1.0 Supplemental Notes on Planar Spiral Inductances

1.1 Better analytical formulas for inductance

By returning to first principles (indeed, by following a prescription outlined by Maxwell himself), Dr. S. S. Mohan has recently developed remarkably simple, but highly accurate, analytical formulas for the inductance of planar spirals. The following equations, developed as part of Mohan's Stanford Ph.D. thesis work, should replace those given in section of the text.

As noted in the book, planar spirals with a variety of geometries have been used (e.g., circular, octagonal, hexagonal and square). The inductance and Q values attainable are very much second-order functions of shape (despite much lore to the contrary), so engineers should feel free to use their favorite shape with relative impunity. A square spiral is the simplest to lay out, and is therefore the overwhelming favorite of lazy engineers (of which the author is a proud member). Octagonal spirals are definitely better (order of 10%), though, and are therefore often favored when layout tools facilitate it.

The formulas for all of these shapes can be cast in a simple unified form:

$$L = \frac{\mu n^2 d_{avg} c_1}{2}\left[ln\left(\frac{c_2}{\rho}\right) + c_3\rho + c_4\rho^2 \right] , \tag{1}$$

where n is the number of turns, d_{avg} is the average of the inner and outer diameters, and ρ is a *fill factor*, defined as

$$\rho \equiv \frac{d_{out} - d_{in}}{d_{out} + d_{in}} . \tag{2}$$

From this last equation, you can see why the term "fill factor" is appropriate: ρ approaches unity as the inductor windings fill the entire space, and approaches zero as the inductor becomes progressively hollower.

The various coefficients c_n are a function of geometry, and are given in the following table:[1]

TABLE 1. Coefficients for inductance formula

Shape	c_1	c_2	c_3	c_4
Square	1.27	2.07	0.18	0.13
Hexagonal	1.09	2.23	0.00	0.17
Octagonal	1.07	2.29	0.00	0.19
Circle	1.00	2.46	0.00	0.20

To an excellent approximation, the coefficient c_1 is simply the area for a given outer dimension, normalized to the area of the largest circle that can be inscribed within the layout. The factor c_2 is the primary term, while c_3 and c_4 may be considered first- and second-order correction factors, respectively. When all four factors are used, the equations are typically accurate to within a couple of percent (and almost never in error by more than five percent), thus generally obviating the need for a full electromagnetic field solver to evaluate the inductance of such structures.

On those rare occasions where other regular polygons are of interest, one may use the following analytical formula:

$$L \approx \frac{\mu n^2 d_{avg} A_{out}}{\pi d_{out}^2}\left[ln\left(\frac{2.46 - \frac{1.56}{N}}{\rho}\right) + \left(0.20 - \frac{1.12}{N^2}\right)\rho^2 \right] , \qquad (3)$$

where A_{out} is the outer area, and N is the number of sides of the polygon. This formula is simply a restatement of Eqn. 1, with analytical approximations used for the coefficients c_1, c_2 and c_4. The coefficient c_3 is set to zero, which is a good approximation for all regular polygons with more than four sides. This analytical formula is only one or two percent more inaccurate than the tabulated one.

The Q of a planar spiral inductor is often estimated roughly by using a simple skin effect formula to compute an approximation of the effective resistance. This approach can fail because it neglects the influence of one turn's field on the current distribution in adjacent turns. Hence, one should expect the estimate to be a rather crude one, on the optimistic side.

Generally, somewhat hollow inductors have the highest Q because the innermost turns tend not to contribute much magnetic flux, but do contribute significant resistance. Hence, removing them is a good idea in general. While there is no simple rule as to what is optimum in all cases, a reasonable rule of thumb is to have a 3:1 ratio between the outer and inner diameters (corresponding to a fill factor of about 0.5). Fortunately, the optimum conditions are relatively flat, so the rule of thumb is satisfactory for many practical cases.

1.2 Better analytical formulas for loss

Mohan's work also provides improved formulas for the effective resistance in series with the inductor. As mentioned in the text, this loss arises from some combination of skin-effect conductor dissipation and eddy-current substrate loss. The formula given in the text,

1. S. S. Mohan et al., "Simple Accurate Inductance Formulas," *IEEE J. Solid-State Circuits*, Feb. 2000.

$$R_S \approx \frac{l}{w \cdot \sigma \cdot \delta\left(1 - e^{-\frac{t}{\delta}}\right)}, \tag{4}$$

accounts only for skin loss from the surface of the conductor that faces the substrate. As mentioned in the previous section, it neglects skin loss from the other surfaces, and also neglects substrate loss. Neglect of the latter is generally justified in those cases where the substrate is lightly doped. However, many IC processes employ quite heavily doped wafers (e.g., 10mΩ-cm), and eddy current loss in the substrate must be considered in such cases.

Although the analysis is quite complicated, and numerous approximations are cascaded to make the analysis tractable, the resulting formula for the resistance is not too unwieldy (again, see reference [1]):

$$R_{eddy} \approx \frac{\sigma_{sub}}{4e}(\mu n f)^2 d_{avg}^3 \rho^{0.7} z_{n,\,ins}^{-0.55} z_{n,\,sub}^{0.1}, \tag{5}$$

where σ_{sub} is the substrate conductivity, d_{avg} is the average of the inner and outer diameters, ρ remains the fill factor, and e is our old friend (2.7182818...). The quantity $z_{n,\,ins}$ is the total thickness of the insulation between the spiral proper and the heavily-doped portion of the substrate, normalized to the average inductor diameter. That insulation is generally a combination of oxide and lightly-doped semiconductor, and can be treated as a uniform, magnetically transparent material. Similarly, $z_{n,\,sub}$ is the substrate skin depth, also normalized to the average inductor diameter.

The total series resistance is then the sum of R_S and R_{eddy}. The other model elements remain unchanged from those given in the text.

It is worthwhile examining Eqn. 5 to extract some intuition from it. First, note that the resistance due to eddy current loss in the substrate is proportional to the square of frequency, and to the square of the number of turns. Perhaps most important is the proportionality to the *cube* of the (average) diameter.[2] It is a natural tendency to use wide conductors to mitigate conductor loss (see Eqn. 4), but we see that beyond a certain point, eddy current loss dominates, and Q actually degrades rapidly with further increases in size. For heavily doped CMOS substrates, then, one must often use inductors with smaller outer diameters (and hence narrower conductors) than is common practice in technologies with semi-insulating substrates (e.g., GaAs). Failure to recognize the existence of this tradeoff has led to a great spread in reported results.

2. For a fixed inductance, one may use Eqn. 4 to deduce that the eddy resistance then grows approximately as the square of the average diameter.