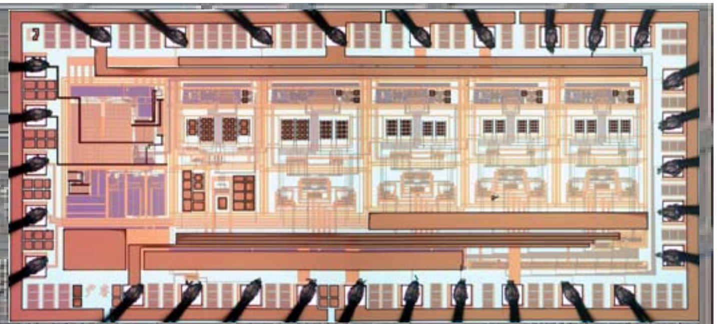

高级模拟集成电路设计
(Analog Design Essentials)

2015年1月8日

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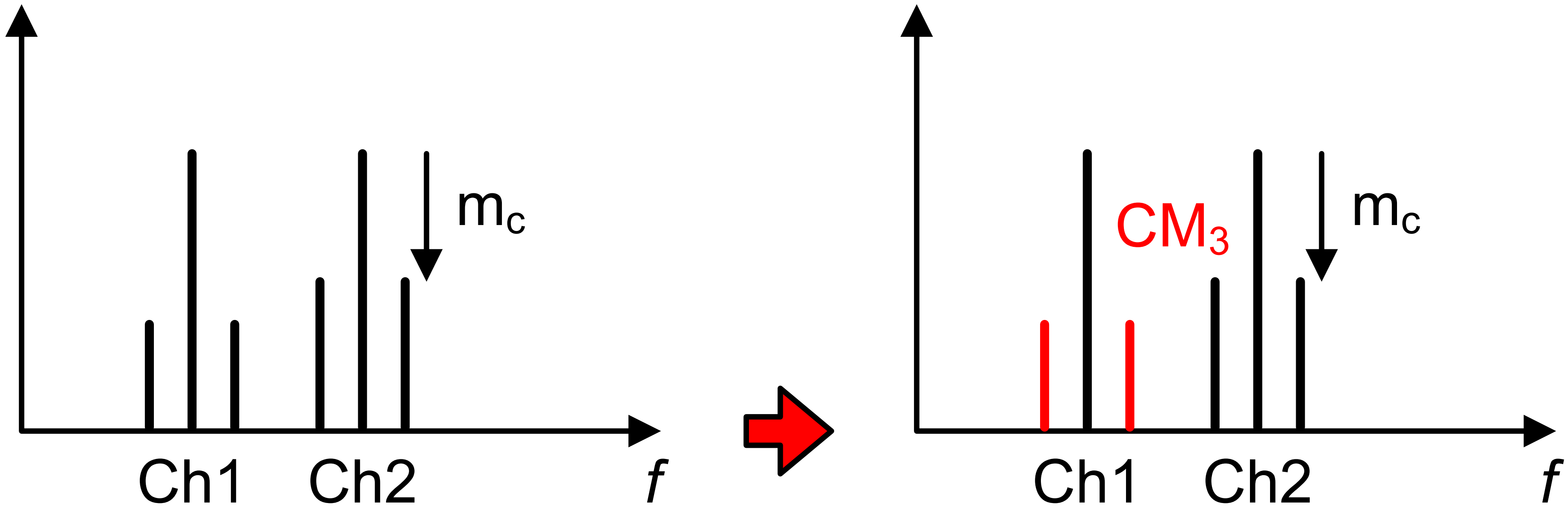
http://rfic.fudan.edu.cn/Courses.htm

复旦大学/微电子学院/射频集成电路设计研究小组

基本晶体管电路的非线性

什么是失真?

失真包括线性失真和非线性失真。

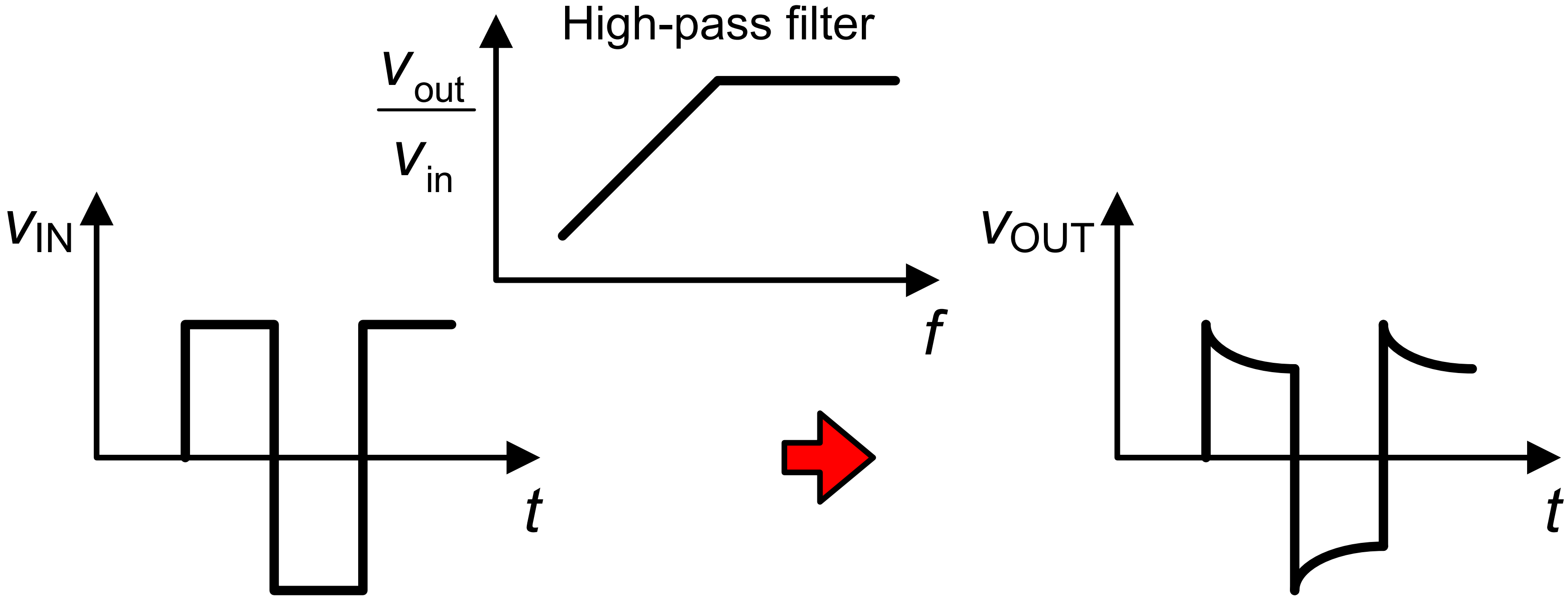


频道串扰!!!

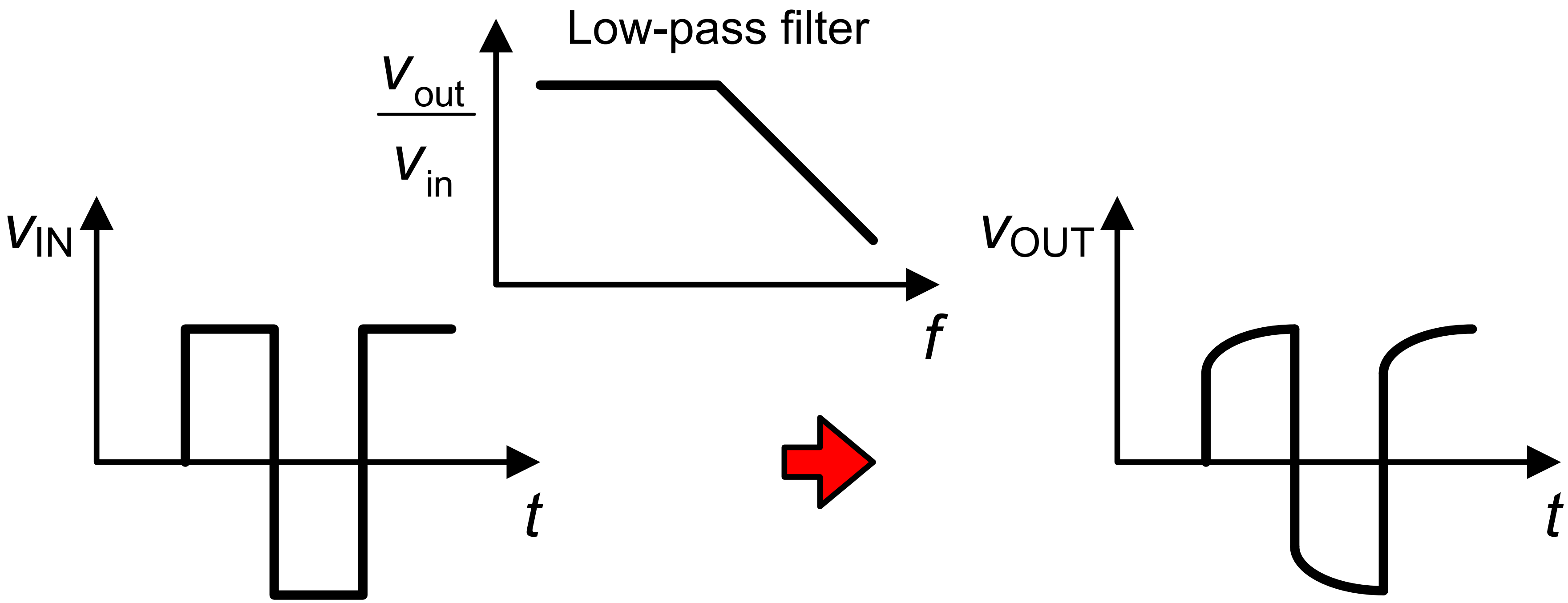
目录

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

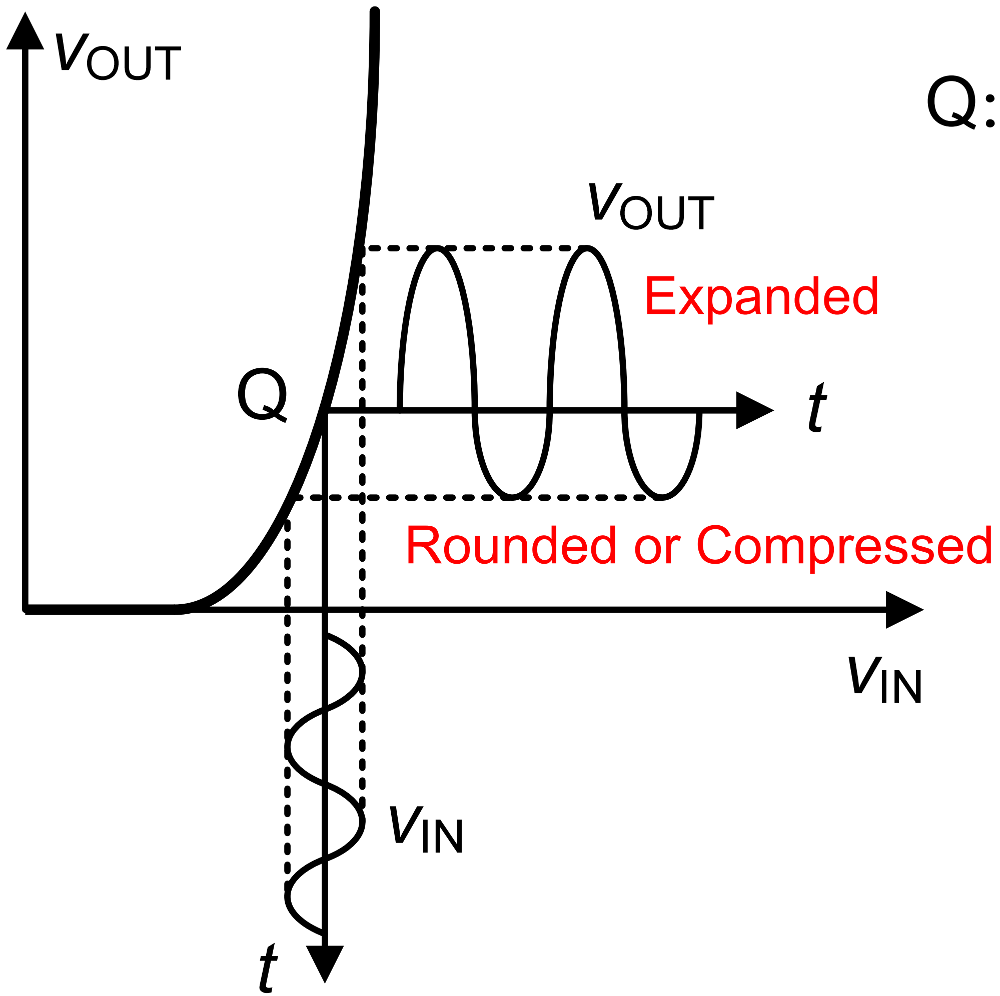
线性失真-1



线性失真-2

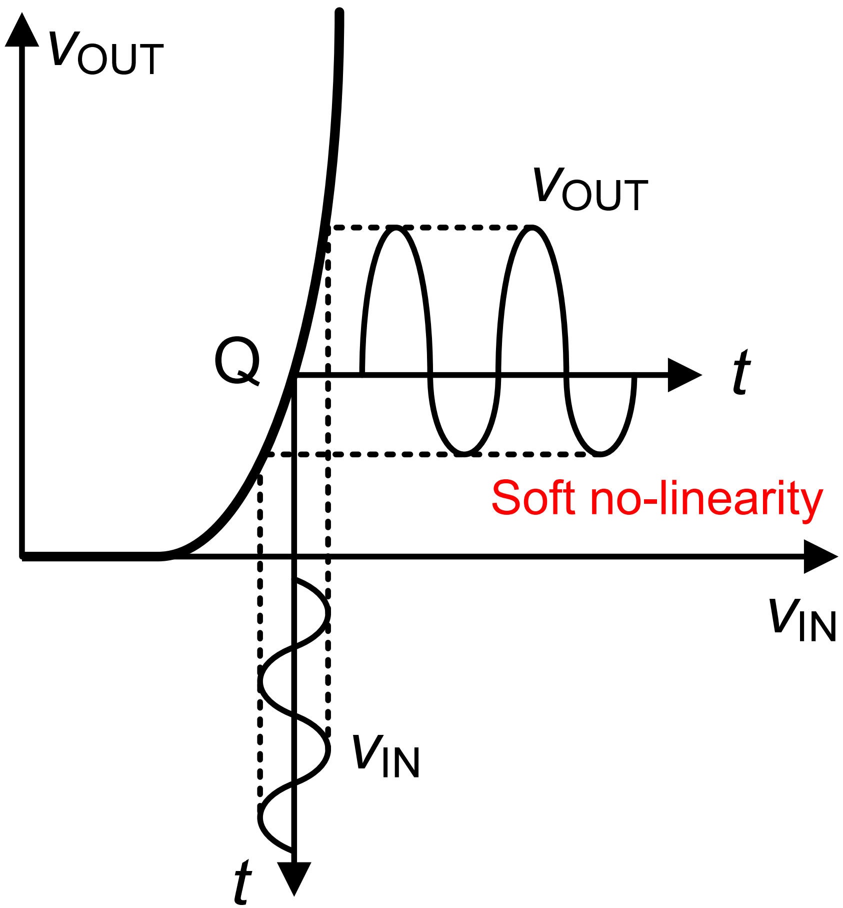


非线性失真

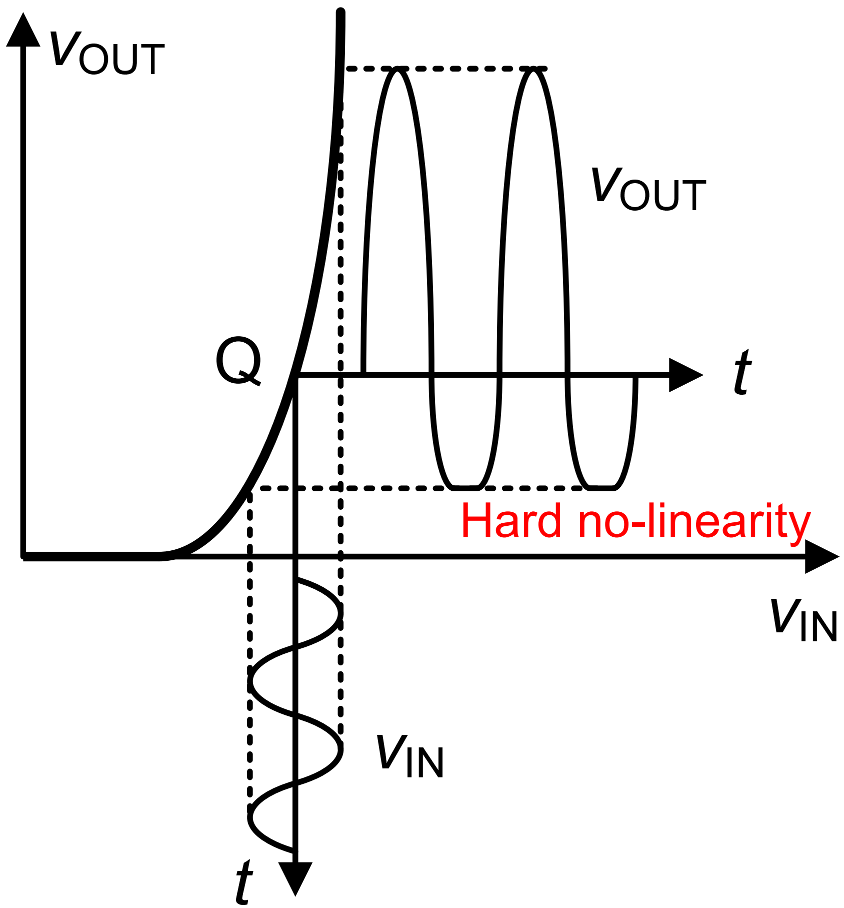


Q: 静态工作点

“软”非线性和“硬”非线性

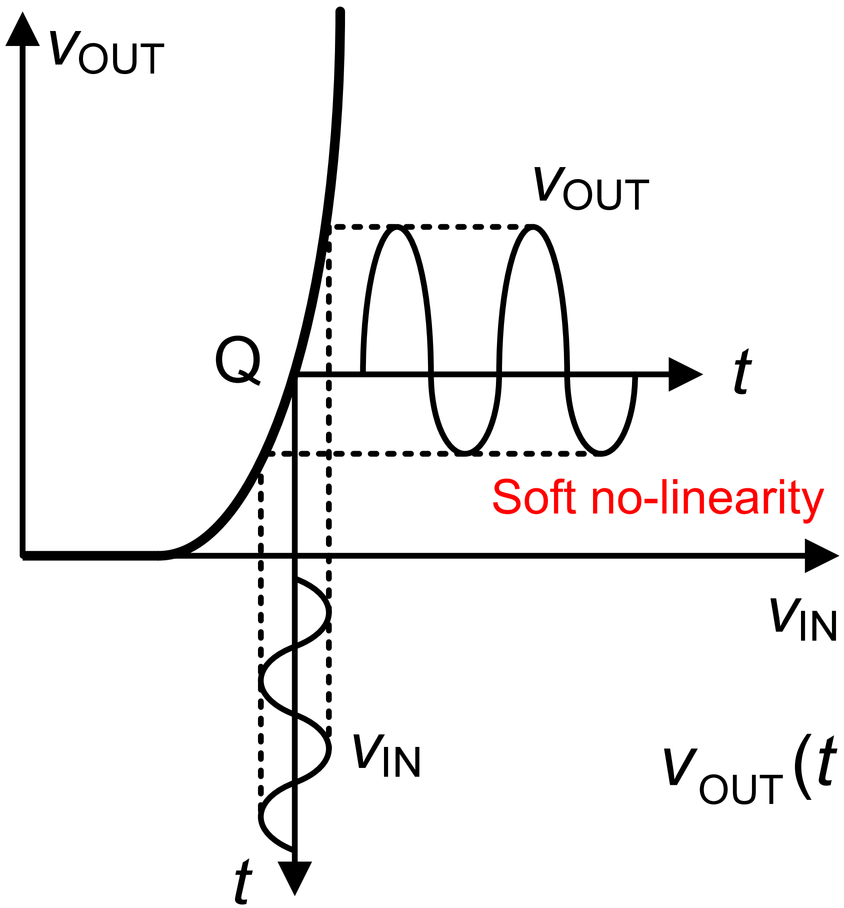


VOUT
VOUT
Q
t
Soft no-linearity
VIN
VIN
t



VOUT
VOUT
Q
t
Hard no-linearity
VIN
VIN
t

非线性：指数表达式



$$v_{IN}(t) \Rightarrow v_{OUT}(t)$$

$$v_{OUT}(t) = \alpha_0 + \alpha_1 v_{IN} + \alpha_2 v_{IN}^2 + \alpha_3 v_{IN}^3 + \ldots$$

系数α_0，α_1，α_2，…

$$y = \alpha_0 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \ldots$$

$$\alpha_0 = y\Big|_{u=0}$$

$$\alpha_1 = \frac{\mathrm{d}y}{\mathrm{d}u}\Big|_{u=0}$$

$$\alpha_2 = \frac{1}{2}\frac{\mathrm{d}^2 y}{\mathrm{d}u^2}\Big|_{u=0}$$

$$\alpha_3 = \frac{1}{6}\frac{\mathrm{d}^3 y}{\mathrm{d}u^3}\Big|_{u=0}$$

谐波失真定义：*HD*

$$y = \alpha_0 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \ldots$$

$$u = U\cos\omega t$$

$$\cos^2 x = 1/2\left(1 + \cos 2x\right)$$

$$\cos^3 x = 1/4\left(3\cos x + \cos 3x\right)$$

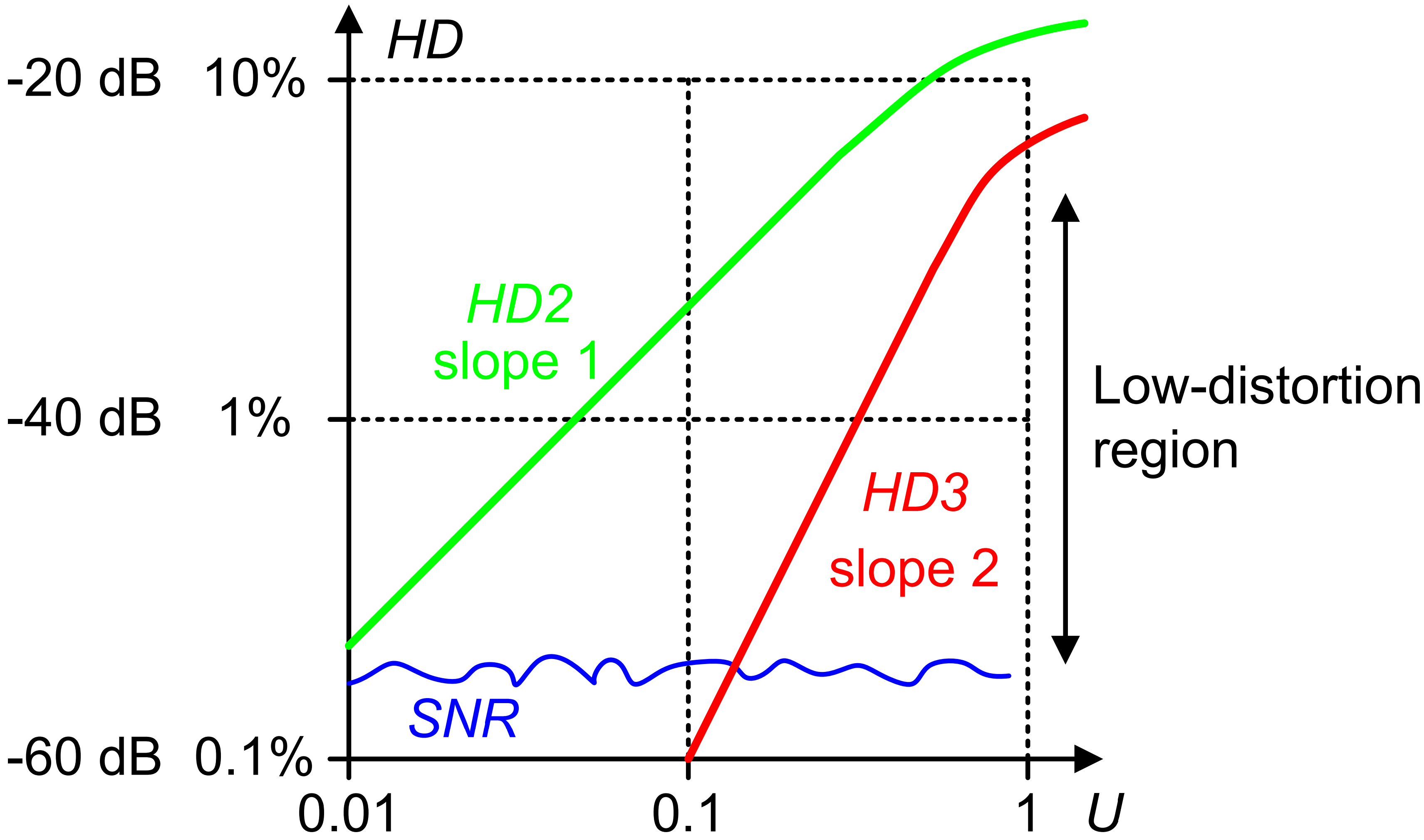
$$y = \alpha_0 + (\alpha_1 + \frac{3}{4}\alpha_3 U^2)U\cos\omega t +$$

$$\frac{\alpha_2}{2}U^2\cos 2\omega t + \frac{\alpha_3}{4}U^3\cos 3\omega t + \ldots$$

