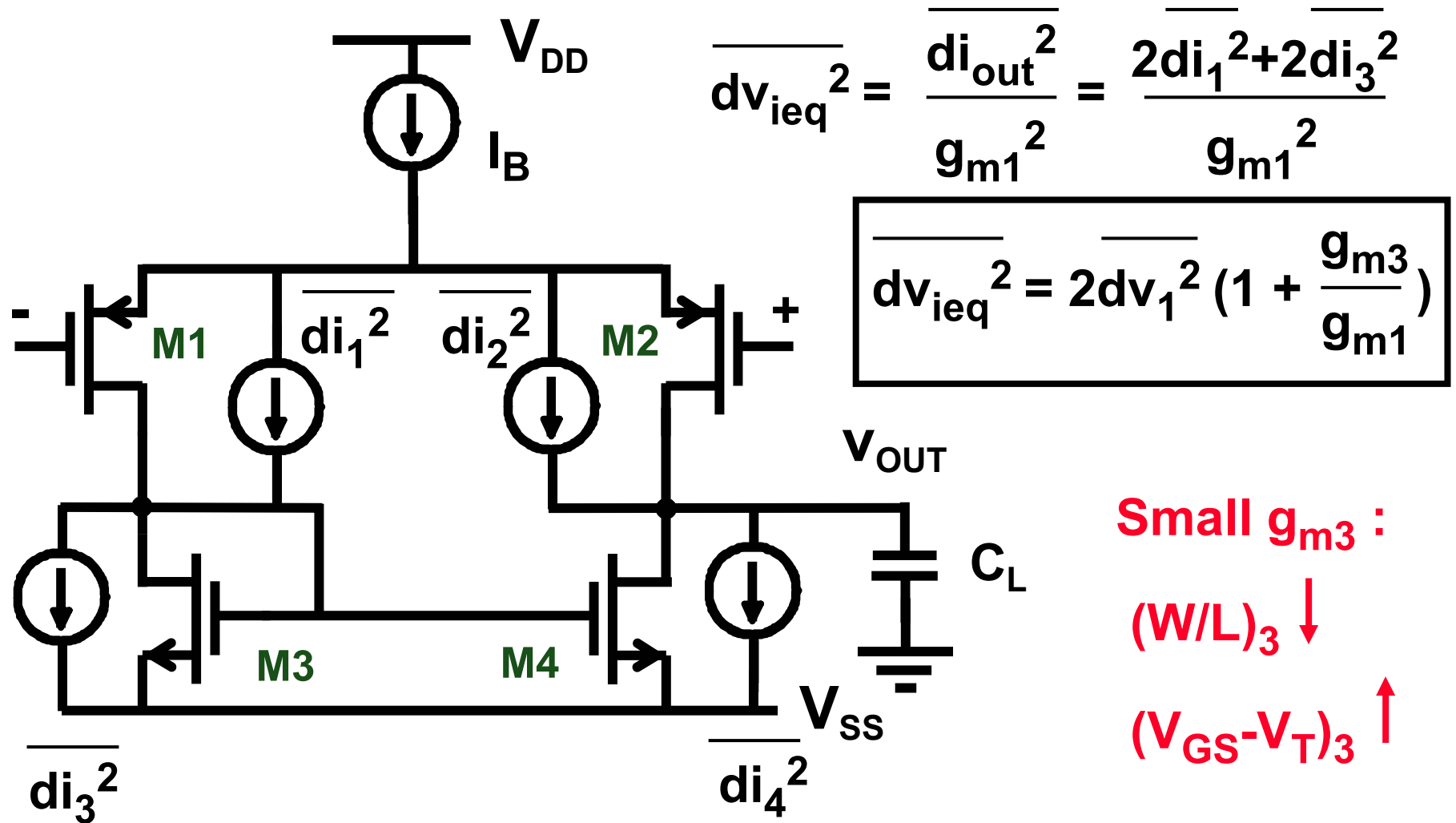
- ◆ Definitions of noise
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- ◆ Noise of a cascode
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Noise of differential pair

