

模拟集成电路设计原理

(Principle of Analog Integrated Circuit Design, INF0130025.02)

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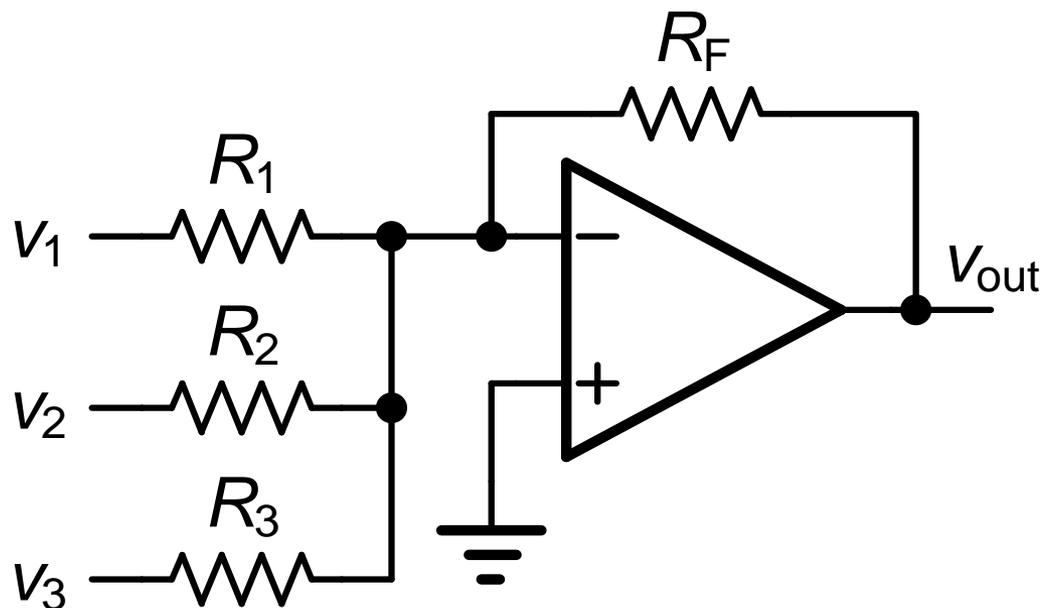
运算放大器的稳定性