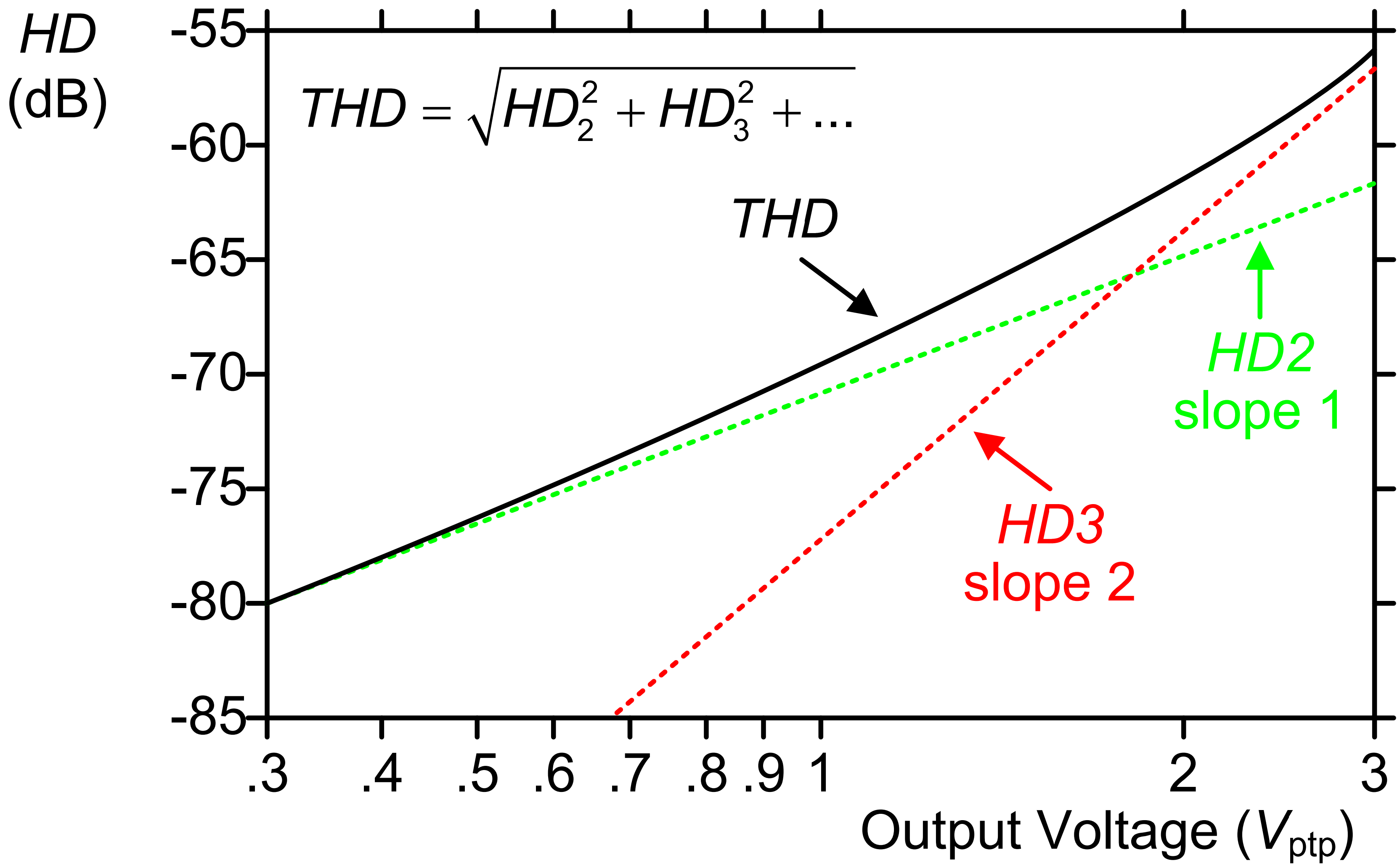
$$HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1}U$$

$$HD_3 = \frac{1}{4}\frac{\alpha_3}{\alpha_1}U^2$$

HD_2，HD_3与输入信号幅度的关系

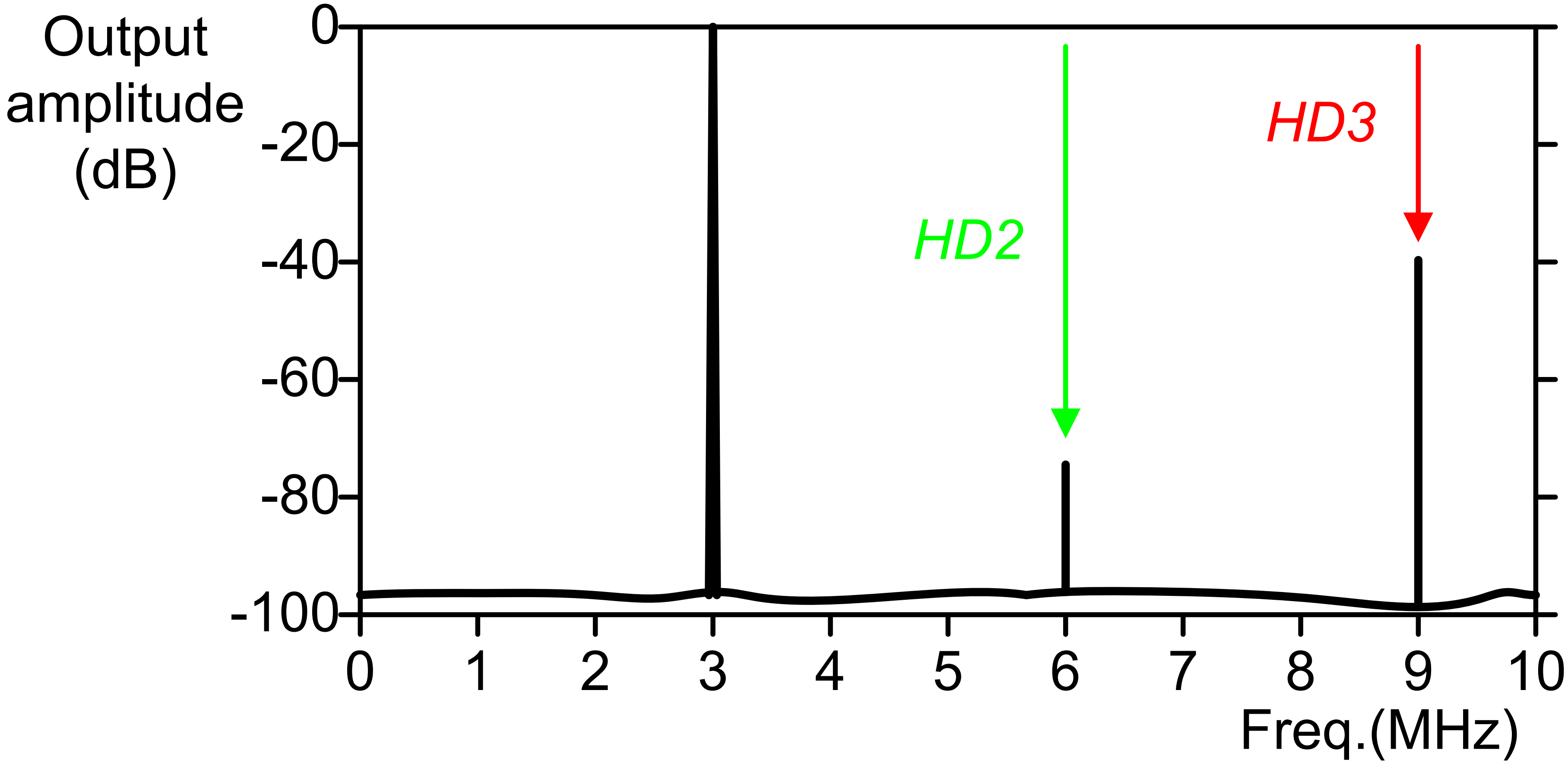


电阻的谐波失真*HD*



HD
(dB)
-55
-60
-65
-70
-75
-80
-85
$THD = \sqrt{HD_2^2 + HD_3^2 + \ldots}$
THD
HD2
slope 1
HD3
slope 2
.3
.4
.5
.6
.7
.8
.9
1
2
3
Output Voltage (V_{ptp})

谐波失真的频谱图



Output amplitude (dB)
0
-20
-40
-60
-80
-100
HD2
HD3
0
1
2
3
4
5
6
7
8
9
10
Freq.(MHz)

交调失真的定义：*IM*

$$y = \alpha_0 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \ldots$$

$$u = U(\cos\omega_1 t + \cos\omega_2 t)$$

$$y = \alpha_0 + \ldots$$

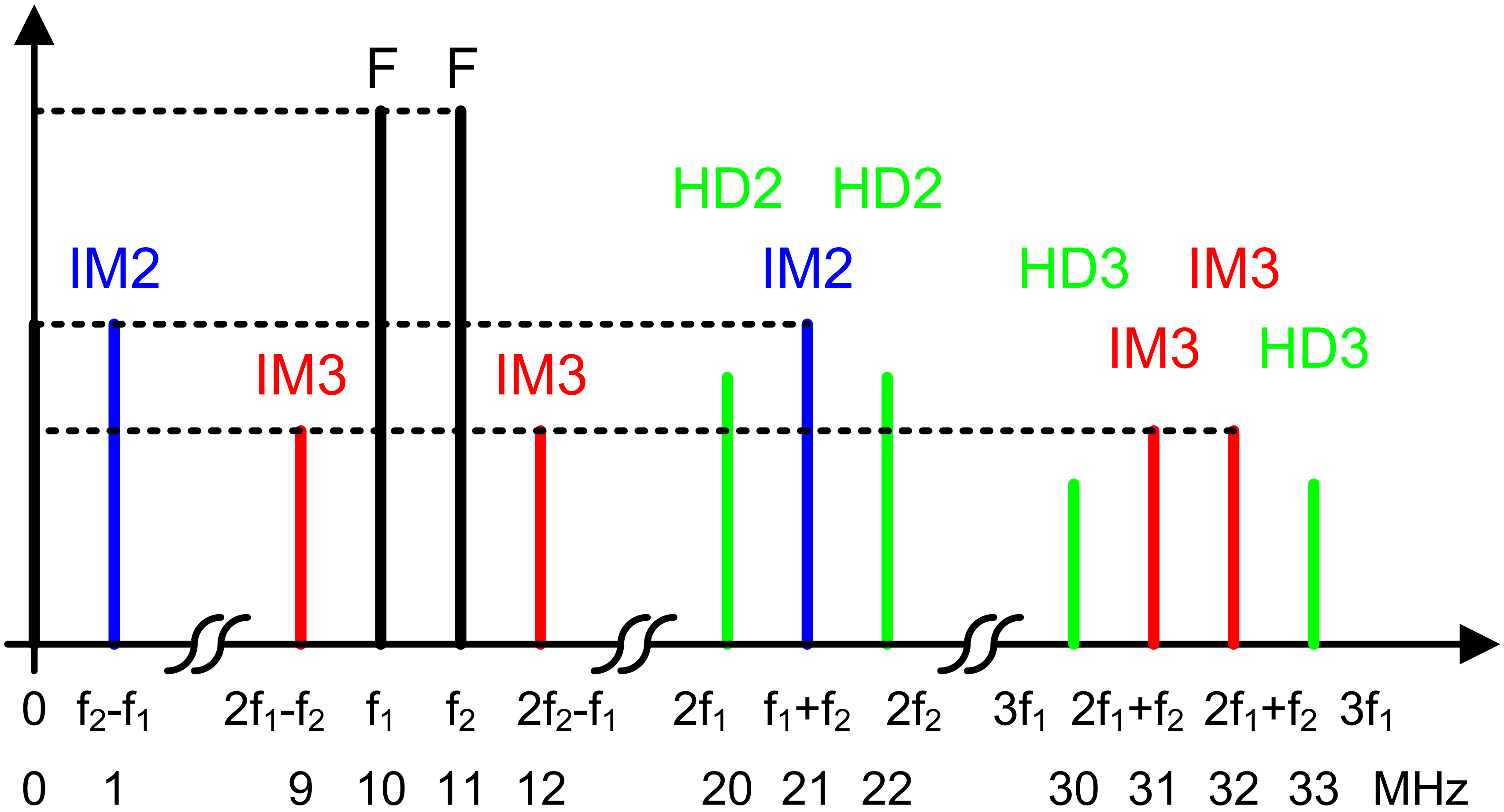
$$IM_2 \ @\ \omega_1 \pm \omega_2$$

$$IM_3 \ @\ 2\omega_1 \pm \omega_2 \text{ and } \omega_1 \pm 2\omega_2$$

$$IM_2 = 2HD_2 = \frac{\alpha_2}{\alpha_1} U$$

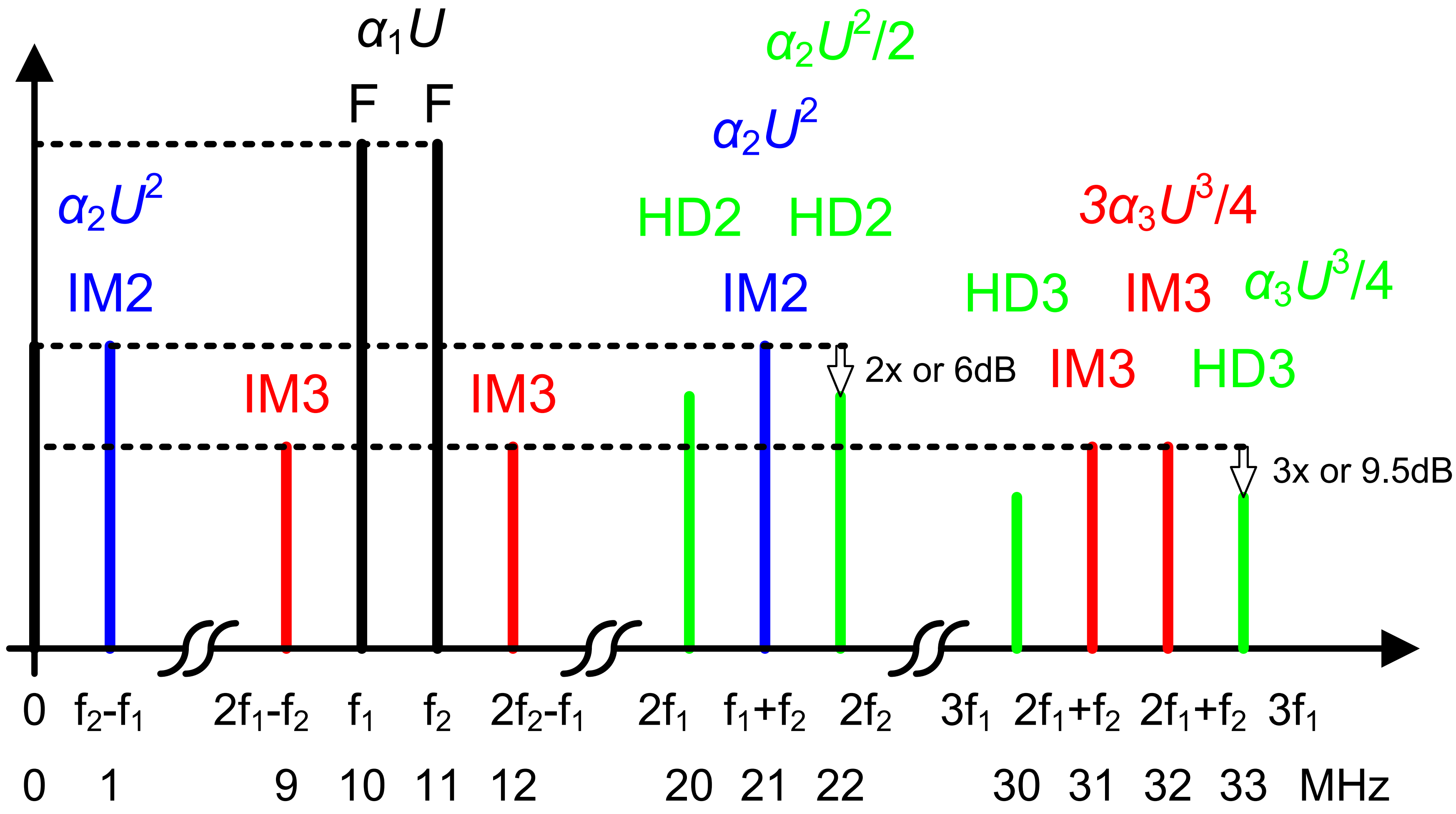
$$IM_3 = 3HD_3 = \frac{3}{4}\frac{\alpha_3}{\alpha_1} U^2$$

IM分量-1



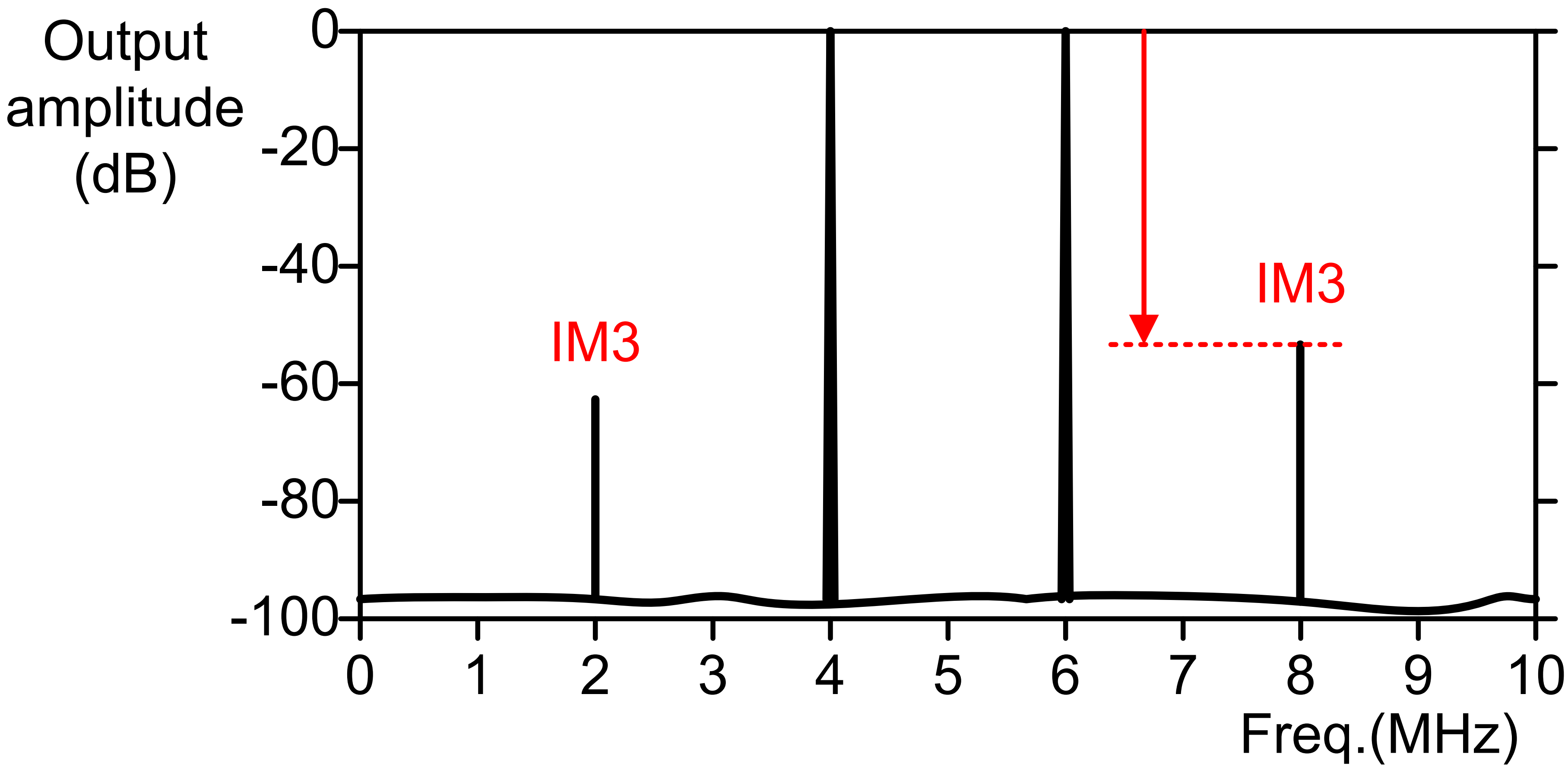
F
F
HD2
HD2
IM2
IM2
HD3
IM3
IM3
HD3
IM3
IM3
0
f2-f1
2f1-f2
f1
f2
2f2-f1
2f1
f1+f2
2f2
3f1
2f1+f2
2f1+f2
3f1
0
1
9
10
11
12
20
21
22
30
31
32
33
MHz

IM分量-2

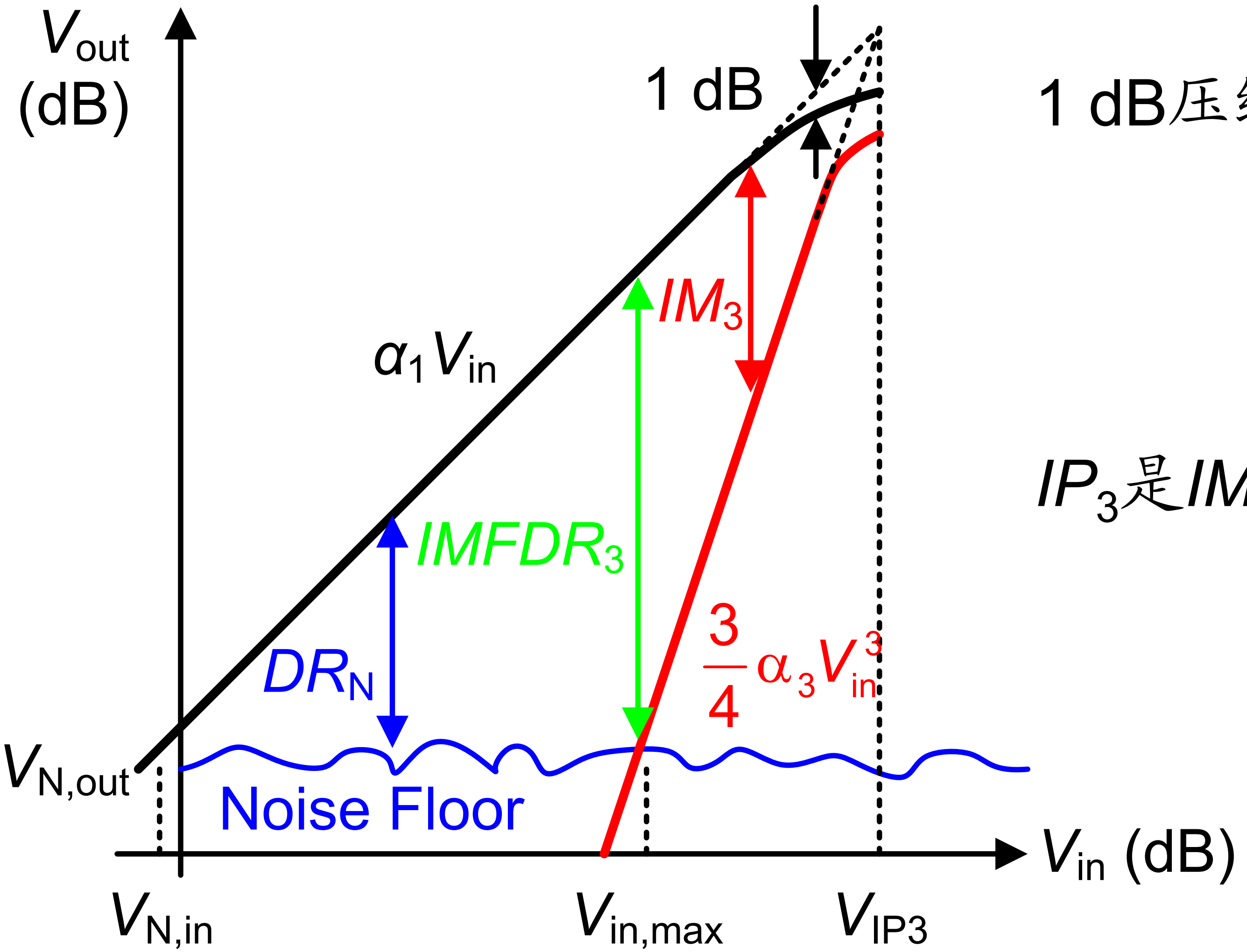


α1U
F F
α2U2/2
α2U2
α2U2
HD2 HD2
3α3U3/4
IM2
IM2
HD3 IM3 α3U3/4
2x or 6dB
IM3 HD3
IM3 IM3
3x or 9.5dB
0 f2-f1 2f1-f2 f1 f2 2f2-f1 2f1 f1+f2 2f2 3f1 2f1+f2 2f1+f2 3f1
0 1 9 10 11 12 20 21 22 30 31 32 33 MHz

交调失真的频谱图


Output
amplitude
(dB)
0
-20
-40
-60
-80
-100
IM3
IM3
0
1
2
3
4
5
6
7
8
9
10
Freq.(MHz)