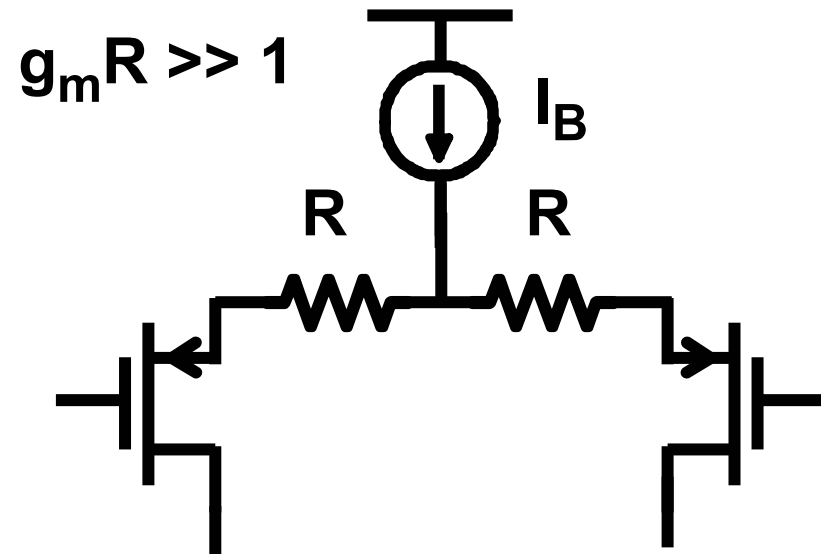


$$\overline{dv_{ieq}^2} = 2 \overline{dv_1^2}$$

Noise of differential pair with active load



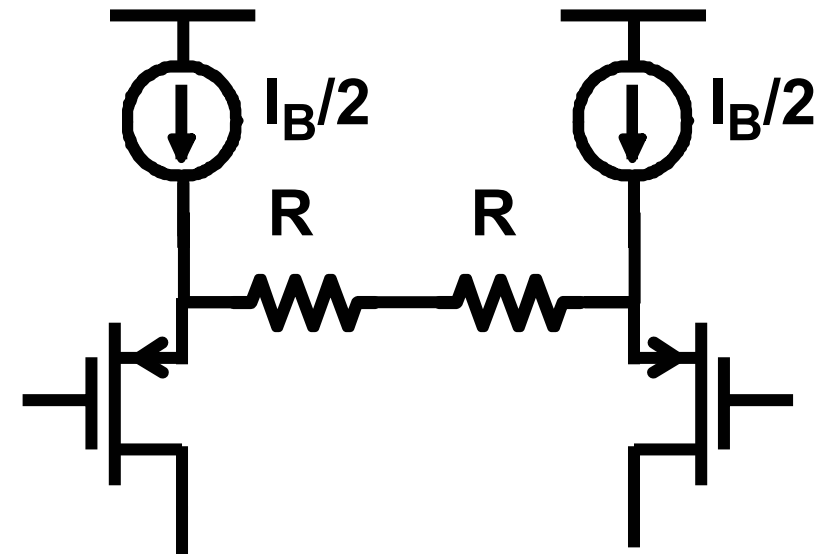
Differential pair with source resistors



$$\overline{di_{out}^2} = 2 \frac{4kT}{R} df$$

$$\overline{dv_{in}^2} = 2 (4kT R df)$$

$$\overline{di_B^2} \text{ is negligible}$$

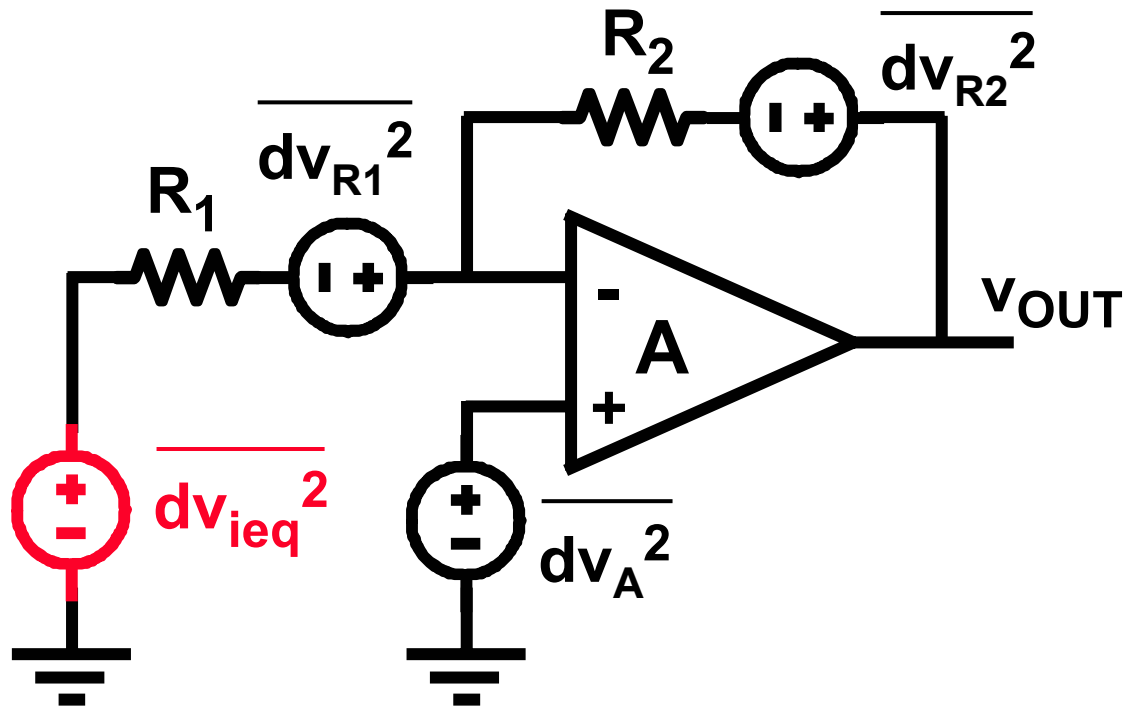