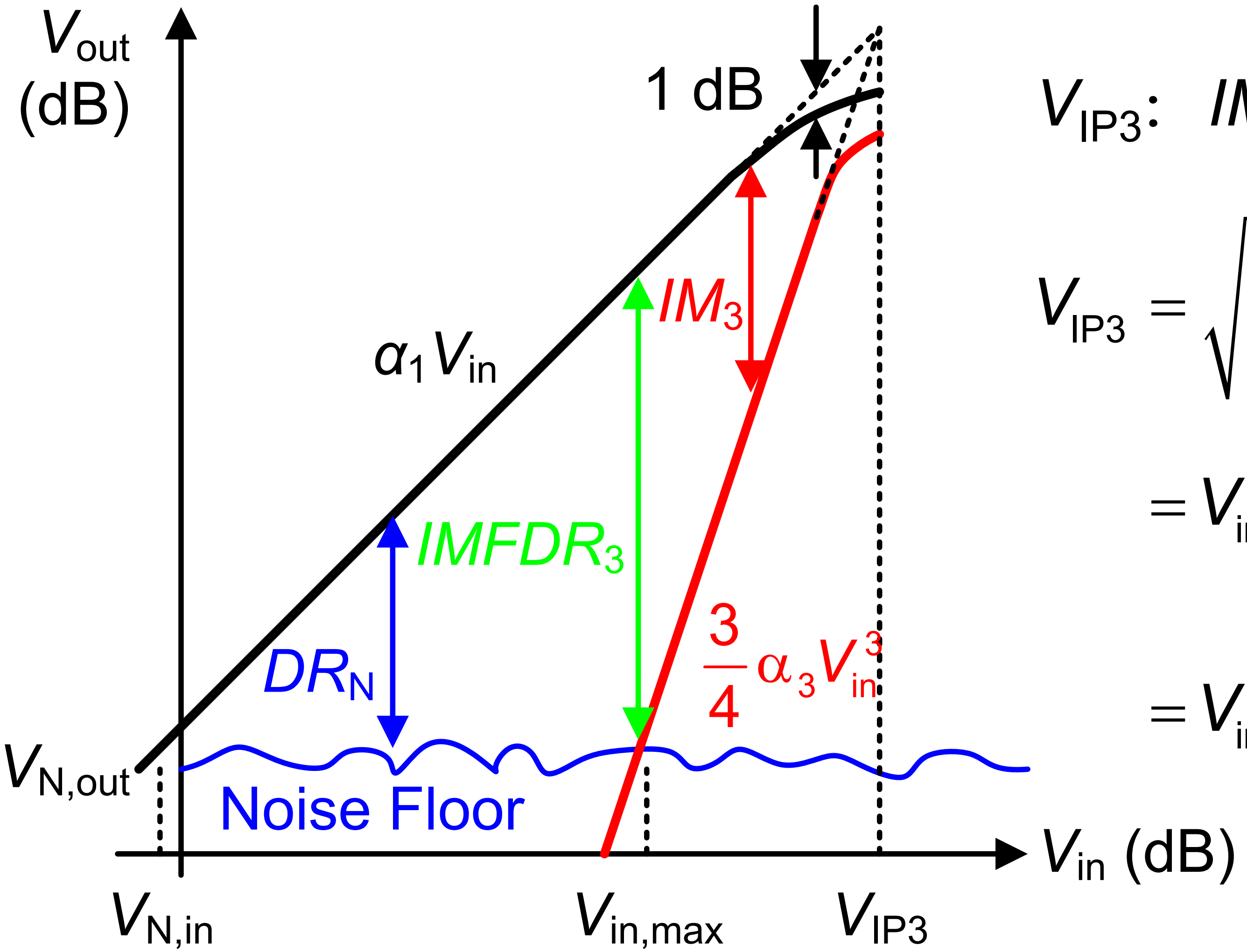
IM_3与输入信号幅度的关系



1 dB压缩点

IP_3是IM_3的交会点

IP_3与IM_3的关系



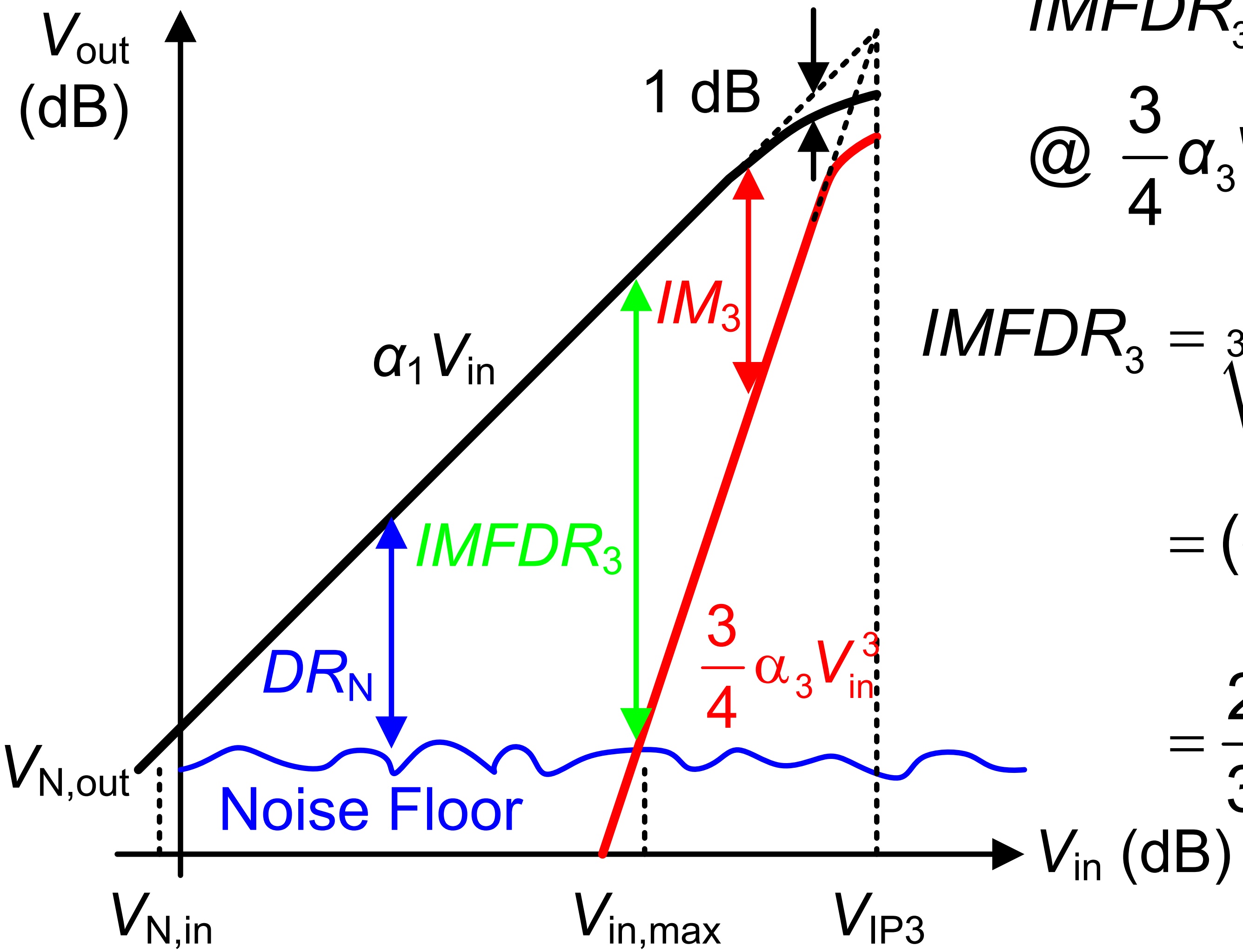
V_{IP3}：　IM_3=F

$$V_{IP3} = \sqrt{\frac{4}{3}\frac{\alpha_1}{\alpha_3}}$$

$$= V_{in}\frac{1}{\sqrt{IM_3}}$$

$$= V_{in,dB} - \frac{1}{2}IM_{3,dB}$$

$IMFDR_3$与IM_3的关系



$$IMFDR_3 = \max \ DR$$

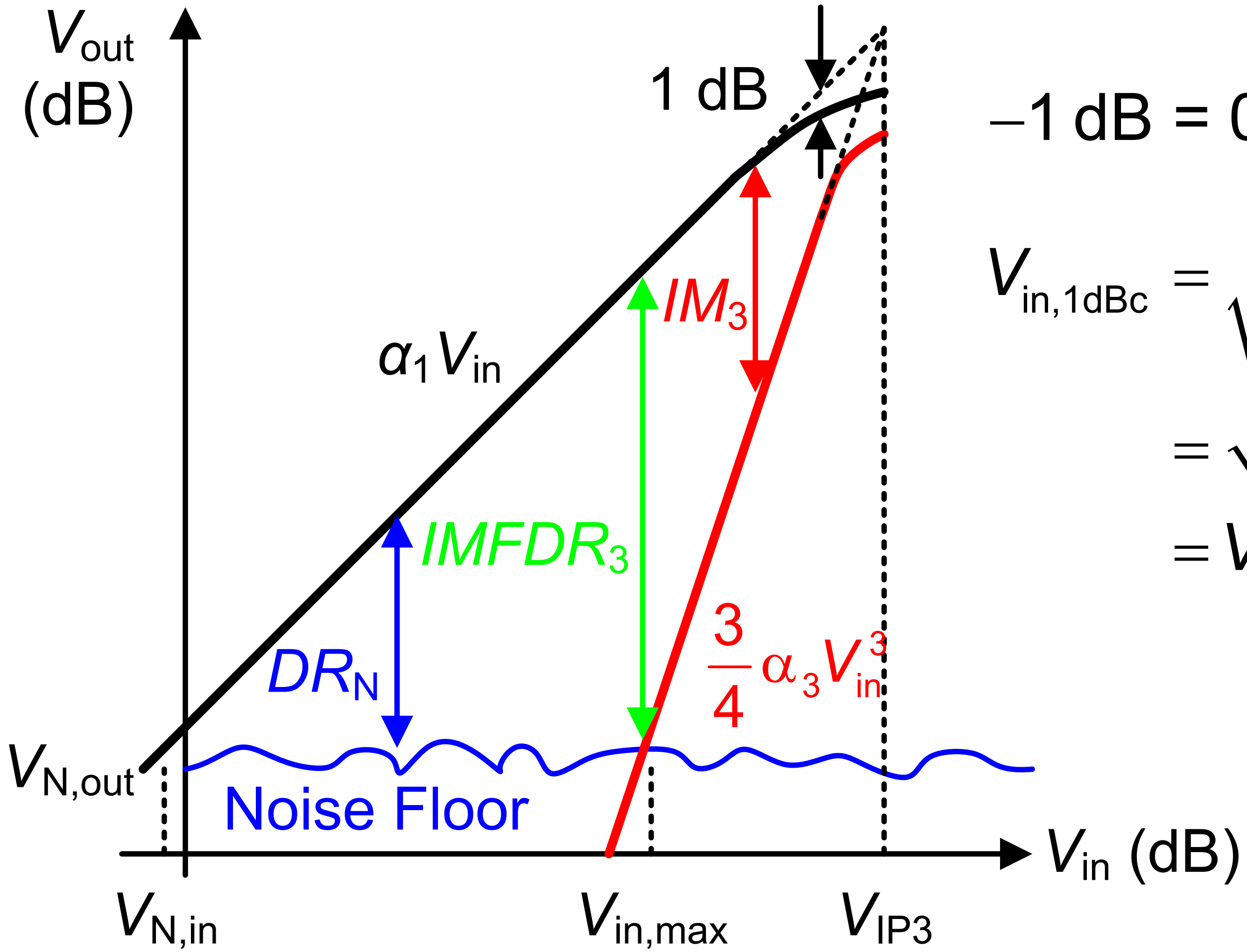
$$@ \ \frac{3}{4}\alpha_3 V_{in}^3 = V_{N,out}$$

$$IMFDR_3 = \sqrt[3]{\frac{4}{3}\frac{\alpha_1}{\alpha_3}\frac{1}{V_{N,in}^2}}$$

$$= \left(\frac{V_{IP3}}{V_{N,in}}\right)^{2/3}$$

$$= \frac{2}{3}\left(V_{IP3,dB} - V_{N,in,dB}\right)$$

IP_3与1 dB压缩点的关系



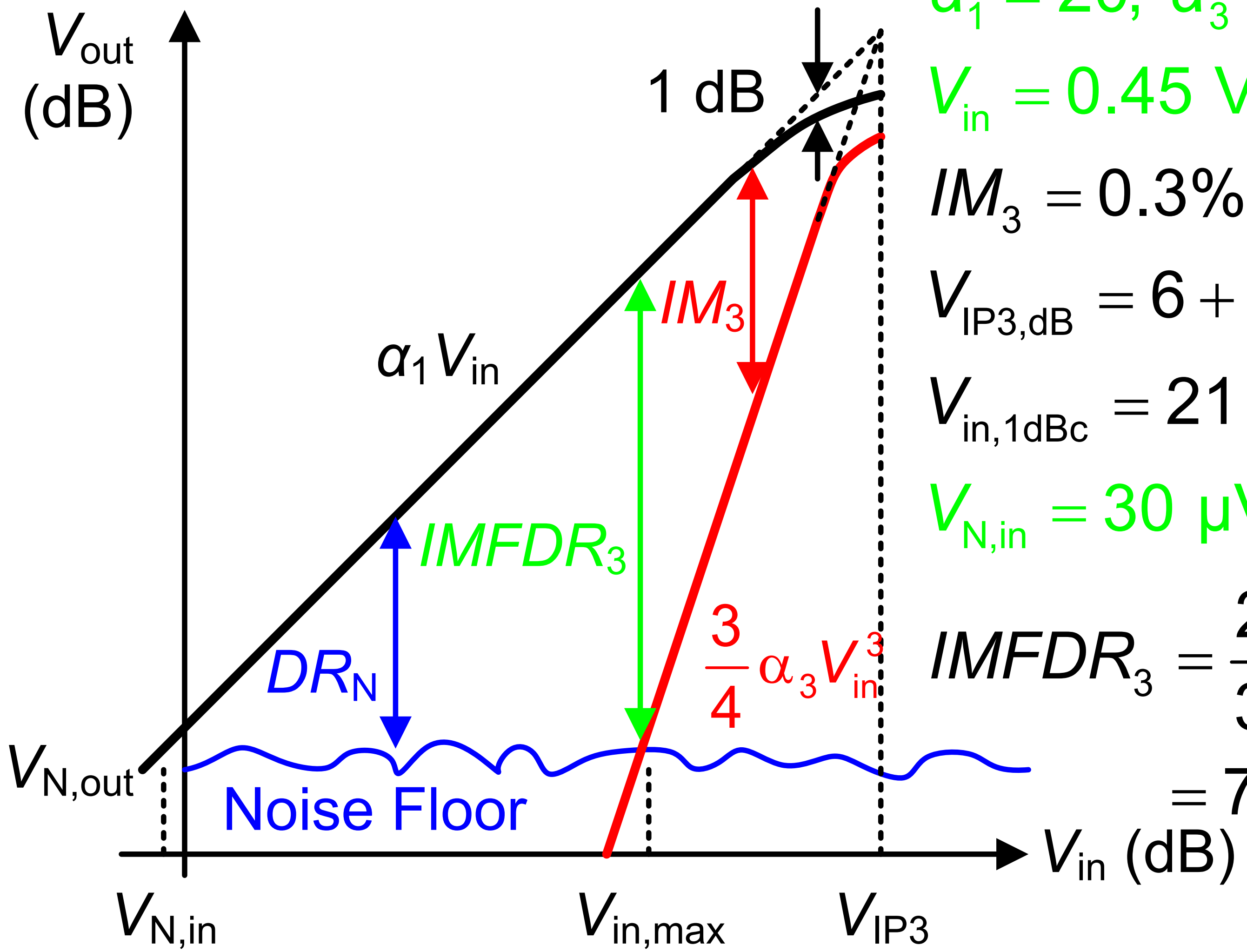
$$-1\text{ dB} = 0.891$$

$$V_{in,1dBc} = \sqrt{0.109\frac{4}{3}\frac{\alpha_1}{\alpha_3}}$$

$$= \sqrt{0.109}V_{IP3}$$

$$= V_{IP3,dB} - 9.6\text{ dB}$$

练习



$\alpha_1 = 20,\ \alpha_3 = 0.4$

$V_{in} = 0.45\ V_{RMS}$ or 6 dBm

$IM_3 = 0.3\%$ or -50 dB

$V_{IP3,dB} = 6 + 25 = 31$ dBm

$V_{in,1dBc} = 21$ dBm

$V_{N,in} = 30\ \mu V_{RMS}$ or -78 dBm

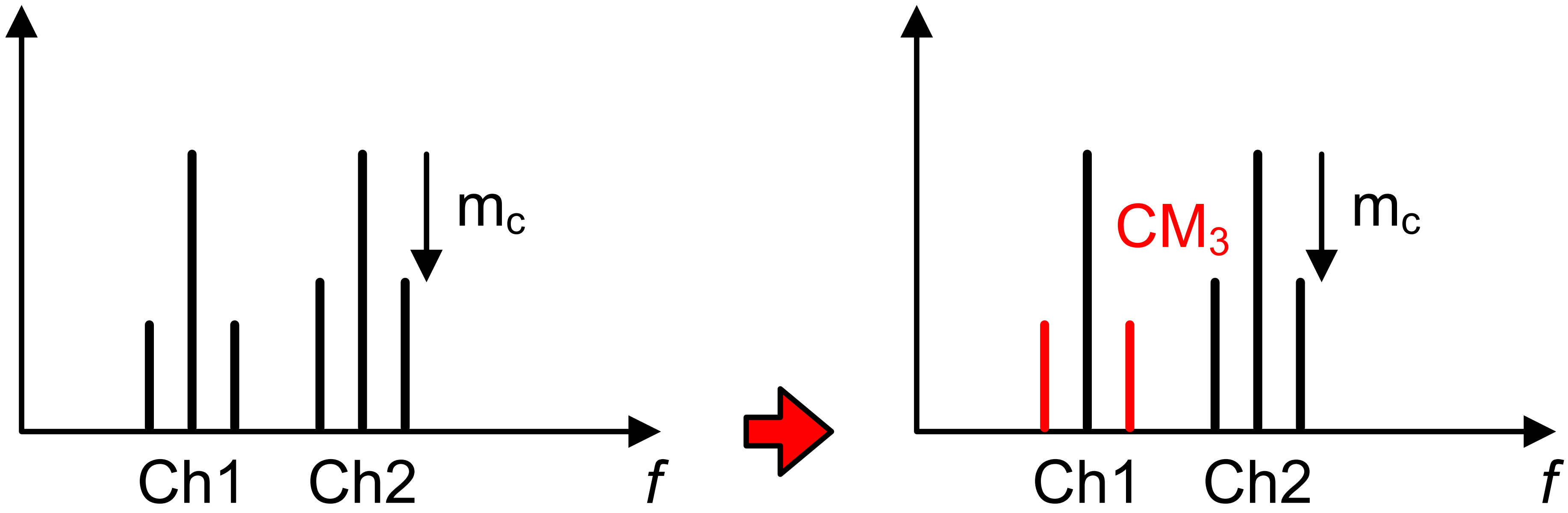
$$IMFDR_3 = \frac{2}{3}(31 + 78)$$

$$= 73 \text{ dB}$$

互交调失真的定义：CM

$$y = \alpha_0 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \ldots$$

$$u = U\cos\omega_1 t + U(1 + m_c\cos\omega_c t)\cos\omega_2 t$$



$$CM_3 = \frac{3}{4} m_c \frac{\alpha_3}{\alpha_1} U^2 = m_c IM_3$$

目录

- 定义：*HD*, *IM*, 交调点
- MOS管的非线性
  - ❑单端放大器
  - ❑差分放大器
- 双极型晶体管的非线性
- 负反馈能够减小非线性
- 运算放大器的非线性
- 其他非线性和准则

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

单MOS管放大器的非线性

$$i_{DS}=K(v_{GS}-V_T)^2 \qquad K=K'\frac{W}{L}$$

$$I_{DS}+i_{ds}=K(V_{GS}+v_{gs}-V_T)^2$$

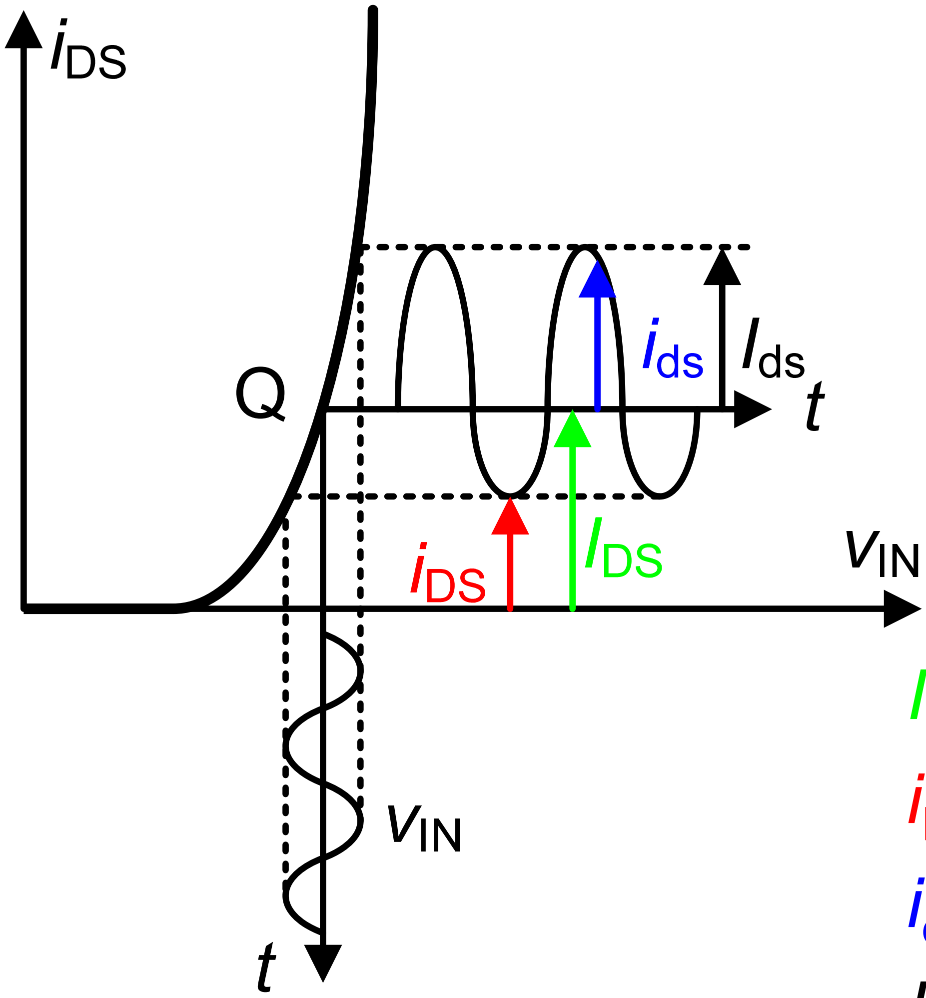
I_{DS}: 直流(DC)分量

i_{DS}: 直流(DC)+交流(ac)分量

i_{ds} : 交流(ac)分量

I_{ds} : 交流(ac)分量的幅度

DC和ac分量



I_{DS}：直流(DC)分量

i_{DS}：直流(DC)+交流(ac)分量

i_{ds} ：交流(ac)分量

I_{ds} ：交流(ac)分量的幅度

单MOS管放大器的非线性

$$I_{DS} = K(V_{GS} - V_T)^2 \qquad K = K'\frac{W}{L}$$

$$I_{DS} + i_{ds} = K(V_{GS} + v_{gs} - V_T)^2$$

$$i_{ds} = K(V_{GS} + v_{gs} - V_T)^2 - K(V_{GS} - V_T)^2$$

$$i_{ds} = 2K(V_{GS} - V_T)v_{gs} + Kv_{gs}^2$$

系数α_0，α_1，α_2，…

$$i_{ds} = 2K(V_{GS} - V_T)v_{gs} + Kv_{gs}^2 \qquad K = K'\frac{W}{L}$$

$$\alpha_1 = 2K(V_{GS} - V_T) = g_m$$

$$\alpha_2 = K$$

$$\alpha_3 = 0$$

$$IM_2 = \frac{\alpha_2}{\alpha_1}V_{gs} = \frac{V_{gs}}{2(V_{GS} - V_T)} \qquad IM_3 = 0$$

归一化电流摆幅

$$i_{ds} = 2K(V_{GS} - V_T)v_{gs} + Kv_{gs}^2 \qquad I_{DS} = K(V_{GS} - V_T)^2$$

$$y = \alpha_1 u + \alpha_1 u^2 + \alpha_3 u^3 + \ldots$$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2v_{gs}}{V_{GS} - V_T} + \frac{1}{4}\left(\frac{2v_{gs}}{V_{GS} - V_T}\right)^2$$

$$y = u + \frac{1}{4}u^2 \qquad u = \frac{v_{gs}}{(V_{GS} - V_T)/2}, \quad U = \frac{V_{gs}}{(V_{GS} - V_T)/2}$$

u: 归一化输入电压摆幅

y: 归一化输出电流摆幅

例子

v_{gs}的峰值电压(电压幅度)V_{gsp} = 100 mV

则均方根电压(有效值) V_{gsRMS} = 100/1.414 = 71 mV

如果 V_{GS}-V_T = 0.5V，

则归一化输入电压摆幅U = 2V_{gsp}/(V_{GS}-V_T) = 0.4

$$IM_2 = \frac{\alpha_2}{\alpha_1}U = \frac{1}{4}\times\frac{100\ \text{mV}}{0.5/2\ \text{V}} = 0.1 \quad IM_3 = 0$$

更复杂的非线性

一般而言，

$$i_{ds} = g_m v_{gs} + K_{2gm} v_{gs}^2 + K_{3gm} v_{gs}^3 +$$
$$g_o v_{ds} + K_{2go} v_{ds}^2 + K_{3go} v_{ds}^3 +$$
$$g_{mb} v_{bs} + K_{2gmb} v_{bs}^2 + K_{3gmb} v_{bs}^3 +$$
$$K_{2,gm\&gmb} v_{gs} v_{bs} + K_{3,2gm\&gmb} v_{gs}^2 v_{bs}$$
$$+ K_{3,gm\&2gmb} v_{gs} v_{bs}^2 +$$
$$..............+$$
$$K_{3,gm\&gmb\&go} v_{gs} v_{ds} v_{bs}$$

二极管连接MOST的非线性

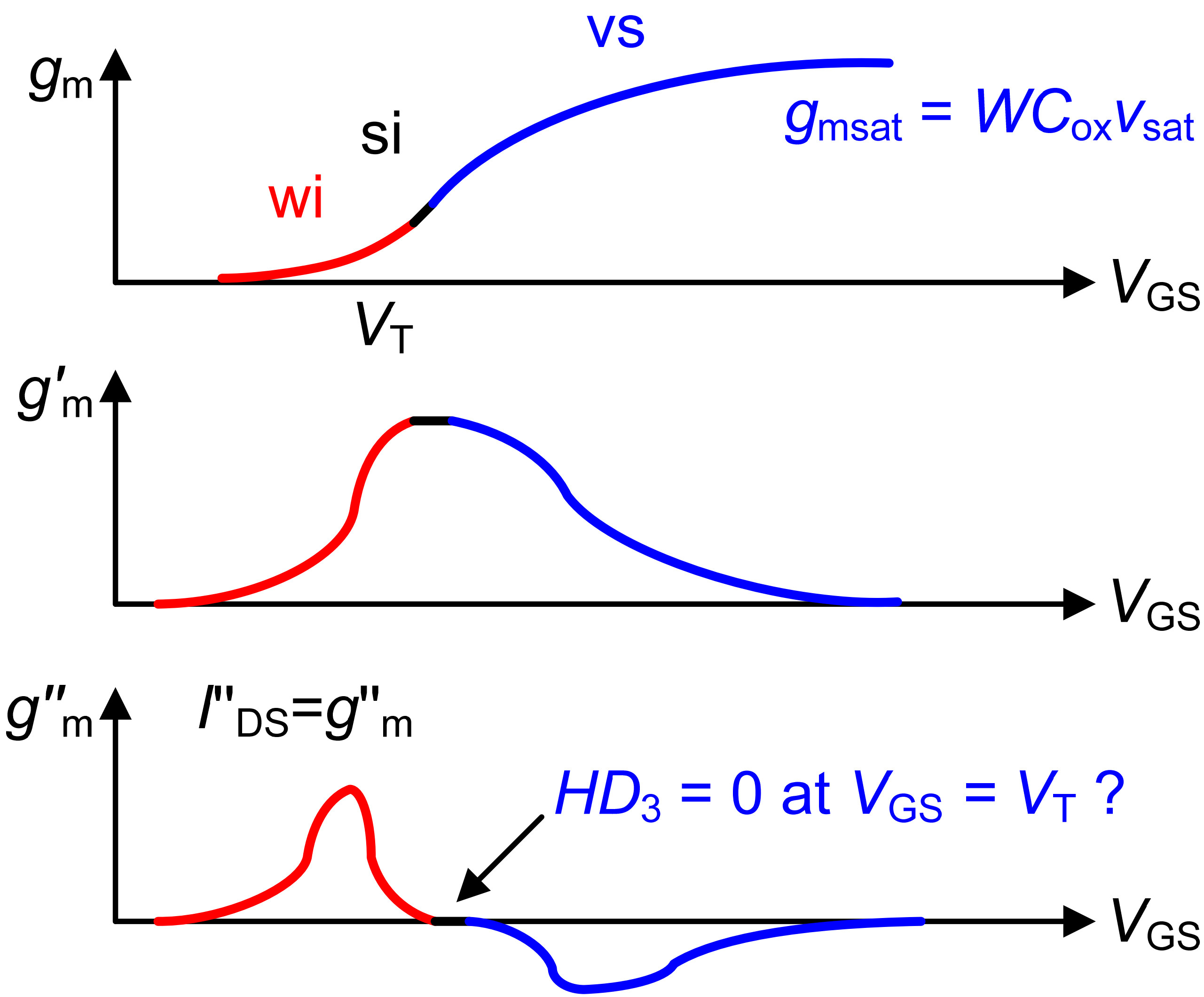
$$i_{DS} = K(v_{DS} - V_T)^2 \qquad K = K'\frac{W}{L}$$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2v_{ds}}{V_{DS} - V_T} + \frac{1}{4}\left(\frac{2v_{ds}}{V_{DS} - V_T}\right)^2$$

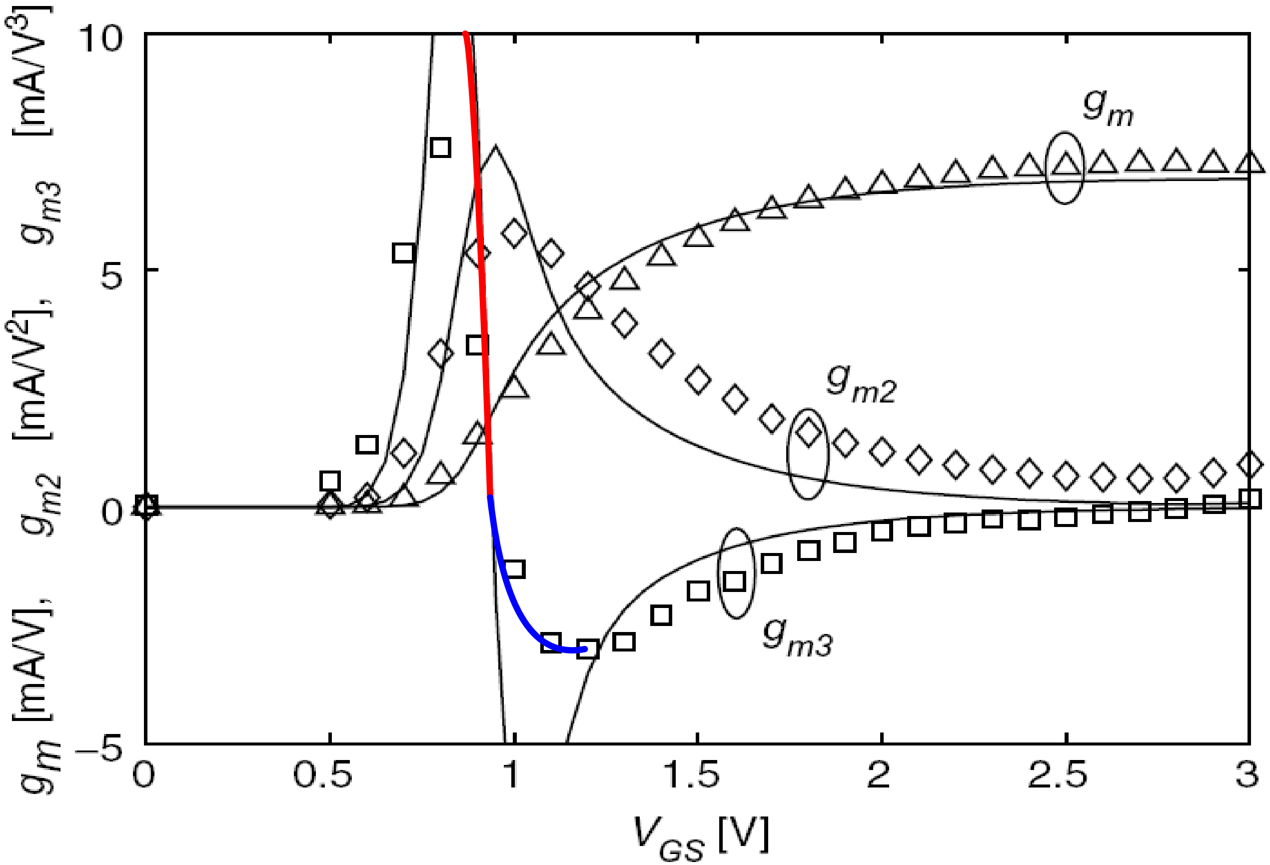
$$y = u + \frac{1}{4}u^2 \qquad u = \frac{v_{ds}}{(V_{DS} - V_T)/2},\ U = \frac{V_{ds}}{(V_{DS} - V_T)/2}$$

二极管连接MOST与单管共源放大器具有相同的非线性！

小尺寸MOS管的零HD_3点



g_m的导数



W = 60 μm

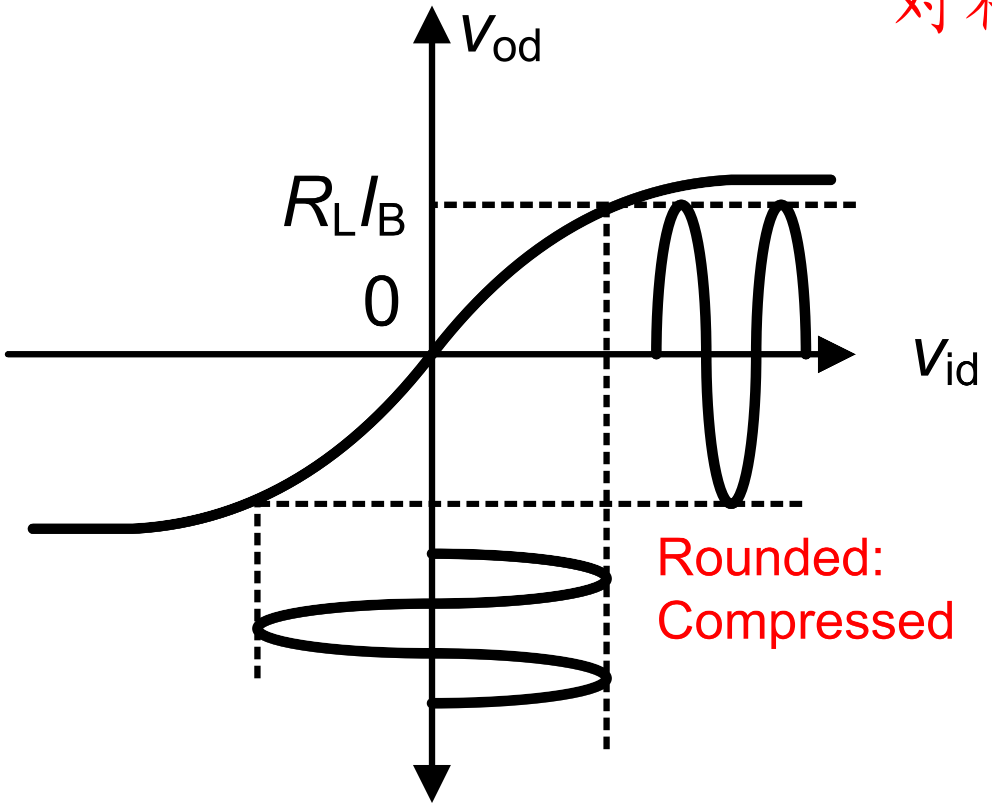
L = 0.6 μm

V_{DS} = 2 V

Ref. Fager JSSC Jan. 2004, 24-33

差分对具有对称特性

对称特性：无二次分量！



MOS管差分对的非线性−1

$$y=\frac{i_{\mathrm{od}}}{I_{\mathrm{B}}}=\frac{v_{\mathrm{ind}}}{V_{\mathrm{GS}}-V_{\mathrm{T}}}\sqrt{1-\frac{1}{4}\left(\frac{v_{\mathrm{ind}}}{V_{\mathrm{GS}}-V_{\mathrm{T}}}\right)^2}$$

v_{ind}为差分输入电压

i_{od}为差分小信号输出电流($g_m v_{\mathrm{ind}}$)

或两倍的环路电流($g_m v_{\mathrm{ind}}/2$)

I_{B}为差分对的总的直流电流

注意：$g_m=\frac{I_{\mathrm{B}}}{V_{\mathrm{GS}}-V_{\mathrm{T}}}=2K'\frac{W}{L}(V_{\mathrm{GS}}-V_{\mathrm{T}})$

MOS管差分对的非线性-2

$$y=\frac{i_{od}}{I_B}=\frac{v_{ind}}{V_{GS}-V_T}\sqrt{1-\frac{1}{4}\left(\frac{v_{ind}}{V_{GS}-V_T}\right)^2}$$

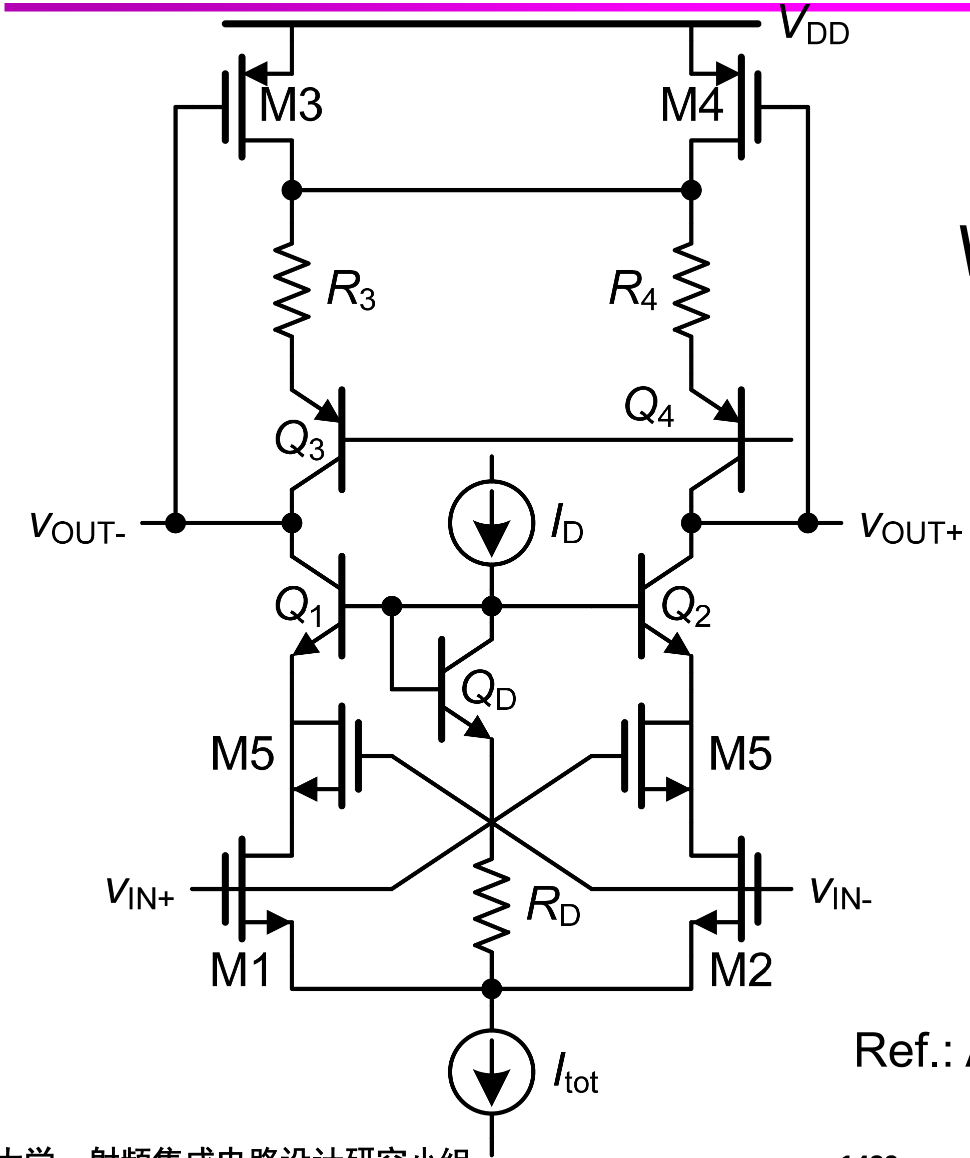
$$\sqrt{1-x}=1-\frac{x}{2}$$

$$y=\frac{i_{od}}{I_B}=u\sqrt{1-\frac{1}{4}u^2}=u-\frac{1}{8}u^3 \qquad u=\frac{v_{id}}{V_{GS}-V_T}$$

$$IM_2=0 \qquad IM_3=\frac{3}{32}U^2 \qquad U=\frac{V_{id}}{V_{GS}-V_T}$$

$$IP_3=4\sqrt{\frac{2}{3}}(V_{GS}-V_T)\approx 3.3(V_{GS}-V_T)$$

线性区MOS管的非线性



$$V_{DS1} = R_D I_D \approx 0.2\ \text{V}$$

$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$g_{m1} = \beta_1 V_{DS1}$$ 为常数

Ref.: Alini, JSSC, Dec.92, pp.1905-1915

目录

- 定义：*HD*, *IM*, 交调点
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  - 单端放大器
  - 差分放大器
- 双极型晶体管的非线性
- 负反馈能够减小非线性
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- 其他非线性和准则

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

单双极型管的非线性-1

$$I_{CE} = I_S \exp(\frac{V_{BE}}{kT/q})$$

$$I_{CE} + i_{ce} = I_S \exp(\frac{V_{BE} + v_{be}}{kT/q})$$

$$1 + y = \exp(\frac{v_{be}}{kT/q})$$

I_{CE}：直流(DC)分量

i_{CE}：直流(DC)+交流(ac)分量

i_{ce} ：交流(ac)分量

I_{ce} ：交流(ac)分量的幅度

$$\exp(u) \approx 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots \quad \text{if } u << 1$$

单双极型管的非线性-2

$$y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots \qquad u = \frac{v_{be}}{kT/q},\ U = \frac{V_{be}}{kT/q}$$

$$\alpha_1 = 1$$

$$\alpha_2 = 1/2$$

$$\alpha_3 = 6$$

$$IM_2 = \frac{\alpha_2}{\alpha_1} U = \frac{1}{2} \frac{V_{be}}{kT/q} \qquad IM_3 = \frac{3}{4} \frac{\alpha_3}{\alpha_1} U^2 = \frac{1}{8} \left(\frac{V_{be}}{kT/q}\right)^2$$

$$IP_3 = \sqrt{\frac{4}{3} \frac{\alpha_1}{\alpha_3}} (kT/q) = \sqrt{8} (kT/q)$$

例子

1. 归一化输入电压摆幅为10%

$U = 0.1$，$IM_2 = 5\%$ ($HD_2 = 2.5\%$)

$IM_3 = 0.125\%$ ($HD_3 = 0.375\%$)

$V_{bep} = U(kT/q) = 2.6\ mV_p$ ($1.8\ mV_{RMS}$)

$IP_3 = \sqrt{8}(kT/q) = 74\ mV_p$ or $50\ mV_p$ or $-13\ dBm$

2. $V_{bep} = 100\ mV$

$y_p = 0.1/0.026 \approx 4$ (不满足<<1 !!)

$IM_2 = ??$ 强烈非线性!!

二极管的非线性

$$i_D = I_S \exp\left(\frac{v_D}{kT/q}\right) \qquad y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots$$

$$y = \frac{i_D}{I_D} = u + \frac{u^2}{2} + \frac{u^3}{6} \qquad u = \frac{v_d}{kT/q},\ U = \frac{V_d}{kT/q}$$

二极管与单管共射放大器具有相同的非线性！

双极型差分对的非线性

$$y=\frac{i_{od}}{I_B}=\tanh\frac{v_{id}}{2kT/q}$$

$$\tanh x=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\approx x-\frac{1}{3}x^{3}$$

$$y=\frac{i_{od}}{I_B}=u-\frac{1}{3}u^{3}\qquad u=\frac{v_{id}}{2kT/q},\ U=\frac{V_{id}}{2kT/q}$$

$$\alpha_1=1$$

$$\alpha_2=0$$

$$\alpha_3=1/3$$

$$IM_2=0\qquad IM_3=\frac{1}{4}U^{2}\qquad IP_3=2kT/q$$

电阻和电容的非线性

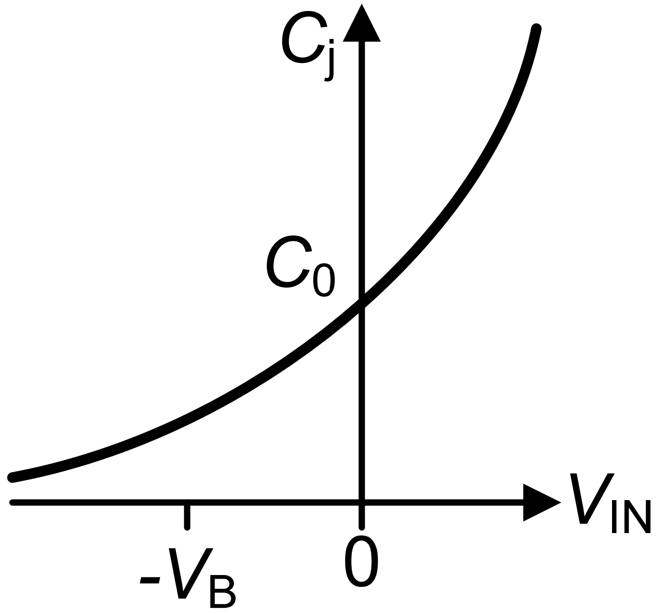
$R = R_0(1 + \alpha_1 V + \alpha_2 V^2 + ...)$ [$\approx$ JFET with large V_P]

For diffused resistors: $\alpha_1 \approx 5\ \text{ppm/V},\ \alpha_2 \approx 1\ \text{ppm/V}^2$

$C = C_0(1 + \alpha_1 V + \alpha_2 V^2 + ...)$

For poly-poly caps: $\alpha_1 \approx 20\ \text{ppm/V},\ \alpha_2 \approx 2\ \text{ppm/V}^2$

耗尽区电容的非线性



$$C_j = \frac{C_0}{\sqrt{1-\frac{v_{IN}}{\Phi}}} \qquad v_{IN} = -V_B + v_{in}$$

$$C_j = \frac{C_0}{\sqrt{1+\frac{V_B}{\Phi}}}\frac{1}{\sqrt{1+\frac{v_{in}}{\underbrace{V_B+\Phi}_{x}}}}$$

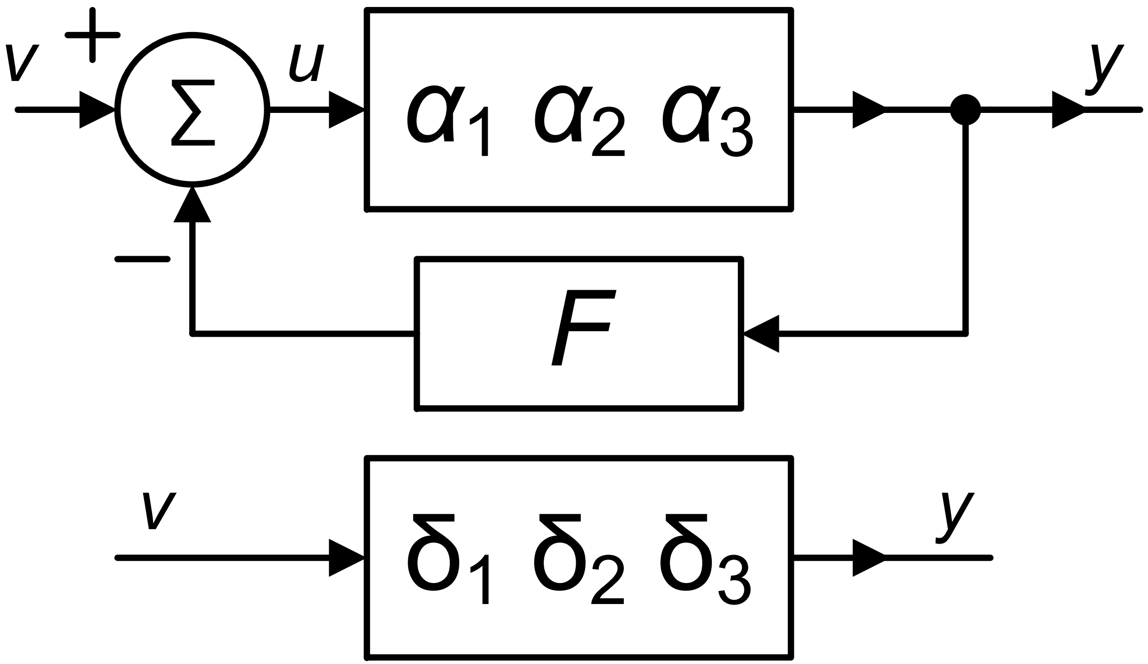
$$C_j = C_{0B}(1+x)^{-1/2} = C_{0B}(1-(1/2)x+(3/8)x^2-(5/16)x^3+\ldots)$$

目录

- 定义： *HD*, *IM*, 交调点
- MOS管的非线性
  - ❑单端放大器
  - ❑差分放大器
- 双极型晶体管的非线性
- 负反馈能够减小非线性
- 运算放大器的非线性
- 其他非线性和准则