$$\overline{di_{out}^2} = 2 \left(\frac{4kT}{R} df + \overline{di_B^2} \right)$$

$$\overline{di_B^2} = 4kT \frac{2}{3} g_{mB} df$$

$$\overline{dv_{in}^2} = 2 (4kT R df) (1 + \frac{2}{3} g_{mB} R)$$

Noise of an opamp



$$dv_{ieq}^2 = \Sigma dv_{out}^2 \left(\frac{R_1}{R_2} \right)^2$$

$$dv_{out}^2 = dv_{R1}^2 \left(\frac{R_2}{R_1} \right)^2$$

$$dv_{out}^2 = dv_{R2}^2$$

$$dv_{out}^2 = dv_A^2 \left(1 + \frac{R_2}{R_1} \right)^2$$

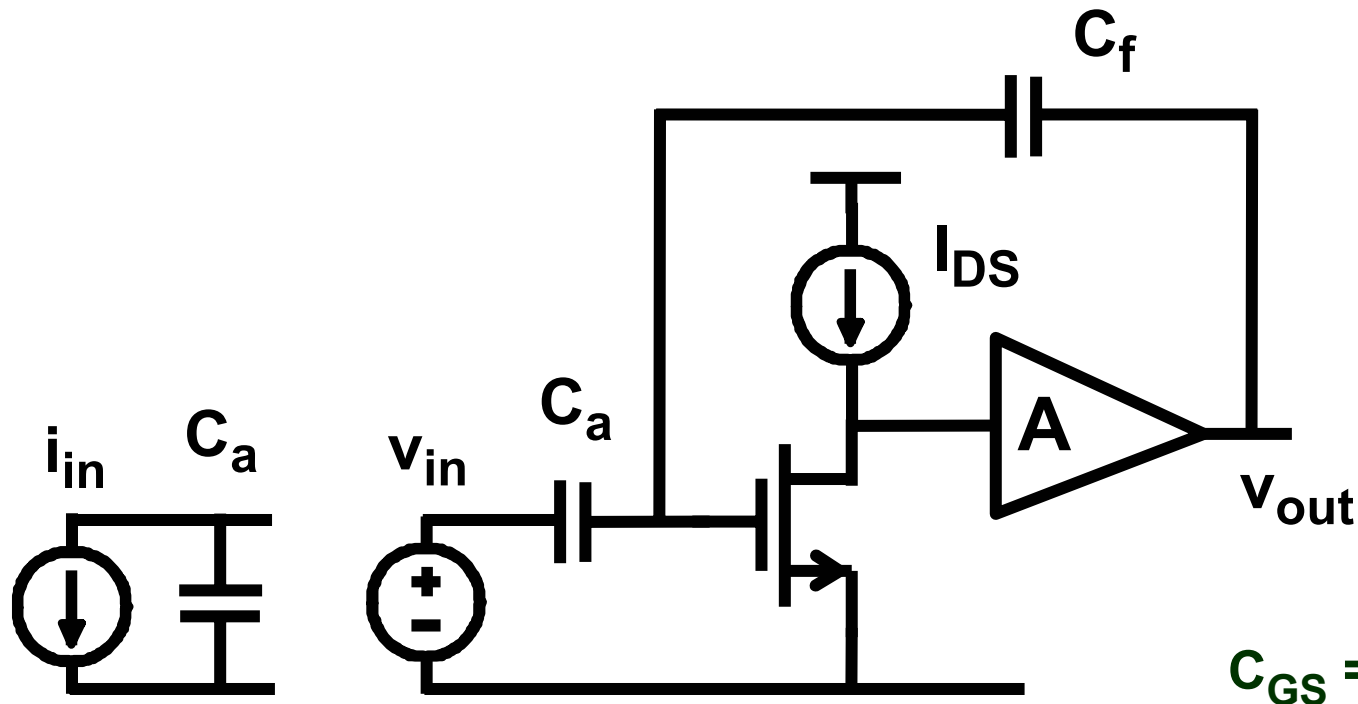
$$dv_{ieq}^2 = dv_{R1}^2 + dv_{R2}^2 \left(\frac{R_1}{R_2} \right)^2 + dv_A^2 \left(1 + \frac{R_1}{R_2} \right)^2 \approx dv_{R1}^2 + dv_A^2$$

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Ref.: Z.Y.Chang, W.Sansen, Low-noise wide-band amplifiers, Kluwer AP, 1991