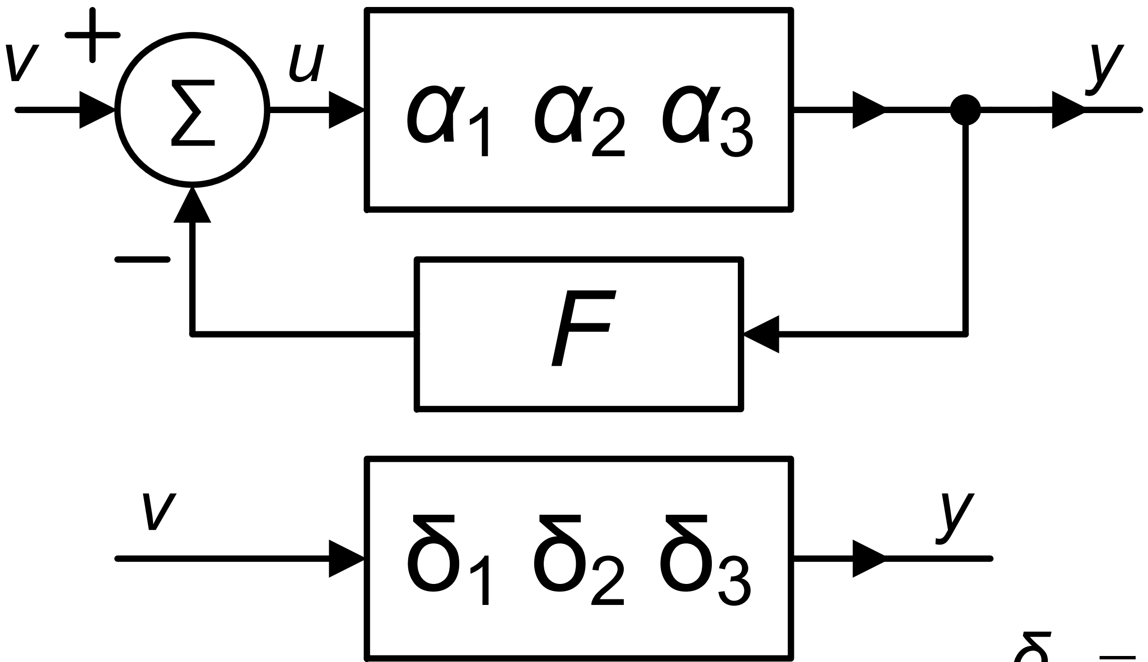
Ref.: W. Sansen : Analog Design Essentials, Springer 2006

负反馈减小非线性−1



$$\left.\left.\begin{array}{l} u = v - Fy \\ y = \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 \end{array}\right\} \text{elim. u} \atop y = \delta_1 v + \delta_2 v^2 + \delta_3 v^3 \right\} \text{elim. y} \begin{cases} \text{coeff. v} & : \delta_1 \\ \text{coeff. } v^2 & : \delta_2 \\ \text{coeff. } v^3 & : \delta_3 \end{cases}$$

负反馈减小非线性-2



$$u = v - Fy$$

$$\delta_1 = \frac{\alpha_1}{1+T} \approx \frac{1}{F}$$

$$\delta_2 = \frac{\alpha_2}{(1+T)^3}$$

$$\delta_3 = \frac{\alpha_3(1+T) - 2F\alpha_2^2}{(1+T)^5}$$

环路增益 $1+T = 1+\alpha_1 F$ $\quad u \approx \dfrac{v}{1+T}$

v的幅度减小了环路增益倍

负反馈系统的非线性

$$IM_{2f}=\frac{\delta_2}{\delta_1}V=\frac{\alpha_2}{\alpha_1}\frac{V}{(1+T)^2}=\frac{\alpha_2}{\alpha_1}\underbrace{\frac{1}{1+T}}_{\text{reduction by Loop Gain}}\underbrace{\frac{V}{1+T}}_{\text{reduction in voltage swing}}$$

$$IM_{3f}=\frac{3}{4}\frac{\delta_3}{\delta_1}V^2=\frac{3}{4}\left[\frac{\alpha_3}{\alpha_1}\frac{1}{1+T}-\left(\frac{\alpha_2}{\alpha_1}\right)^2\frac{2T}{(1+T)^2}\right]\underbrace{\frac{V^2}{(1+T)^2}}_{\text{reduction in voltage swing}}$$

compression　　expansion

例子：IM_{3f}

$$IM_{3f} = \frac{3}{4}\frac{\delta_3}{\delta_1}V^2 = \frac{3}{4}\underbrace{\left[\frac{\alpha_3}{\alpha_1}\frac{1}{1+T} - \left(\frac{\alpha_2}{\alpha_1}\right)^2\frac{2T}{(1+T)^2}\right]}\frac{V^2}{(1+T)^2}$$

$$\text{For large } T\text{: } \frac{\alpha_3\alpha_1 - 2\alpha_2^2}{\alpha_1^2}\frac{1}{T} = \frac{\alpha_3}{\alpha_1}\left(1 - \frac{2\alpha_2^2}{\alpha_1\alpha_3}\right)\frac{1}{T}$$

MOST: $\alpha_3 = 0,\ \ \alpha_2$ dominant

Bipolar: $\alpha_1 = 1,\ \ \alpha_2 = 1/2,\ \ \alpha_3 = 1/6,\ \ \alpha_2$ dominant

Diff. pair: $\alpha_2 = 0,\ \ \alpha_3$ dominant

射极电阻减小非线性IM_{2f}

$$T = g_m R_E = \frac{V_{RE}}{kT/q},\quad V_{RE} \approx I_{CE} R_E \qquad \frac{\alpha_2}{\alpha_1} = \frac{1}{2}$$

$$IM_{2f} = \frac{1}{2}\frac{1}{(1+T)^2}\frac{V_{in}}{kT/q} = \frac{1}{2}\frac{1}{1+T}\left(\frac{1}{1+T}\frac{V_{in}}{kT/q}\right)$$

$$= \frac{1}{1+T}\frac{U}{2}$$

$$U = \frac{1}{1+T}\frac{V_{in}}{kT/q}$$

IM_{2f}随着T的增大而线性减小！

射极电阻减小非线性IM_{3f}

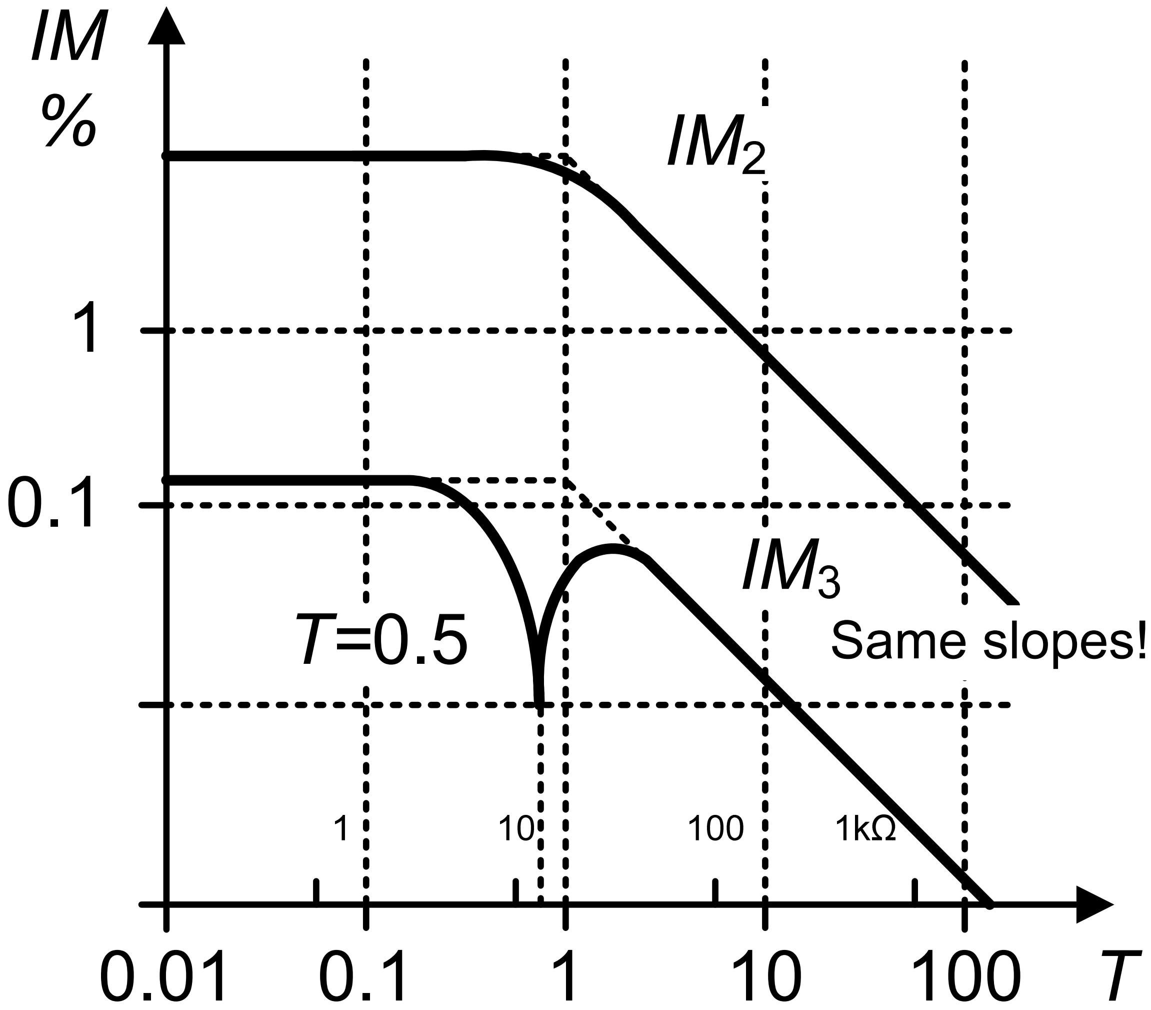
$$IM_{3f}=\frac{1-2T}{(1+T)^2}\frac{U^2}{8}\qquad \frac{\alpha_2}{\alpha_1}=\frac{1}{2},\ \frac{\alpha_3}{\alpha_1}=\frac{1}{6}$$

$$U=\frac{1}{1+T}\frac{V_{in}}{kT/q}$$

$T = 0.5$时，$IM_{3f} = 0$!!!

IM_{2f}随着T的增大而减小！

调整R_E使得IM_3=0 (BJT: I_{CE} = 1mA)



$$IM_3 = 0$$

$$\alpha_3(1+T) = 2F\alpha_2^2$$

$$\alpha_3(1+T) = 2T\frac{\alpha_2^2}{\alpha_1}$$

$$T = \frac{1}{\frac{2\alpha_2^2}{\alpha_1\alpha_3} - 1} = 0.5$$

$$U = \frac{1}{1+T}\frac{V_{in}}{kT/q}$$

$$= \text{const.}$$

射极电阻R_E减小非线性

$$U=\frac{1}{1+T}\frac{V_{in}}{kT/q}\approx\frac{V_{in}}{R_E I_{CE}} \qquad T=g_m R_E=\frac{R_E I_{CE}}{kT/q}$$

$$IM_{2f}=\frac{U}{2}\frac{1}{1+T}=\frac{V_{in}}{kT/q}\frac{1}{2(1+T)^2}\approx\frac{V_{in}\,kT/q}{2(R_E I_{CE})^2}$$

$$IM_{3f}=\frac{U^2}{4}\frac{1}{1+T}=\left(\frac{V_{in}}{kT/q}\right)^2\frac{1}{4(1+T)^3}\approx\frac{V_{in}^2\,kT/q}{4(R_E I_{CE})^3}$$

源极电阻R_S减小非线性

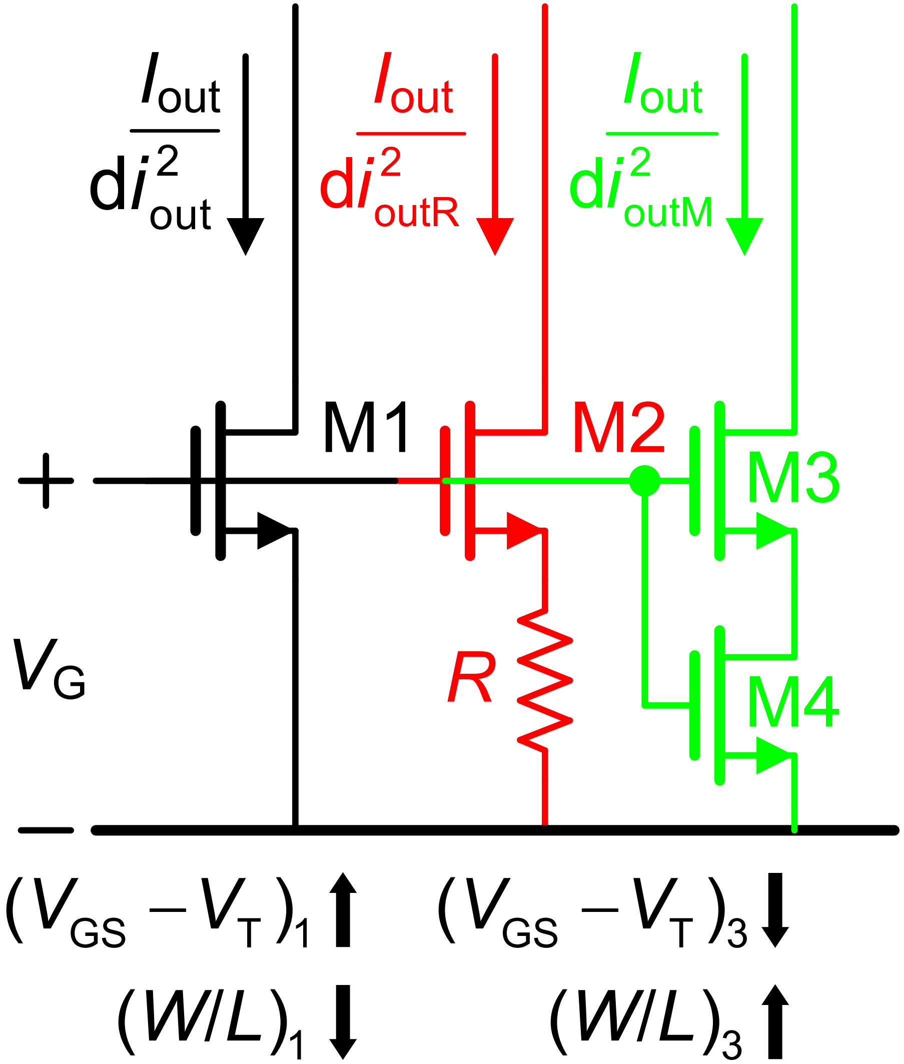
$$T = g_m R_S = \frac{R_S I_{DS}}{(V_{GS} - V_T)/2} \qquad \frac{\alpha_2}{\alpha_1} = \frac{1}{4},\ \alpha_3 = 0$$

$$U = \frac{1}{1+T} \frac{V_{in}}{(V_{GS} - V_T)/2}$$

$$IM_{2f} = \frac{U}{4} \frac{1}{1+T} = \frac{V_{in}}{(V_{GS} - V_T)/2} \frac{1}{4(1+T)^2} \approx \frac{V_{in}(V_{GS} - V_T)/2}{4(R_S I_{DS})^2}$$

$$IM_{3f} = \frac{3U^2}{32} \frac{1}{(1+T)^2} = \frac{V_{in}^2}{(V_{GS} - V_T)^2/4} \frac{3}{32(1+T)^3} \approx \frac{3V_{in}^2(V_{GS} - V_T)/2}{32(R_S I_{DS})^3}$$

源极接串联电阻的电流镜



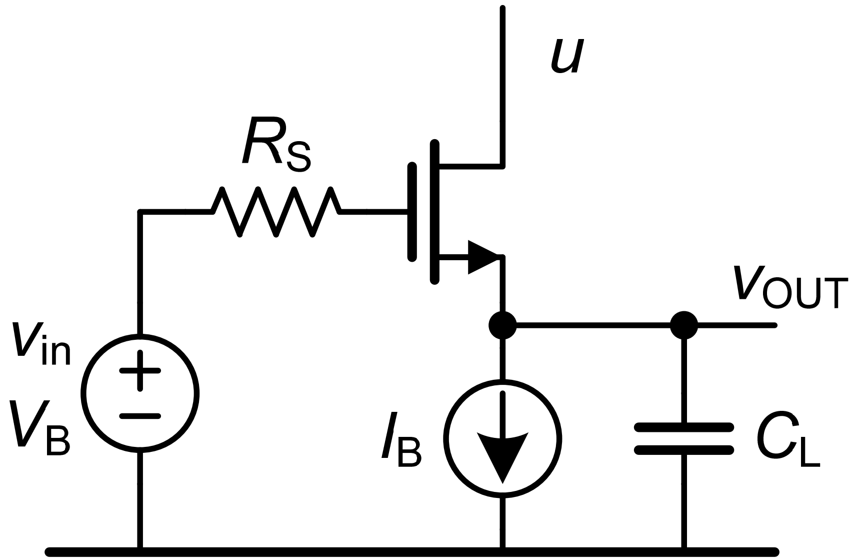
相同的 I_{out} 和相同的 V_G：

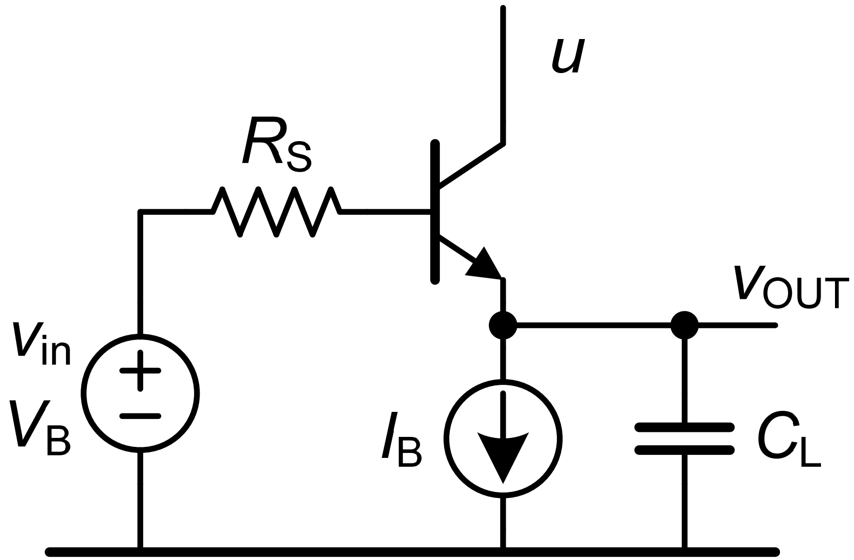
相同的增益！
相同的输出噪声！

相同的非线性？

$$\frac{IM_{2f}}{IM_2} = \frac{1-\dfrac{V_R}{V_{GST1}}}{\left(1+\dfrac{V_R}{V_{GST1}}\right)^2}$$

源极/射极跟随器



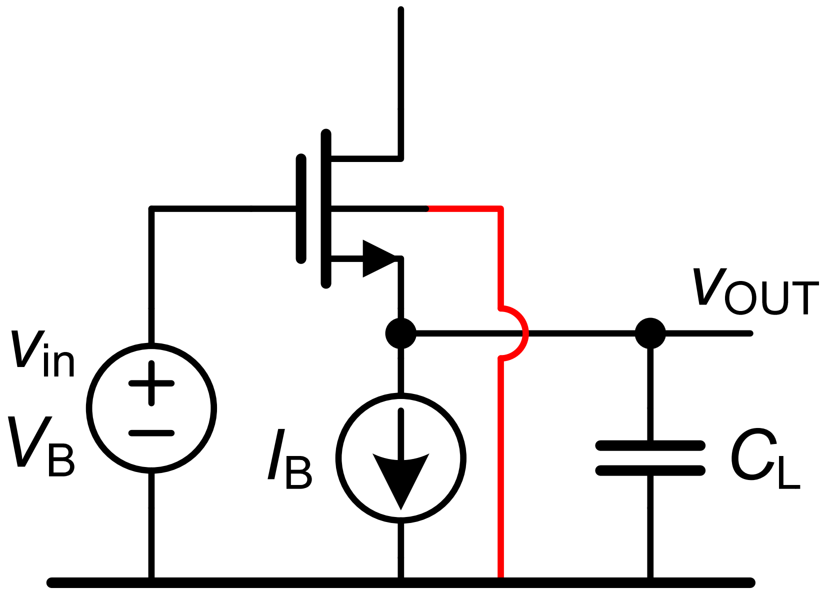


$$U=\frac{1}{g_m r_{DS}}\frac{V_{in}}{(V_{GS}-V_T)/2}=\frac{V_{in}}{V_{En}L}$$

$$U=\frac{1}{g_m r_o}\frac{V_{in}}{kT/q}=\frac{V_{in}}{V_E}$$

if $V_{BS}=0$

衬偏效应的源极跟随器的非线性



$$v_{OUT} = v_{IN} - v_{GS}$$

$$v_{GS} = V_T + \sqrt{\frac{I_B}{K' W/L}}$$

$$V_T = V_{T0} + \gamma\left[\sqrt{|2\Phi_F| + v_{OUT}} - \sqrt{|2\Phi_F|}\right]$$

$$v_{IN} = v_{OUT} + V_{T0} + \gamma\left[\sqrt{|2\Phi_F| + v_{OUT}} - \sqrt{|2\Phi_F|}\right] + \sqrt{\frac{I_B}{K' W/L}}$$

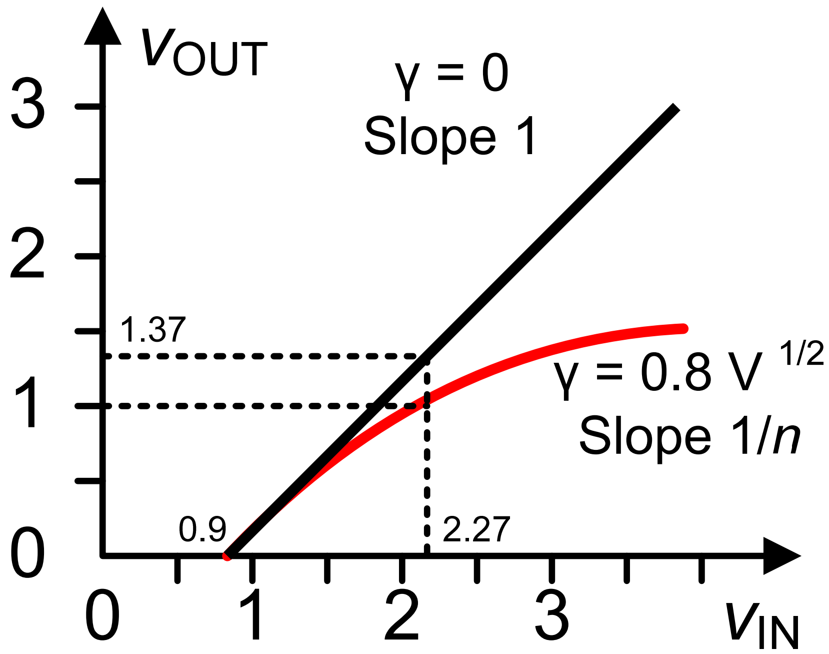
例子：源极跟随器的非线性

$$v_{IN} = u^2 + \gamma u + B$$

$$u^2 = v_{OUT} + |2\Phi_F|$$

$$B = v_{GS0} - |2\Phi_F| - \gamma\sqrt{|2\Phi_F|}$$

$$v_{GS0} = v_{T0} + \sqrt{\frac{I_B}{K' W/L}}$$

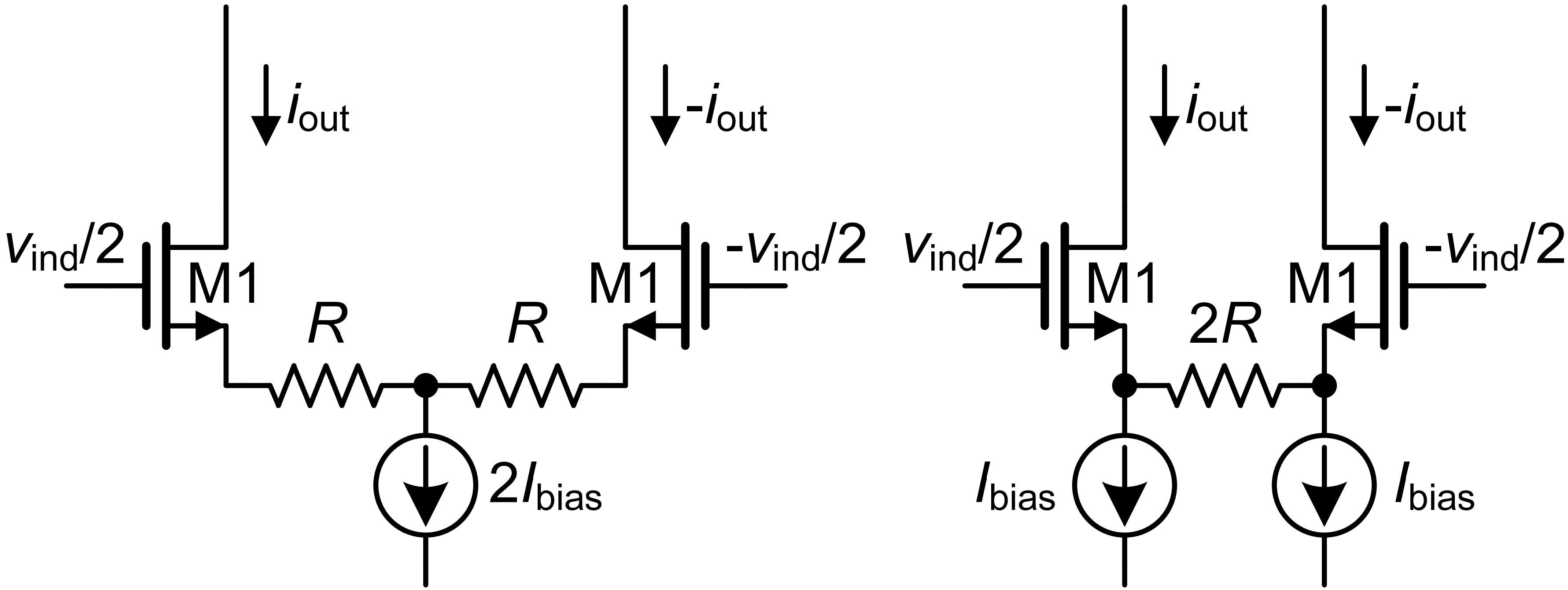


$$v_{T0} = 0.6\text{ V};\ v_{GS0} = 0.9\text{ V};\ 2\Phi_F = 0.7\text{ V};\ B = -0.47\text{ V};\ 1/n = 0.73$$