Capacitive-source amplifier



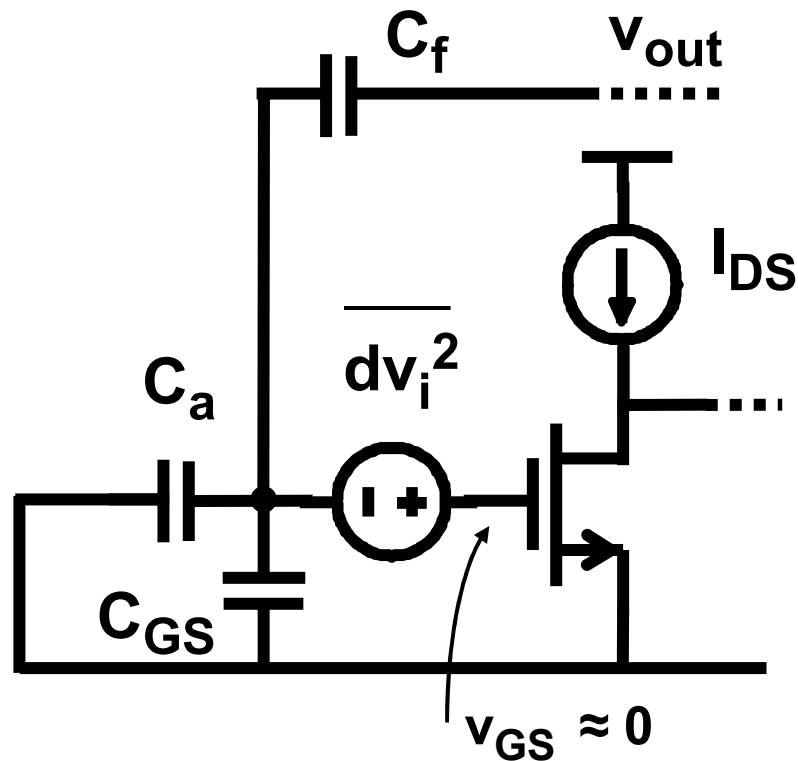
$$A_v = \frac{C_a}{C_f}$$

$$C_a = 5 \text{ pF}$$
$$C_f = 1 \text{ pF}$$

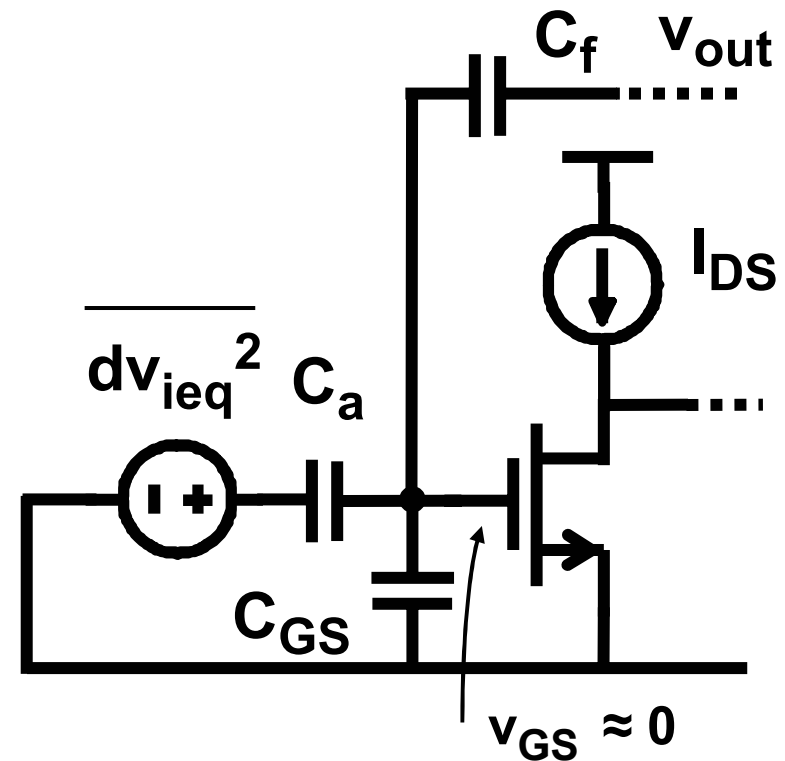
$$C_{GS} = kW \quad k \approx 2 \text{ fF}/\mu\text{m}$$

W_{opt} ? I_{DSopt} ? S/N_{opt} for $V_{in} = 10 \text{ mV}_{RMS}$?