$$\alpha_1 = 0.765;\ \alpha_2 = 0.02;\ \alpha_3 = -0.0035$$

$$V_{inp} = 1\text{ V}_p;\ HD_2 = 1.32\%;\ HD_3 = -0.114\%$$

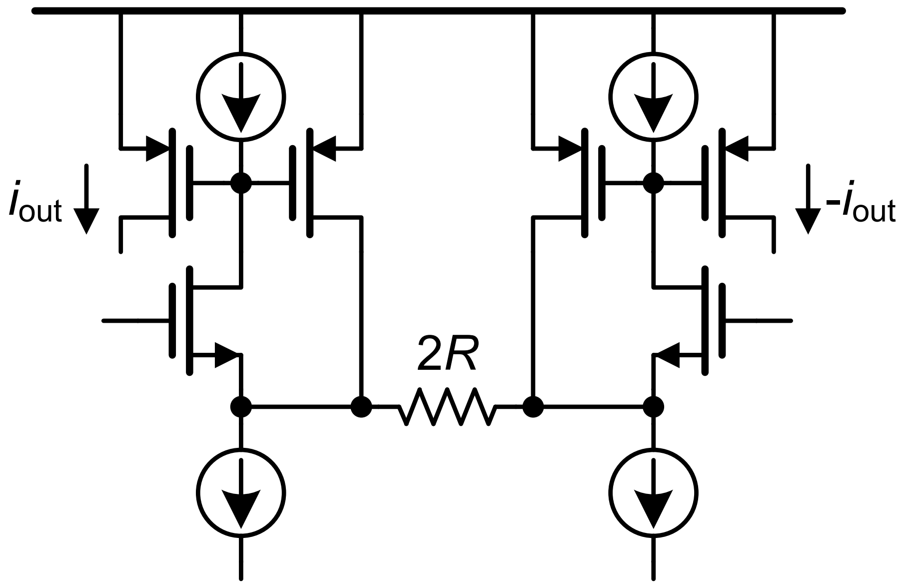
通过负反馈提高IP_3 –1



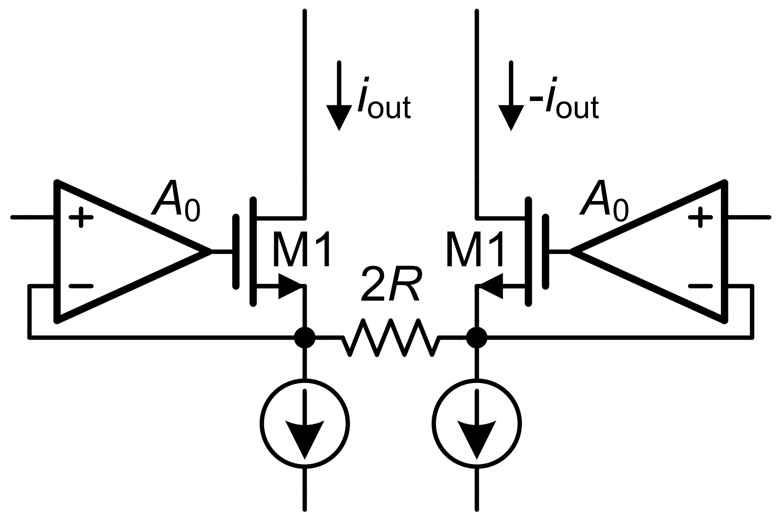
$$IP_3 \approx 3.3(V_{GS} - V_T)(1 + g_{m1}R)^{3/2} \qquad HD_3 = \frac{1}{32}\frac{U^2}{(1+g_{m1}R)^3}$$

$$HD_3 = -60\text{ dB and } V_{id} = 1 \rightarrow V_{GS} - V_T = 0.7\text{ V and } g_{m1}R = 3!!!$$

通过负反馈提高IP_3 −2

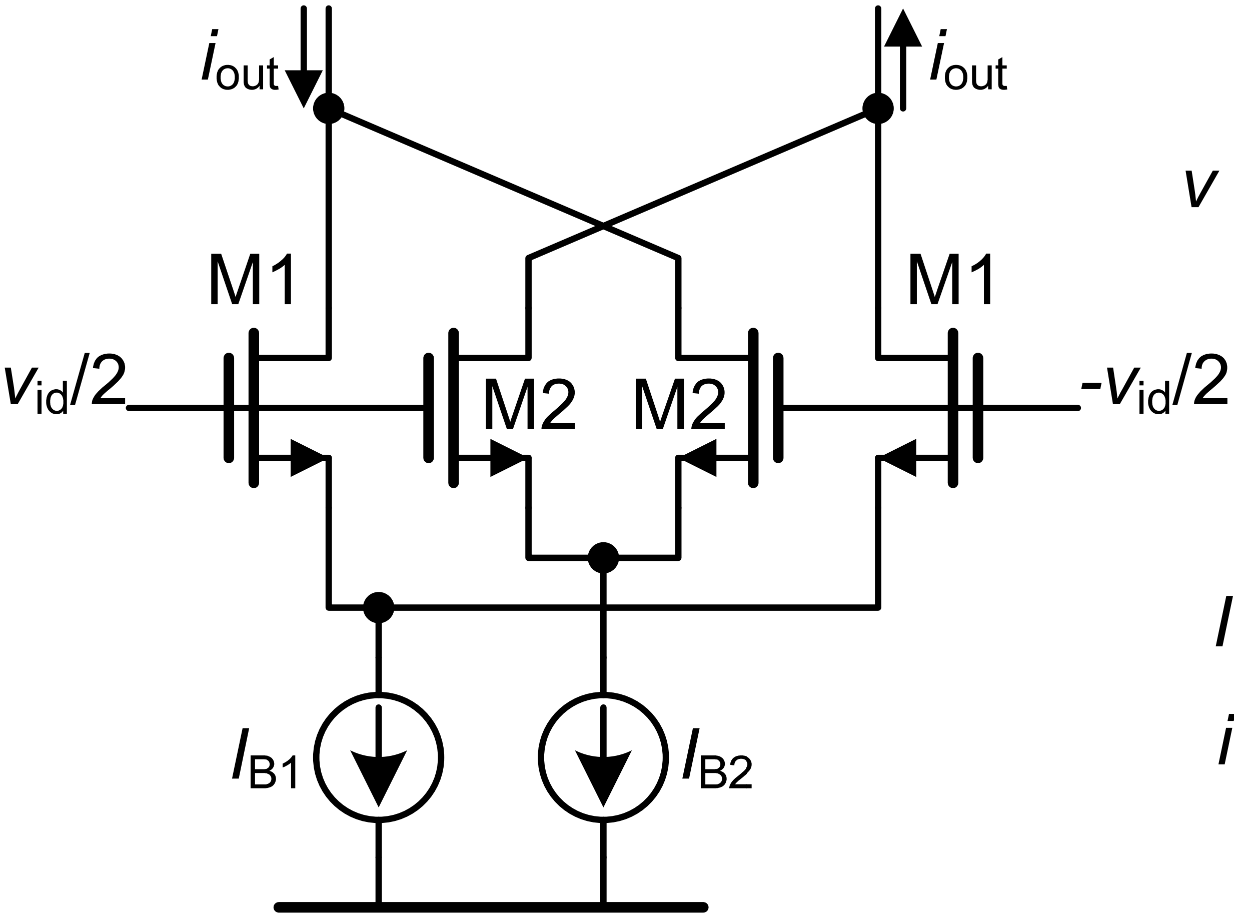


附加局部反馈



运放构建反馈

非线性抵消技术-1



$$\alpha = I_{B2}/I_{B1} \approx 0.25$$

$$v = V_{GST1}/V_{GST2} \approx 1.6$$

$$V_{GST} = V_{GS} - V_T$$

$$IM_3 = 0 \rightarrow v = \alpha^{-1/3},$$

$$i_{out} = g_{m1} v_{id} (1 - \alpha^{2/3})$$

非线性抵消技术-2

$$i_{out} = i_{od1} - i_{od2}$$

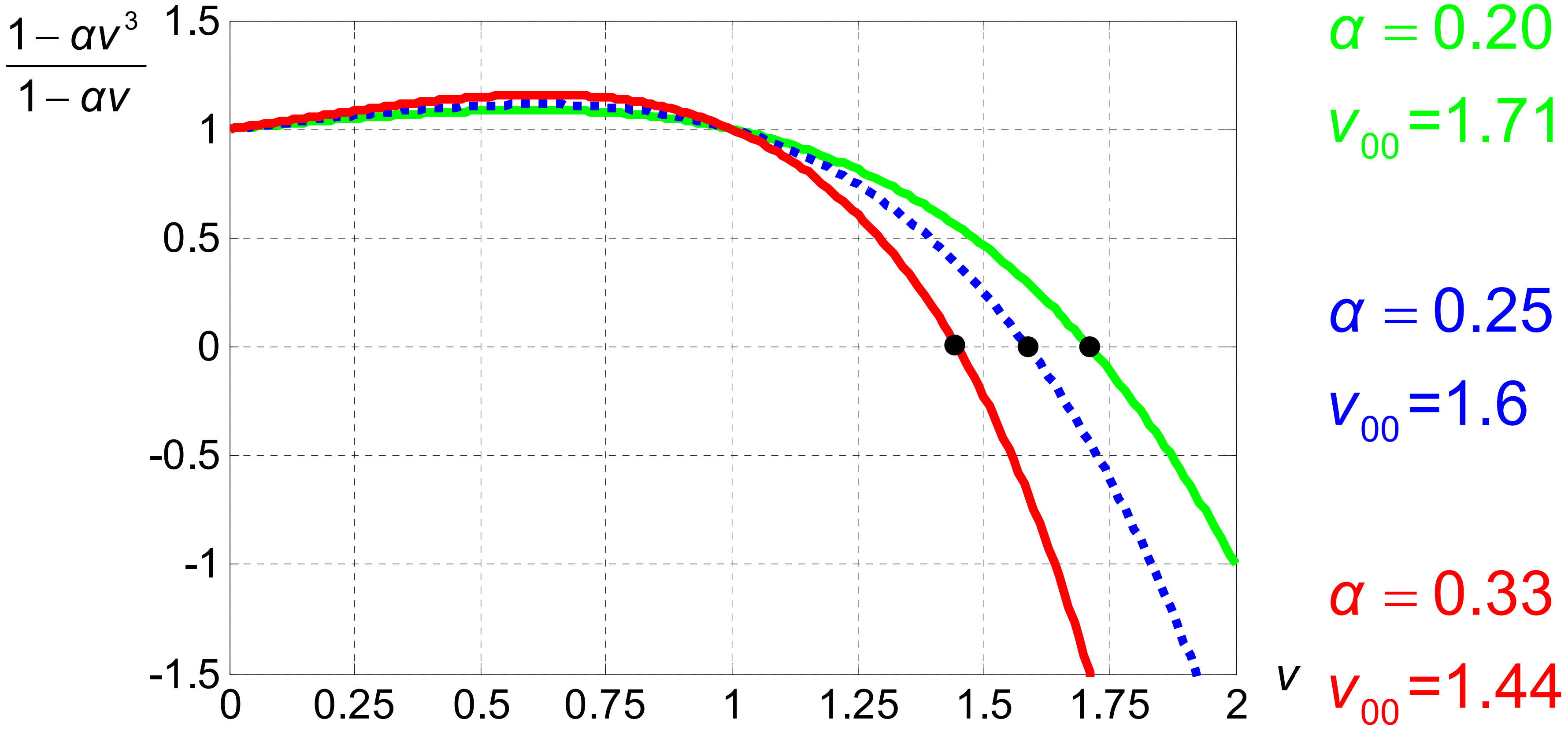
$$\frac{i_{od}}{I_B} = u - \frac{1}{8}u^2 \quad u = \frac{v_{id}}{V_{GS} - V_T},\ U = \frac{V_{id}}{V_{GS} - V_T} \quad IM_3 = \frac{3}{32}U^2$$

$$IM_{3,dc} = \frac{3}{32}\left(\frac{V_{id}}{V_{GS} - V_T}\right)^2 \frac{1-\alpha v^3}{1-\alpha v}$$

$$IM_{3,dc} = 0 \quad \rightarrow$$

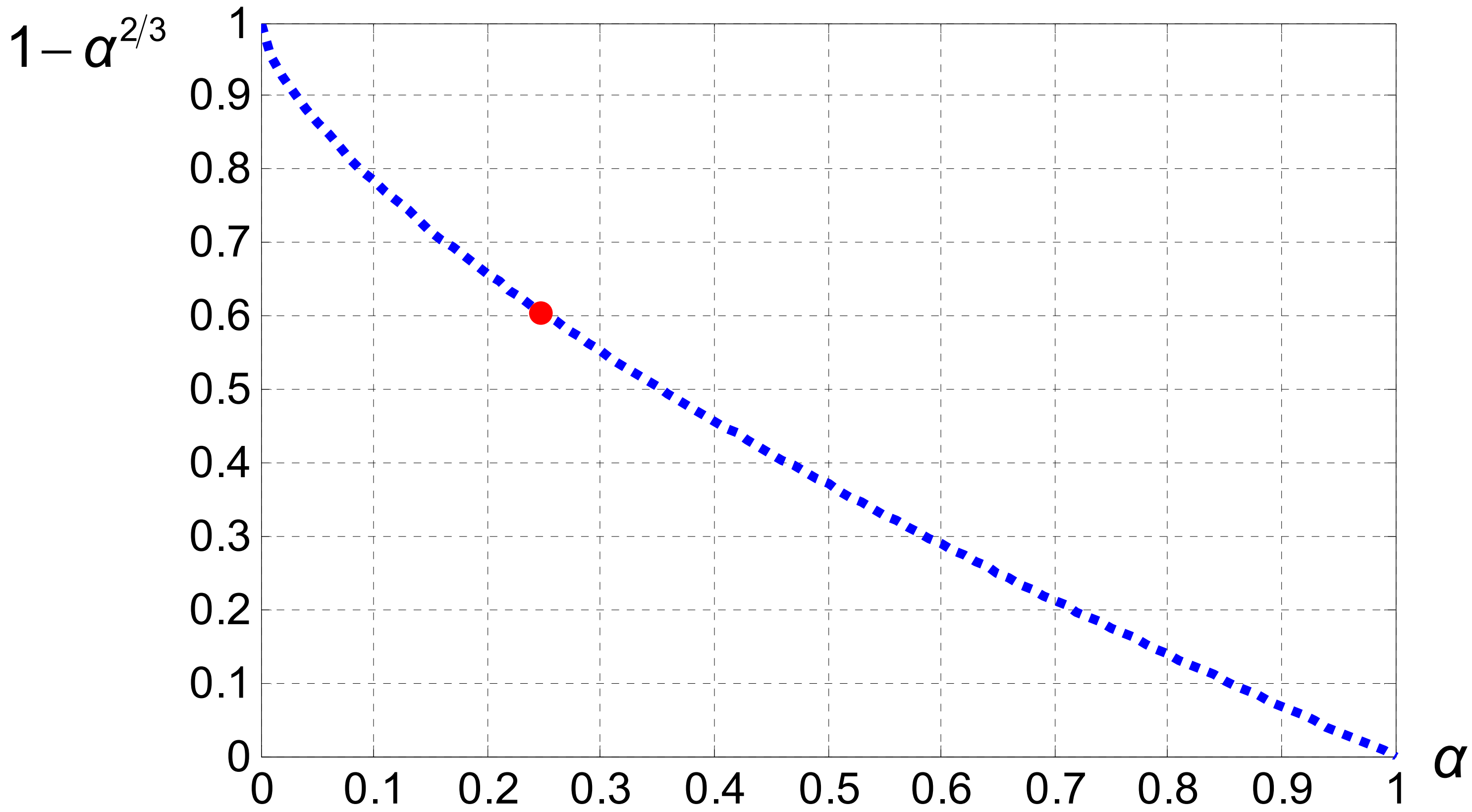
$$v_{00} = \alpha^{-1/3} \quad \& \quad i_{out} = g_{m1} v_{id} (1 - \alpha^{2/3})$$

IM_3的抵消系数



$\frac{1-\alpha v^3}{1-\alpha v}$
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
v
$\alpha = 0.20$
v_{00}=1.71
$\alpha = 0.25$
v_{00}=1.6
$\alpha = 0.33$
v_{00}=1.44

有效跨导减小



$\alpha = 0.25$

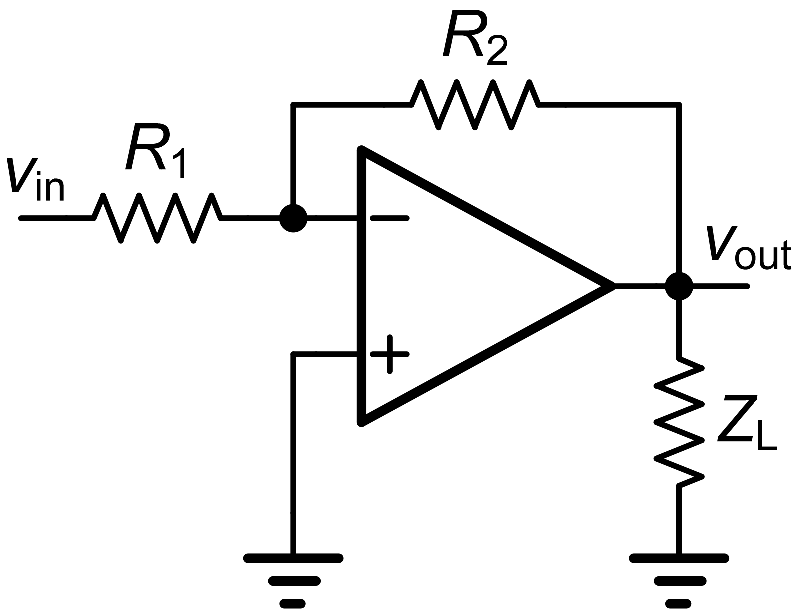
v_{00}=1.6

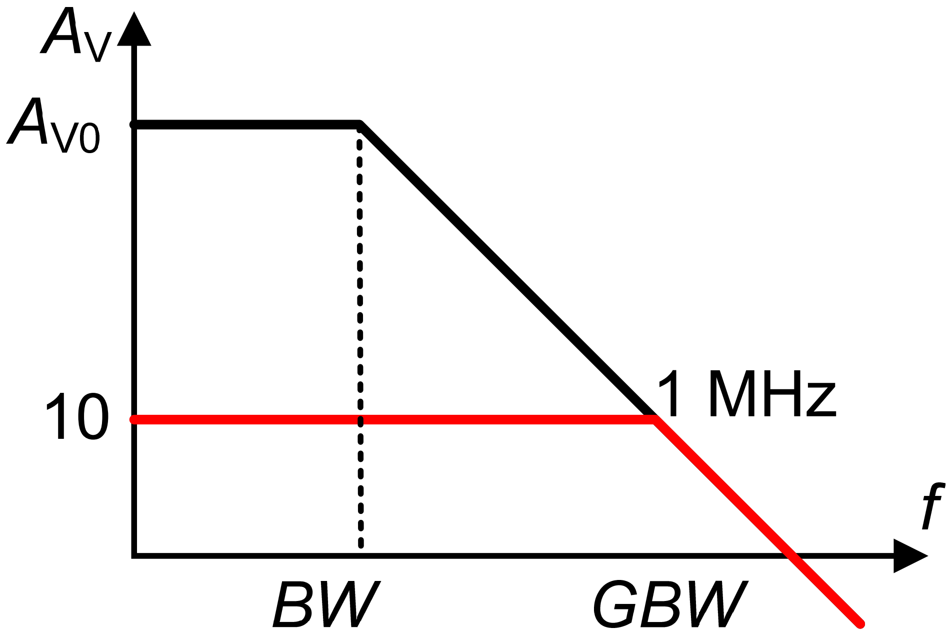
$\times$0.6

目录

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

带负反馈的密勒CMOS运算放大器

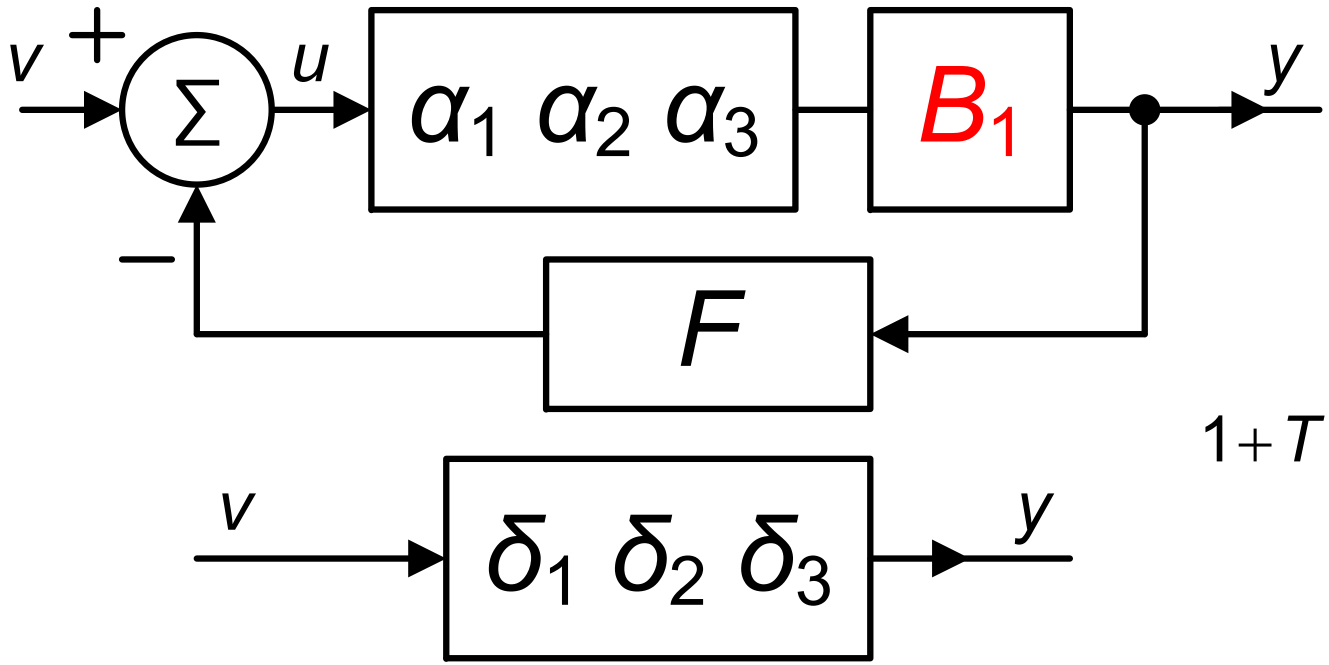




$$GBW = 10\ \text{MHz} \ \ \& \ A_{VC} = 10$$

$$Z_L = 100\ \text{k}\Omega\ //\ 5\ \text{pF}$$

输入级的非线性-1



$$1+T=1+B_1\alpha_1 F$$

$$\left.\begin{array}{l} u = v - Fy \\ y = B_1(\alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3) \end{array}\right\} \text{elim. u}$$

$$y = \delta_1 v + \delta_2 v^2 + \delta_3 v^3$$

elim. y $\begin{cases} \text{coeff. v} : \delta_1 \\ \text{coeff. v}^2 : \delta_2 \\ \text{coeff. v}^3 : \delta_3 \end{cases}$

输入级的非线性-2

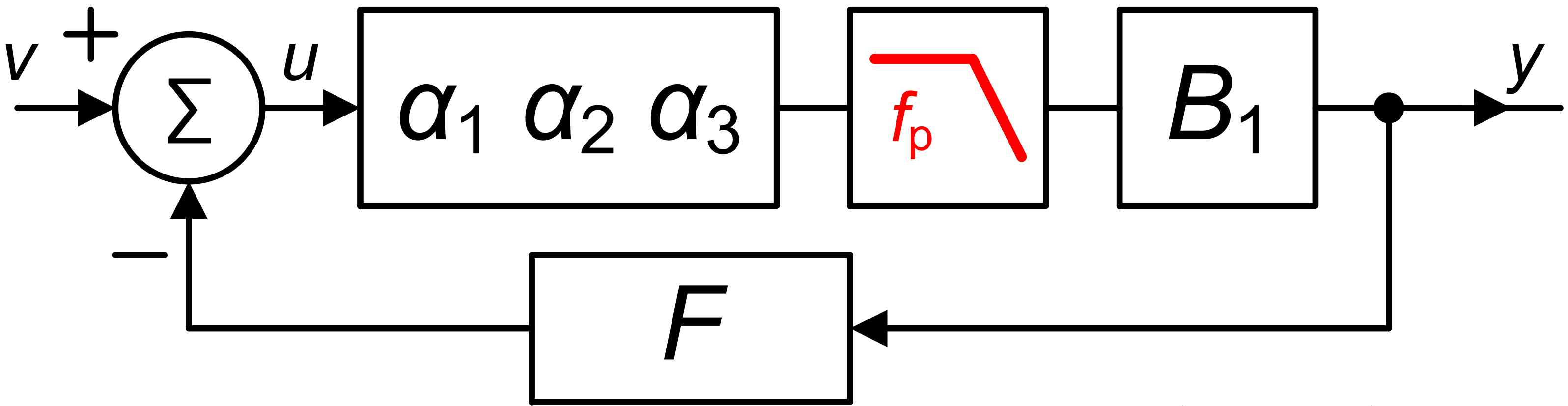
$$IM_{2f}=\frac{\delta_2}{\delta_1}V=\frac{\alpha_2}{\alpha_1}\frac{V}{(1+T)^2}=\frac{\alpha_2}{\alpha_1}\frac{1}{1+T}\frac{V}{1+T}$$

$$IM_{3f}=\frac{3}{4}\frac{\delta_3}{\delta_1}V^2=\frac{3}{4}\left[\frac{\alpha_3}{\alpha_1}\frac{1}{1+T}-(\frac{\alpha_2}{\alpha_1})^2\frac{2T}{(1+T)^2}\right]\frac{V^2}{(1+T)^2}$$