Capacitive noise matching - 1



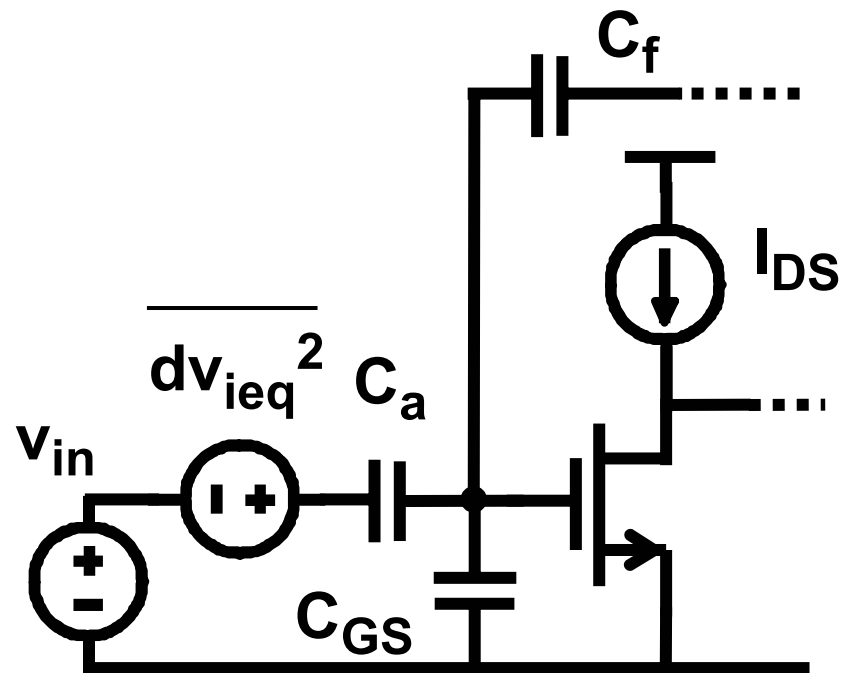
$$\frac{v_{out}}{v_i} = \frac{C_f + C_a + C_{GS}}{C_f}$$



$$\frac{v_{out}}{v_{ieq}} = \frac{C_a}{C_f}$$

**No Miller
with C_{DG} !!!**

Capacitive noise matching - 2



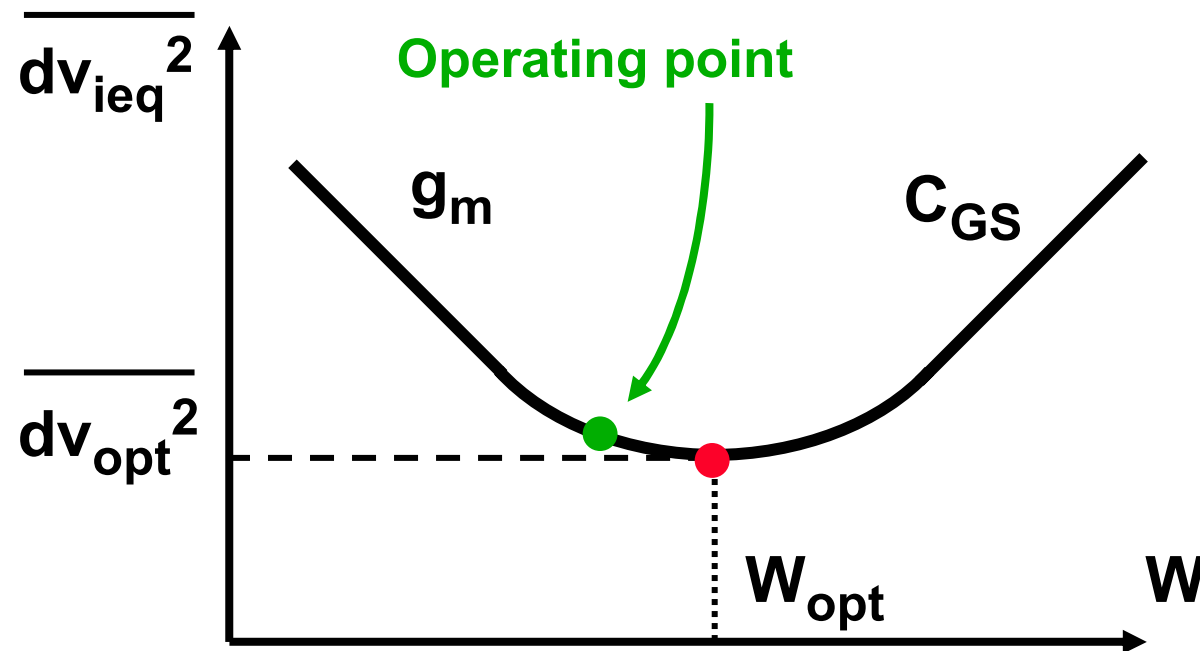
$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + C_{GS})^2}{C_a^2} \overline{dv_i^2}$$

$$\overline{dv_i^2} = \frac{8kT}{3} \frac{1}{g_m} df$$

$$g_m = 2 K'_n \frac{W}{L} (V_{GS} - V_T)$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + k\mathbf{W})^2}{C_a^2} \frac{\mathbf{L}}{\mathbf{W}} \frac{8kT}{3} \frac{1}{2 K'_n (V_{GS} - V_T)}$$