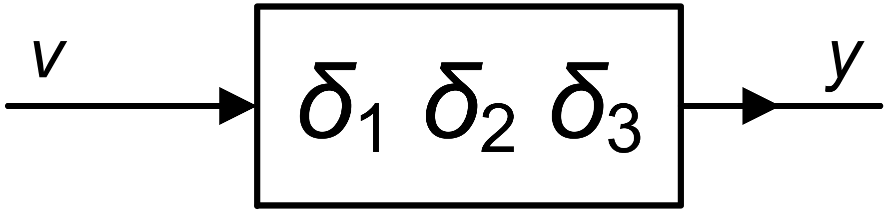
只是环路增益不同，其他未发生变化！

$$1+T=1+B_1\alpha_1 F$$

具有低通滤波输入级的非线性-1



$$1+T=1+B_{1p}\alpha_1 F$$



$$u=v-Fy$$

$$y=B_{1p}(\alpha_1 u+\alpha_2 u^2+\alpha_3 u^3)$$

elim. u

$$y=\delta_1 v+\delta_2 v^2+\delta_3 v^3$$

elim. y

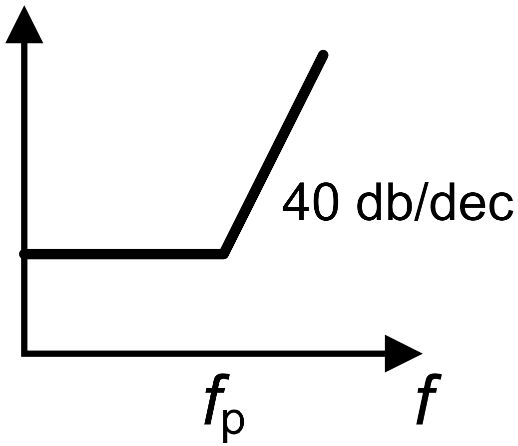
coeff. v : δ_1

coeff. v^2 : δ_2

coeff. v^3 : δ_3

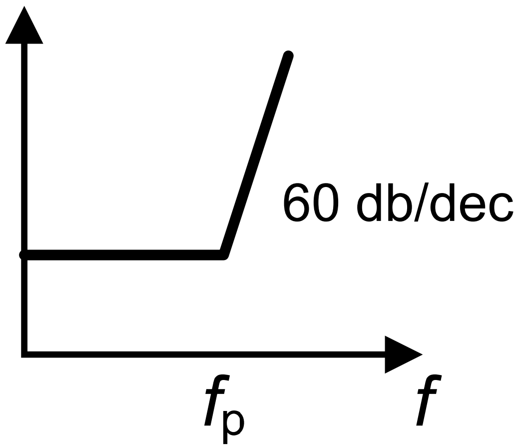
具有低通滤波输入级的非线性-2

$$IM_{2f} = \frac{\alpha_2}{\alpha_1}\frac{V}{(1+T)^2} \approx \frac{\alpha_2}{\alpha_1}\frac{V}{(B_{1p}\alpha_1 F)^2}$$

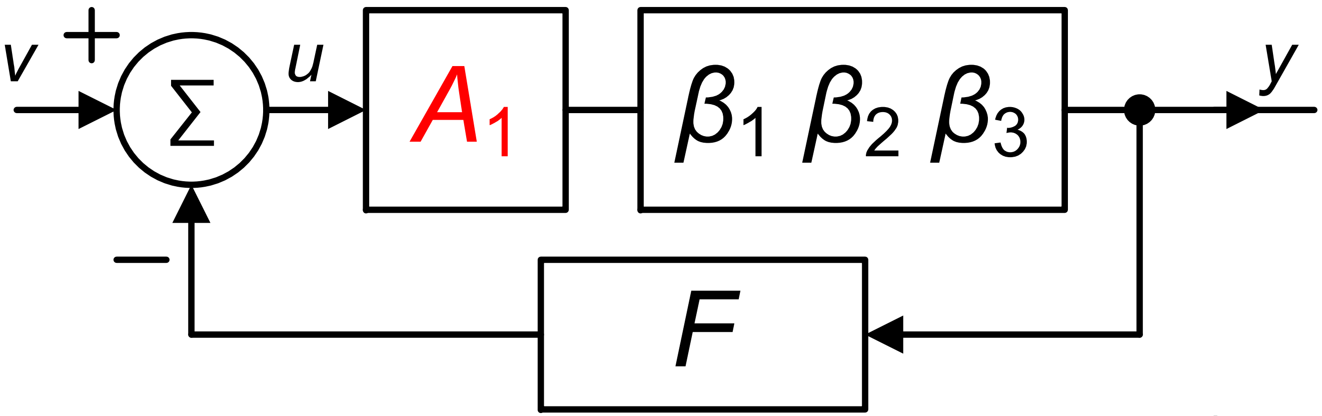


$$IM_{3f}\Big|_{\text{diff. pair}} = \frac{3}{4}\frac{\alpha_3}{\alpha_1}\frac{V^2}{(1+T)^3} \approx \frac{3}{4}\frac{\alpha_3}{\alpha_1}\frac{V^2}{(B_{1p}\alpha_1 F)^3}$$

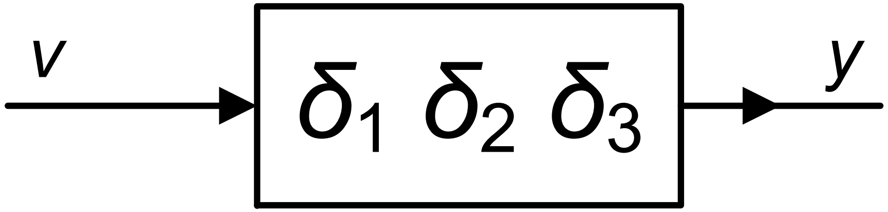
$$IM_{3f}\Big|_{\text{single trans.}} \approx \frac{3}{4}\frac{\alpha_2^2}{\alpha_1^2}\frac{2V^2}{(B_{1p}\alpha_1 F)^3}$$



输出级的非线性-1



$$1+T=1+A_1\beta_1F$$



$$u=v-Fy$$

$$y=A_1\beta_1u+A_1^2\beta_2u^2+A_1^3\beta_3u^3$$

} elim. u

$$y=\delta_1v+\delta_2v^2+\delta_3v^3$$

} elim. y

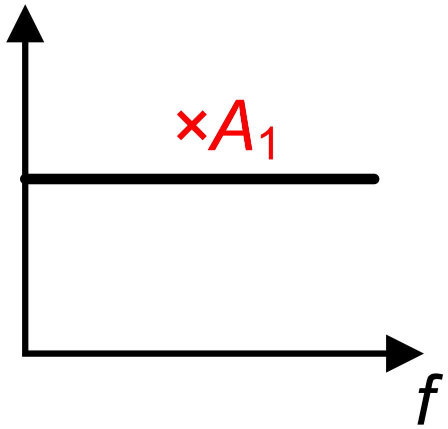
coeff. v : δ_1

coeff. v^2 : δ_2

coeff. v^3 : δ_3

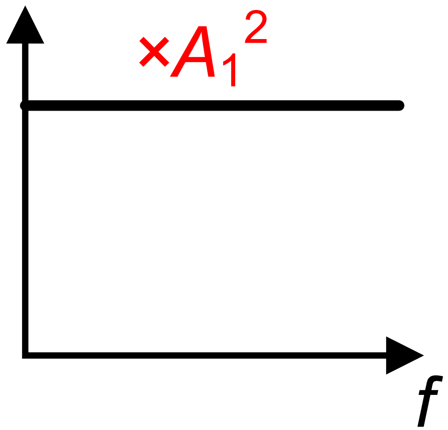
输出级的非线性-2

$$IM_{2f} = \frac{\beta_2}{\beta_1}\frac{A_1 V}{(1+T)^2} \approx \frac{\beta_2}{\beta_1}\frac{A_1}{(A_1\beta_1 F)^2}V$$



$$IM_{3f}\Big|_{\text{diff. pair}} = \frac{3}{4}\frac{\beta_3}{\beta_1}\frac{(A_1 V)^2}{(1+T)^3} \approx \frac{3}{4}\frac{\beta_3}{\beta_1}\frac{A_1^2}{(A_1\beta_1 F)^3}V^2$$

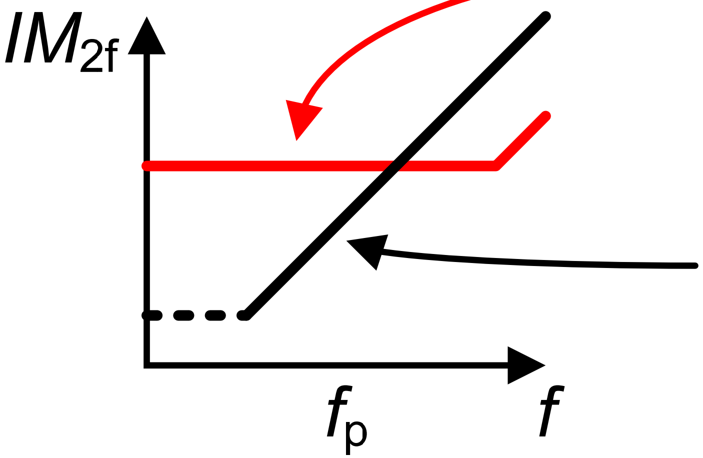
$$IM_{3f}\Big|_{\text{single trans.}} \approx \frac{3}{4}\frac{\beta_2^2}{\beta_1^2}\frac{2A_1^2}{(A_1\beta_1 F)^3}V^2$$



两级运算放大器：α & β

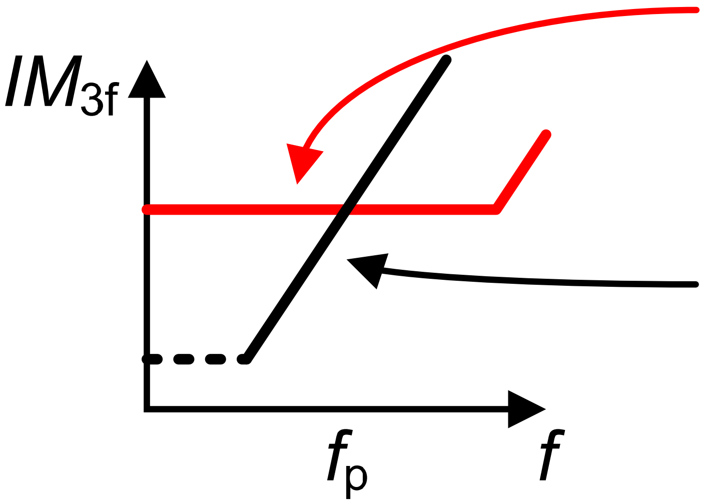
$$T=\alpha_1\beta_1 F$$

$$\frac{\beta_2}{\beta_1}\alpha_1\frac{V}{T^2}$$



$$\frac{\alpha_2}{\alpha_1}\frac{V}{T^2}(\frac{f}{f_p})^2$$

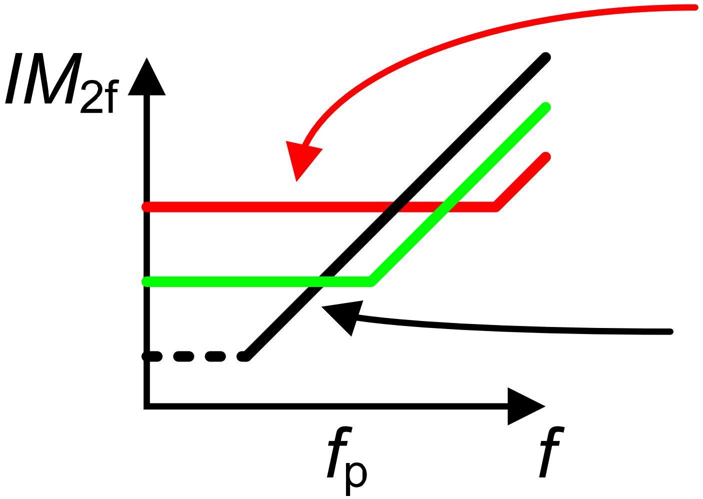
$$\frac{3}{4}\frac{\beta_3}{\beta_1}\alpha_1^2\frac{V^2}{T^3}(\frac{\beta_1\beta_3-2\beta_2^2}{\beta_1\beta_3})$$



$$\frac{3}{4}\frac{\alpha_3}{\alpha_1}\frac{V^2}{T^3}(\frac{f}{f_p})^3(\frac{\alpha_1\alpha_3-2\alpha_2^2}{\alpha_1\alpha_3})$$

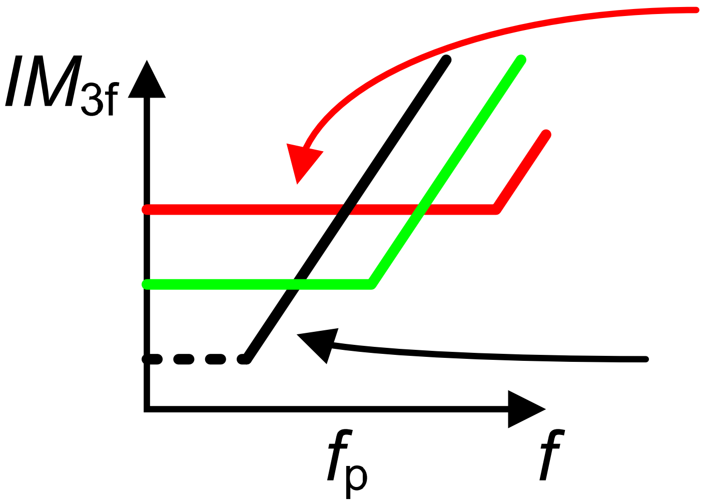
三级运算放大器：α & β & γ

$$T = \alpha_1 \beta_1 \gamma_1 F$$



$$\frac{\gamma_2}{\gamma_1}\alpha_1\beta_1\frac{V}{T^2}$$

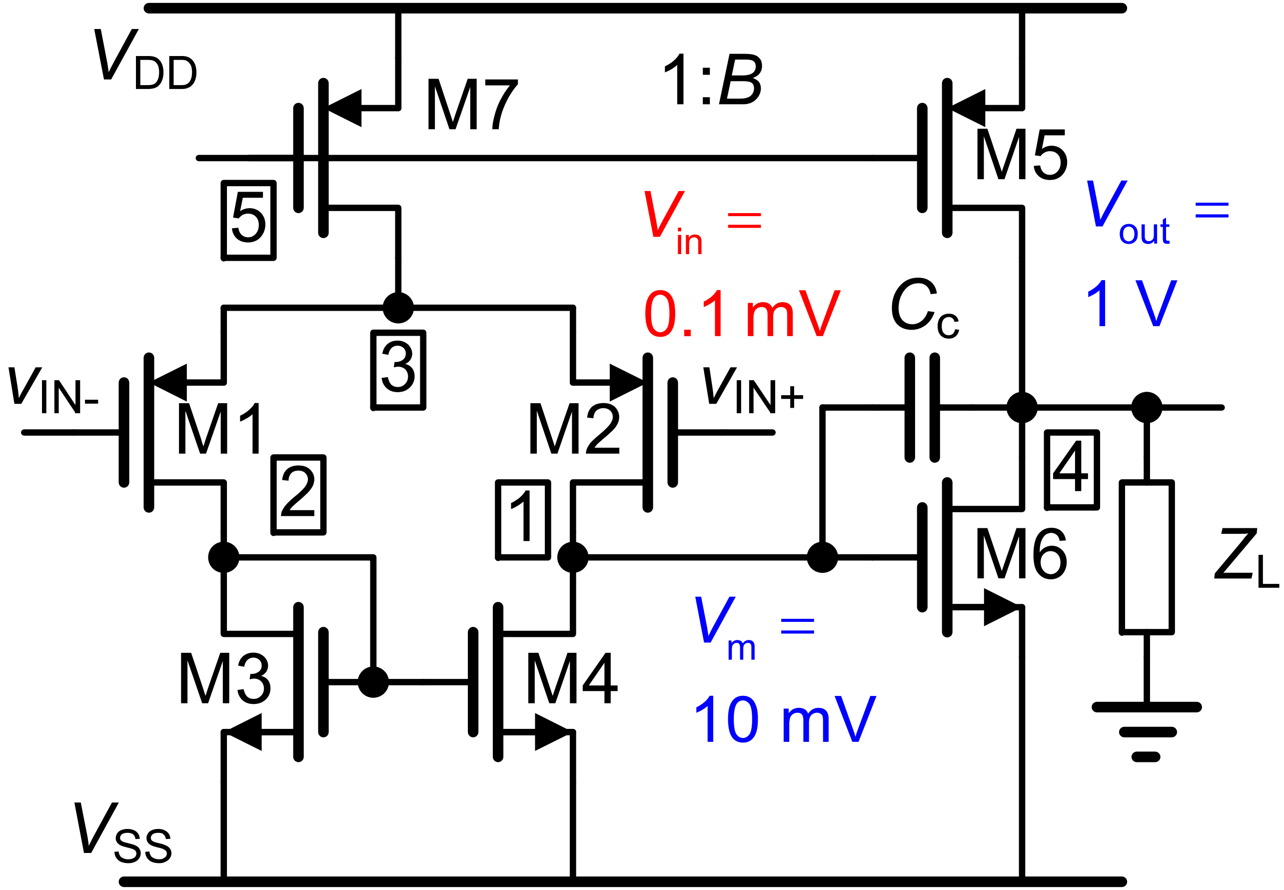
$$\frac{\alpha_2}{\alpha_1}\frac{V}{T^2}\left(\frac{f}{f_p}\right)^2$$



$$\frac{3}{4}\frac{\gamma_3}{\gamma_1}\alpha_1^2\beta_1^2\frac{V^2}{T^3}\left(\frac{\gamma_1\gamma_3 - 2\gamma_2^2}{\gamma_1\gamma_3}\right)$$

$$\frac{3}{4}\frac{\alpha_3}{\alpha_1}\frac{V^2}{T^3}\left(\frac{f}{f_p}\right)^3\left(\frac{\alpha_1\alpha_3 - 2\alpha_2^2}{\alpha_1\alpha_3}\right)$$

两级运放的非线性



$GBW = 10\ \text{MHz}$

$A_{V0} = 10{,}000$

$BW = 1\ \text{kHz}$

$A_{V0} = 10$

$I_{DS1} = 6\ \mu A$

$g_{m1} = 60\ \mu S$

$I_{DS6} = 120\ \mu A$

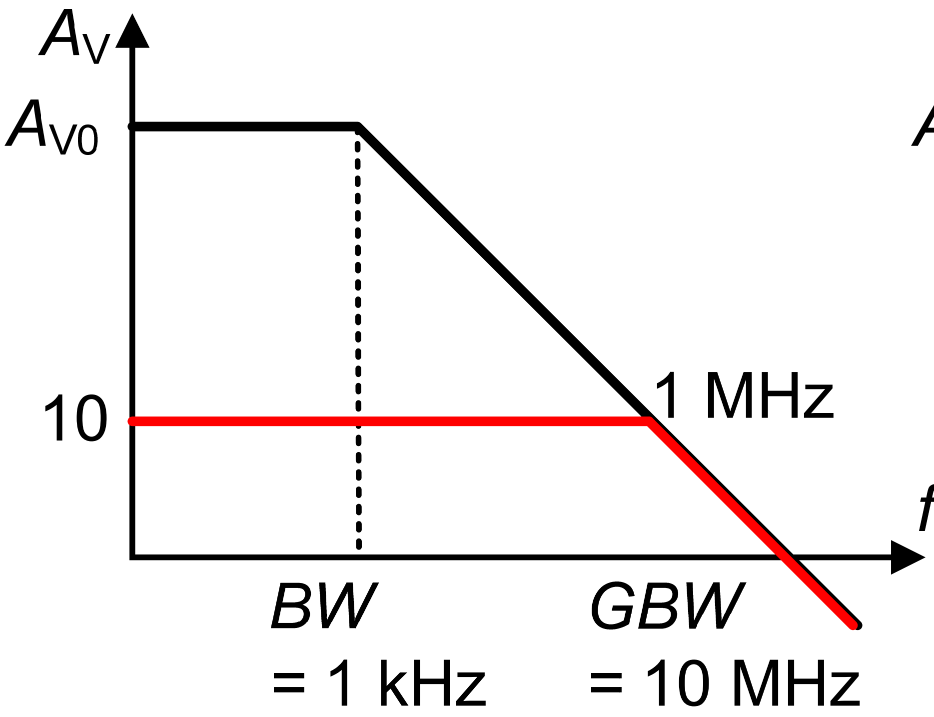
$g_{m6} = 1.2\ mS$

$R_L = 100\ k\Omega$

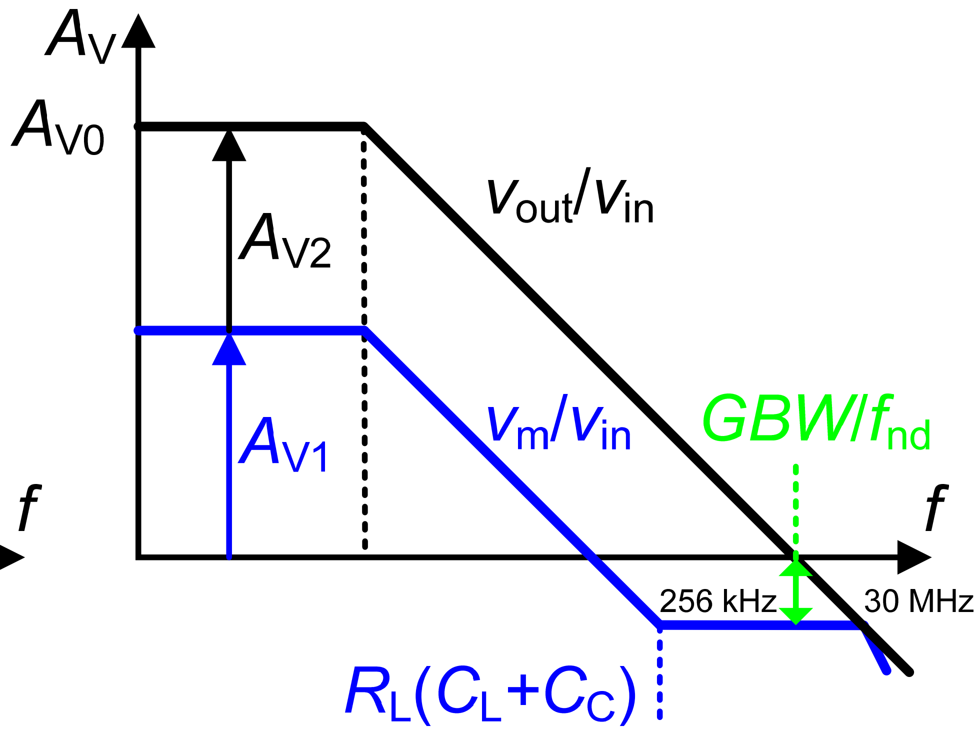
$C_L = 5\ pF$

$C_c = 1\ pF$

两级运放的频响特性

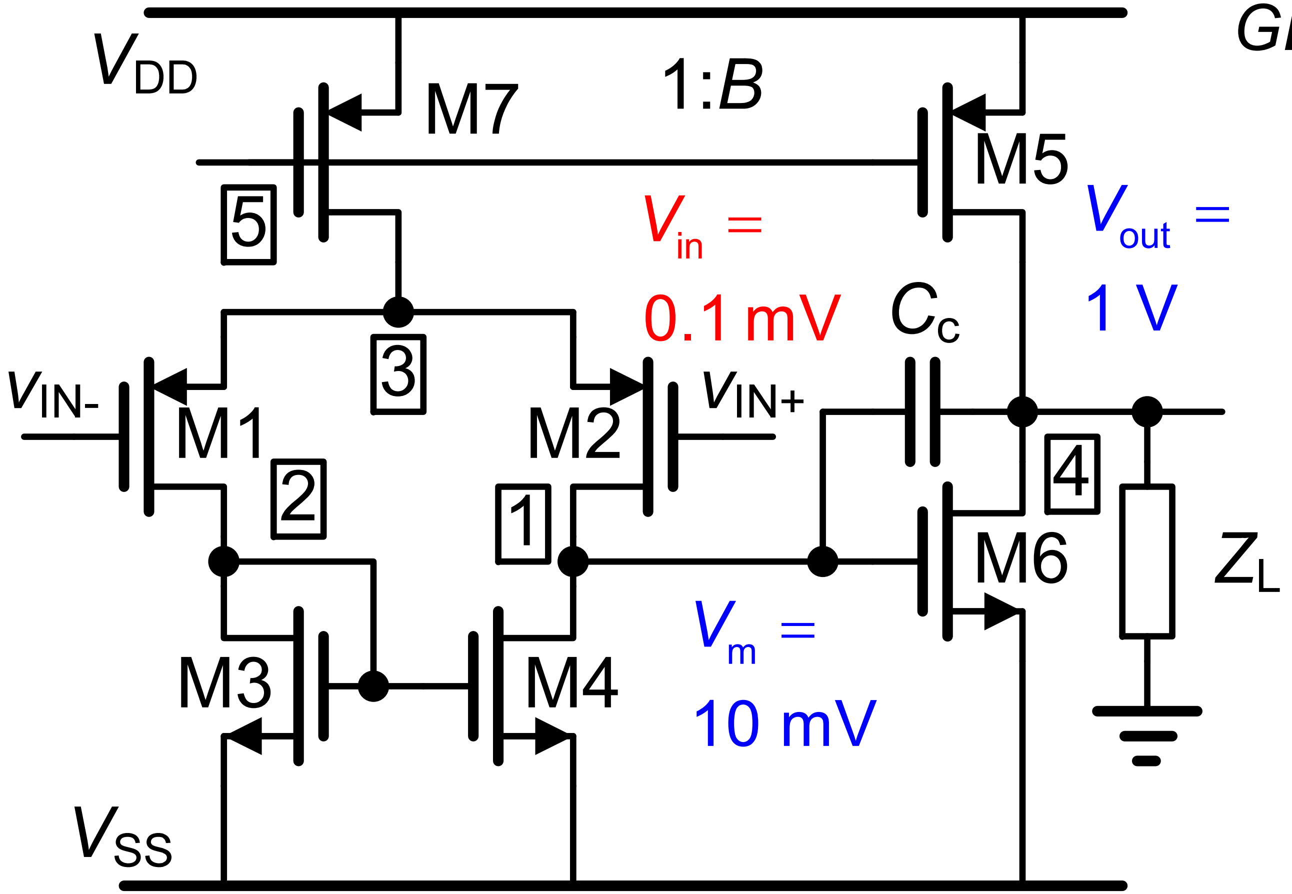


A_V
A_{V0}
10
1 MHz
f
BW
= 1 kHz
GBW
= 10 MHz



A_V
A_{V0}
A_{V2}
A_{V1}
v_{out}/v_{in}
v_m/v_{in}
GBW/f_{nd}
f
256 kHz
30 MHz
$R_L(C_L+C_C)$