Capacitive noise matching - 3



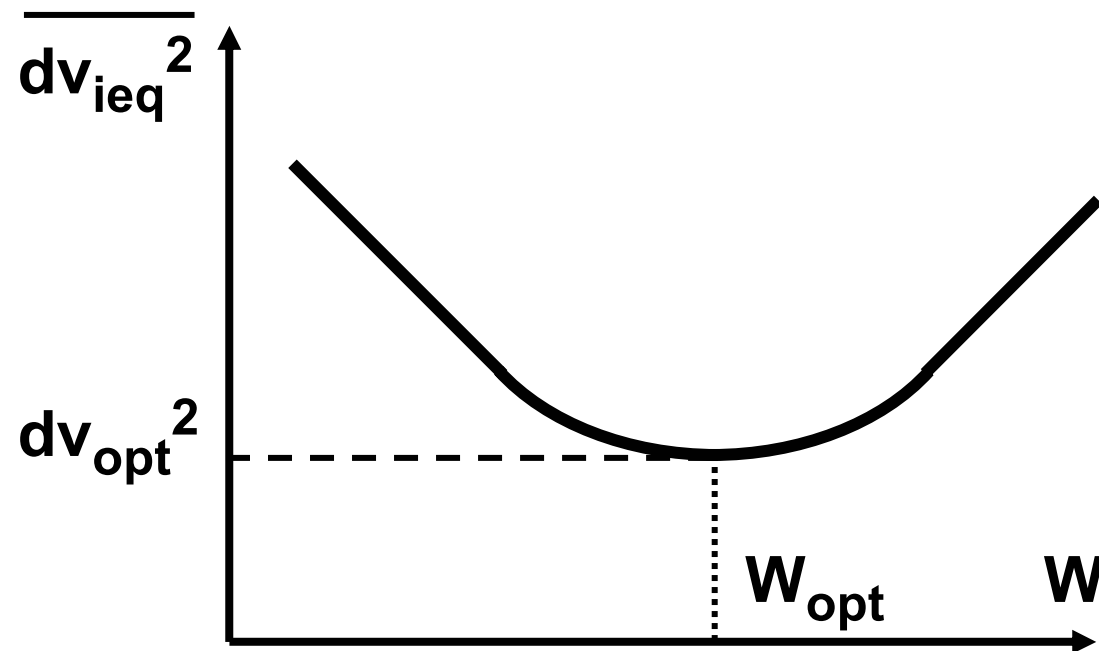
Noise matching
where

$$C_{GS} = C_f + C_a$$

$$W_{opt} = \frac{C_f + C_a}{k}$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2} \frac{L}{W} \frac{8kT}{3} \frac{1}{2 K'_n (V_{GS} - V_T)}$$

Capacitive noise matching - 4



$$W_{opt} = \frac{C_f + C_a}{k}$$

$$C_{GSopt}$$

$$I_{DSopt}, g_{mopt}$$

$$\overline{dv_{opt}^2} = 4 \frac{8kT}{3} \frac{df}{g_{mopt}}$$

$$BW_n = \frac{\pi}{2} BW = \frac{\pi}{2} \frac{f_T}{A_v} = \frac{1}{4A_v} \frac{g_{mopt}}{C_{GSopt}}$$

$$\frac{S}{N_{opt}} = \frac{10 \text{ mV}_{RMS}}{\sqrt{\overline{dv_{opt}^2} BW_n}}$$

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