两级运放的低频非线性–1



$$GBW = 10\ \text{MHz}$$

$$A_{V0} = 10$$

$$I_{DS1} = 6\ \mu\text{A}$$

$$I_{DS6} = 120\ \mu\text{A}$$

$$g_{m1} = 60\ \mu\text{S}$$

$$g_{m6} = 1.2\ \text{mS}$$

$$f = 100\ \text{Hz}$$

$$U_1 = \frac{g_{m1}V_{in}}{2I_{DS1}} = 5\ 10^{-4}$$

$$U_6 = \frac{g_{m6}V_{m}}{I_{DS6}} = 0.1$$

两级运放的低频非线性-2

输出级产生的非线性

$$U_6 = \frac{V_m}{(V_{GS} - V_T)/2} = \frac{g_{m6} V_m}{I_{DS6}} = 0.1$$

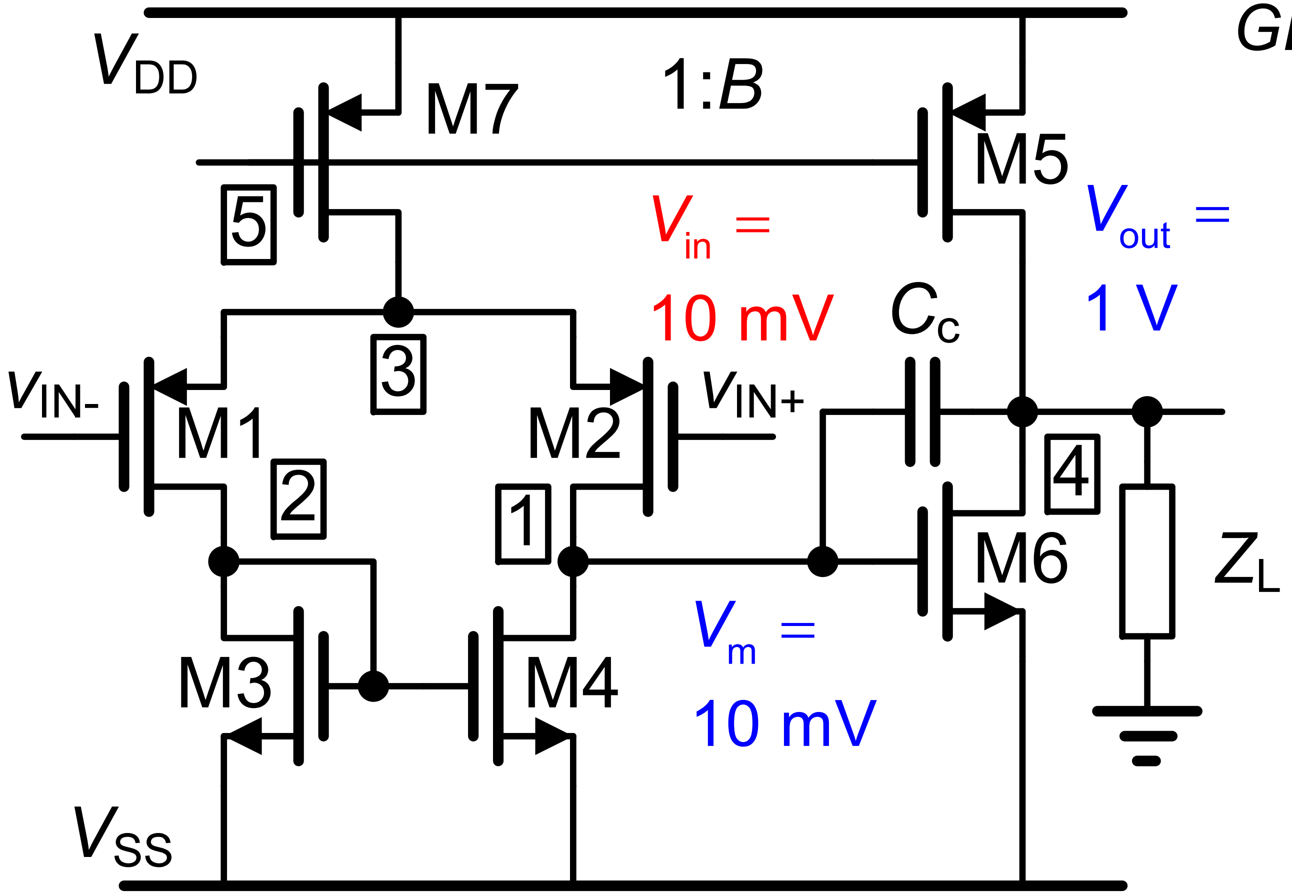
$$IM_2 = U_6/4 = 0.25 \times 0.1 = 2.5\%$$

负反馈减小非线性

$$T = A_V / A_{VC} = 1000$$

$$IM_{2f} = IM_2 \frac{A_{V1}}{(T)^2} = 2.5\% \frac{100}{1000^2} = 0.00025\%$$

两级运放的高频非线性-1



$$GBW = 10\ \text{MHz}$$

$$A_{V0} = 10$$

$$I_{DS1} = 6\ \mu\text{A}$$

$$I_{DS6} = 120\ \mu\text{A}$$

$$g_{m1} = 60\ \mu\text{S}$$

$$g_{m6} = 1.2\ \text{mS}$$

$$f = 100\ \text{kHz}$$

$$U_1 = \frac{g_{m1} V_{in}}{2 I_{DS1}} = 5\ 10^{-2}$$

$$U_6 = \frac{g_{m6} V_m}{I_{DS6}} = 0.1$$

两级运放的高频非线性-2

- 输出级产生的非线性

$$U_6 = g_{m6}V_m / I_{DS6} = 0.1 \quad IM_2 = U_6/4 = 0.25 \times 0.1 = 2.5\%$$

- 输入级产生的非线性

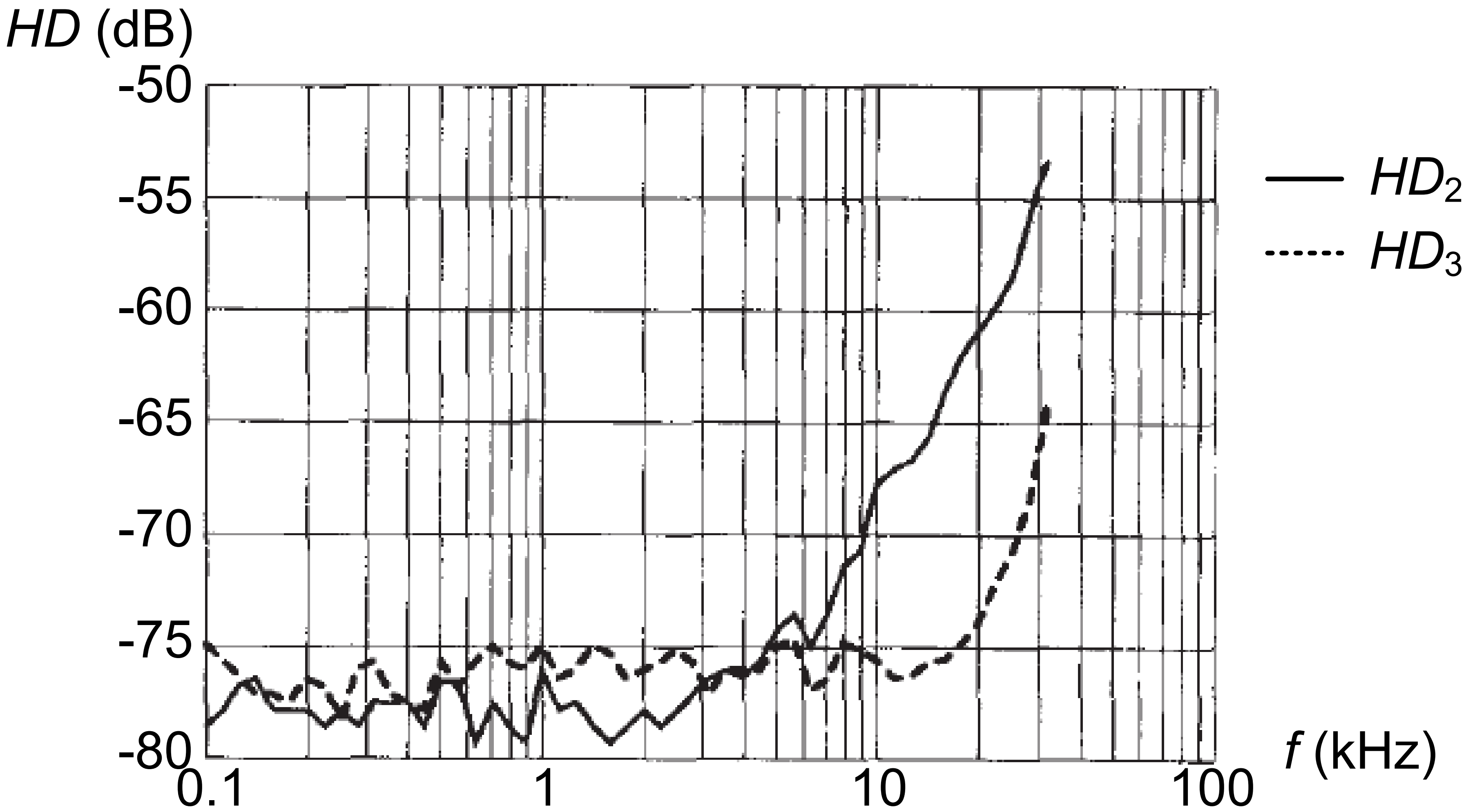
$$U_1 = \frac{V_{in}}{V_{GS} - V_T} = \frac{g_{m1}V_m}{2I_{DS6}} = 0.05 \quad IM_3 = 3U_1^2/32 = 0.023\%$$

- 负反馈减小非线性

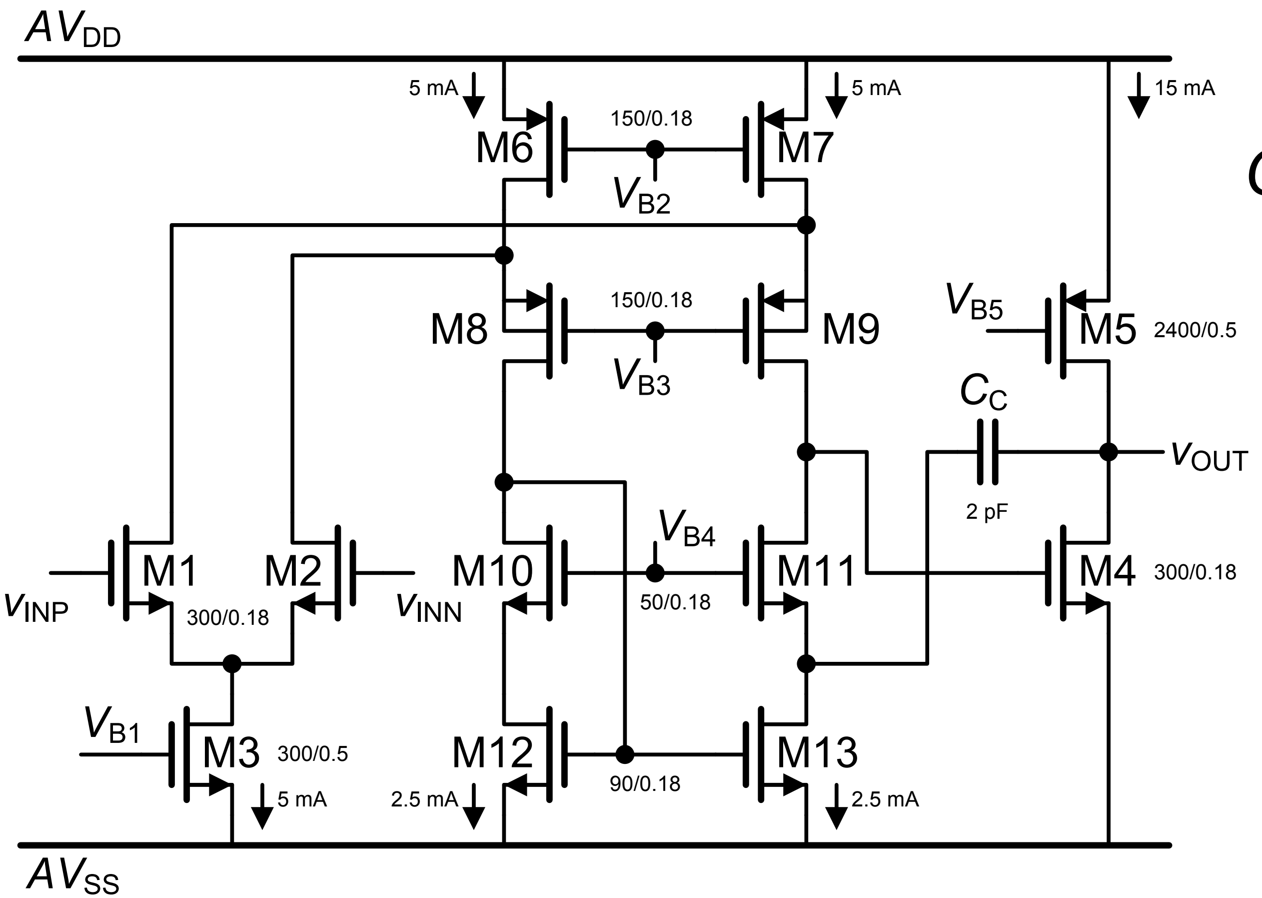
$$T = A_V / A_{VC} = 10$$

$$IM_{2f} = IM_2 \frac{A_{V1}}{(T)^2} = 2.5\% \frac{1}{10^2} = 0.025\%$$

密勒CMOS运放的非线性测量结果



1.8 V 低非线性CMOS运放



$$GBW = 3\ \text{GHz}$$

$$C_L = 8\ \text{pF}$$

$$SR = 900\ \text{V}/\mu\text{s}$$

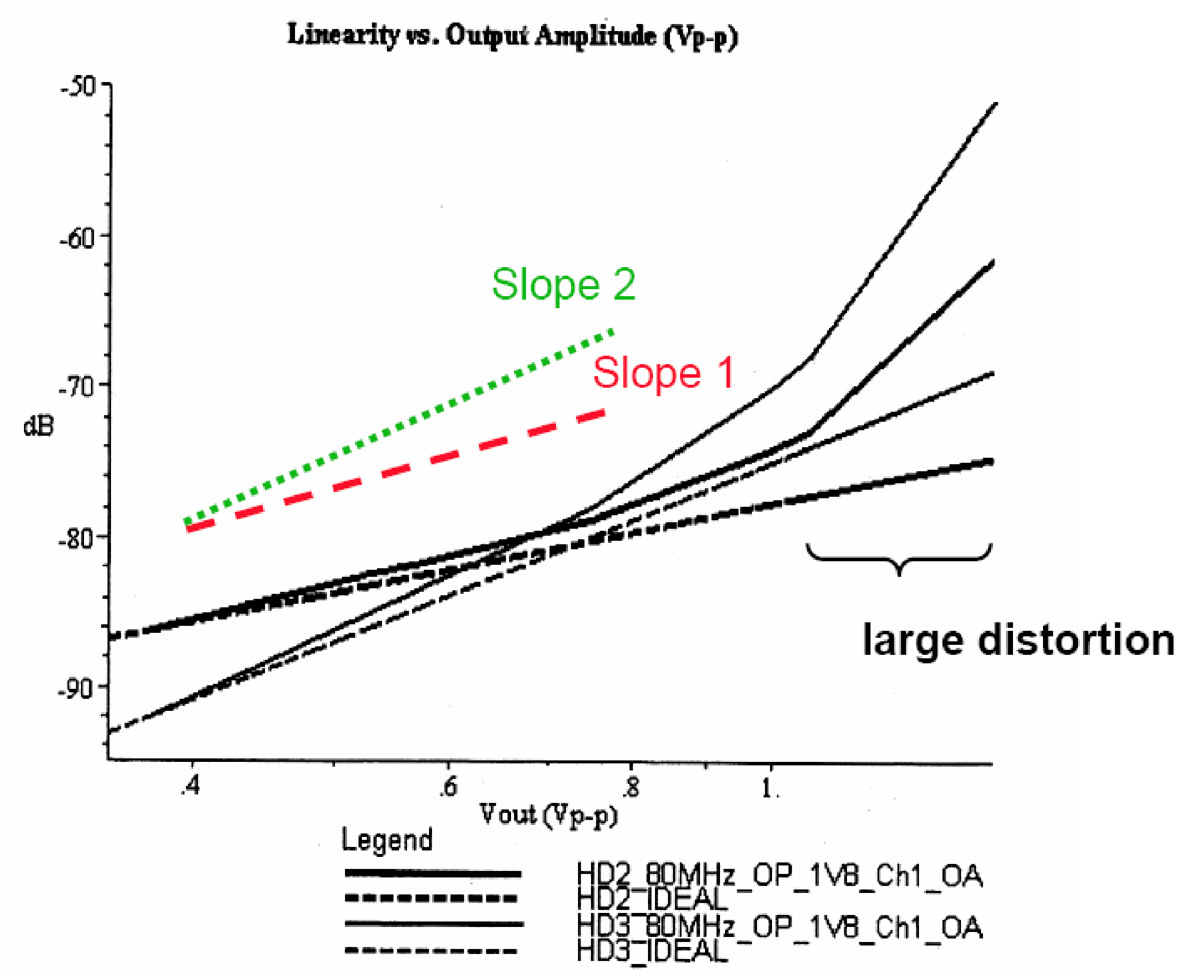
$$f_p = \frac{SR}{2\pi V_{peak}}$$

$$f_p = 300\ \text{MHz}$$

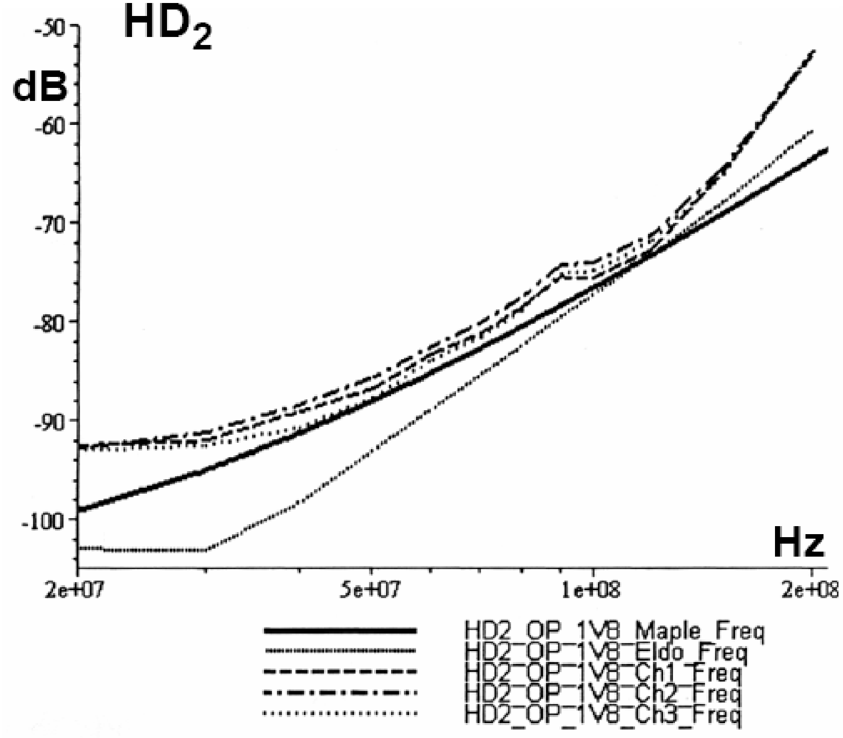
at 0.38 V_{peak}

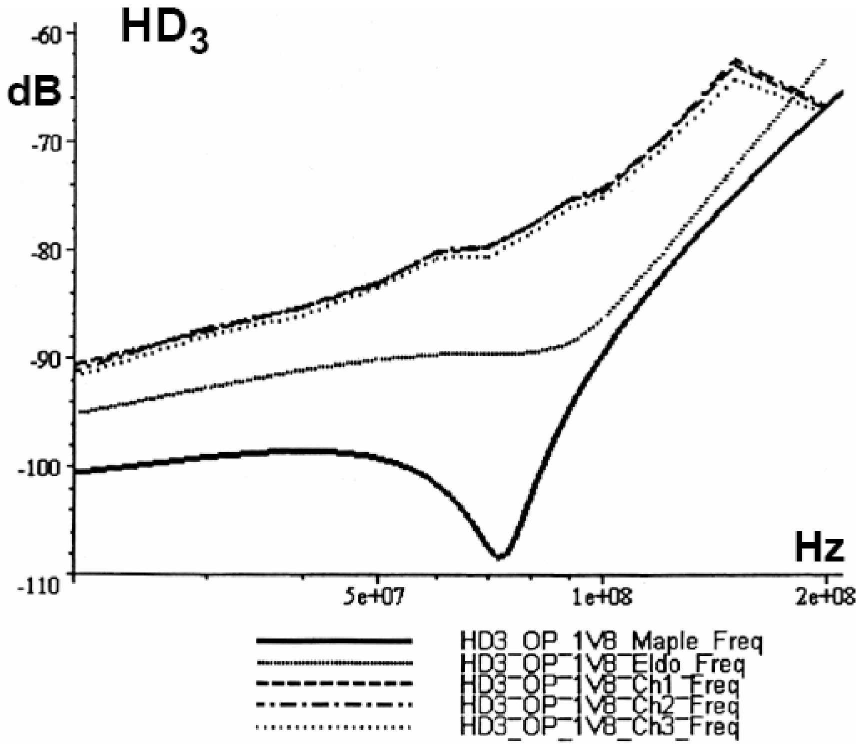
Ref.: Hernes Kluwer2003

HD_2和HD_3随幅度的变化



HD_2和HD_3随频率的变化





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- MOS管的非线性
  - ❑单端放大器
  - ❑差分放大器
- 双极型晶体管的非线性
- 负反馈能够减小非线性
- 运算放大器的非线性
- 其他非线性和准则

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

其他非线性

- 有限的压摆率造成的失真
- 开关的非线性
- 高频非线性:
  - ❑指数级数不使用，采用伏尔特拉级数
- 连续时间滤波器的非线性

准则

- 减小输入归一化电压幅度
- 减小输出归一化电流幅度
- 采用负反馈结构
- 使用全差分结构

非线性分量

非线性分量 ($U_p=V_{ip}/V_{ref}$)	IM_2 $\times U_p$	IM_3 $\times U_p^2$	V_{ref}
双极型	1/2	1/8	kT/q
MOS	1/4	0	$(V_{GS}-V_T)/2$
双极型差分对	0	1/4	$2kT/q$
MOS差分对	0	3/32	$(V_{GS}-V_T)$

采用负反馈的非线性分量(T>5)

非线性分量 ($U_p=V_{ip}/V_{ref}$)	IM_2 $\times U_p$	IM_3 $\times U_p^2$	V_{ref}
双极型	$1/(2T)$	$1/(4T)$	$kT/q \times T$
MOS	$1/(4T)$	0	$(V_{GS}-V_T)/2 \times T$
双极型差分对	0	1/(4T)	$2kT/q \times T$
MOS差分对	0	3/(32T)	$(V_{GS}-V_T) \times T$