目录

- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

运算放大器的运算功能

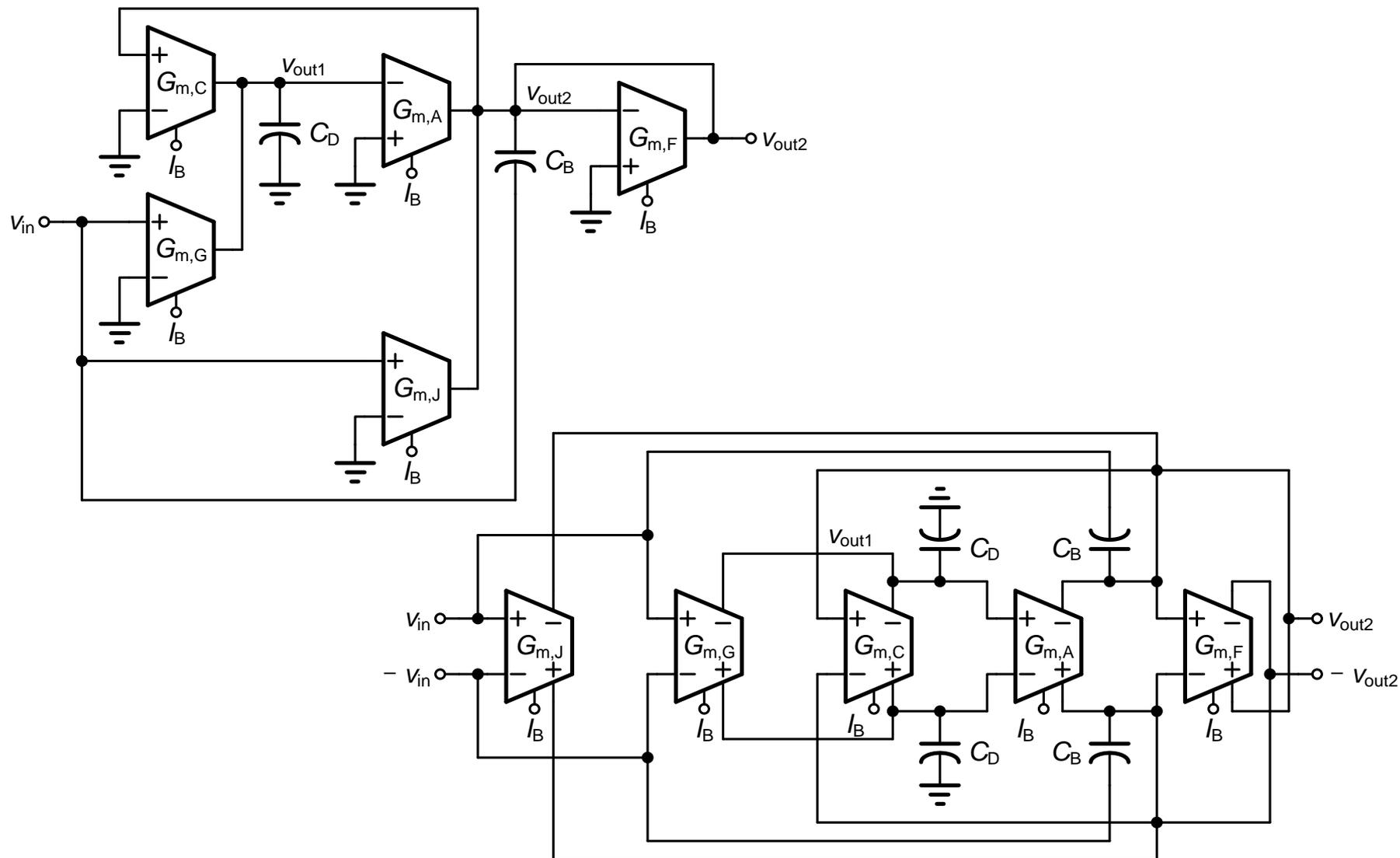


$$-\frac{V_{out}}{R_F} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

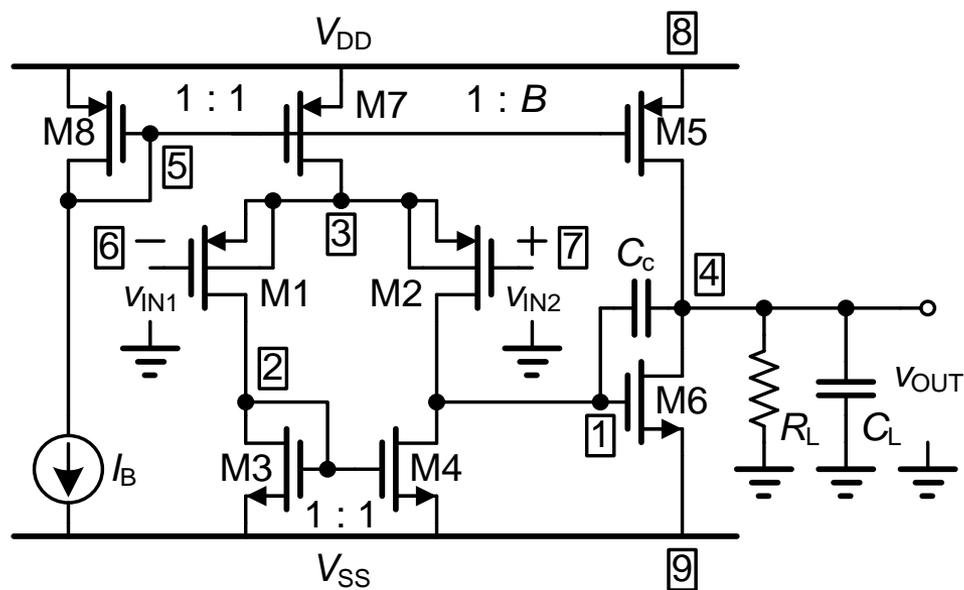
要求：高增益
高速
低噪声
低功耗

运放性能指标：
 高电压增益
 差分输入电压 ≈ 0
 输入电流 = 0
 高带宽
 增益带宽乘积 **GBW** 非常，非常高！

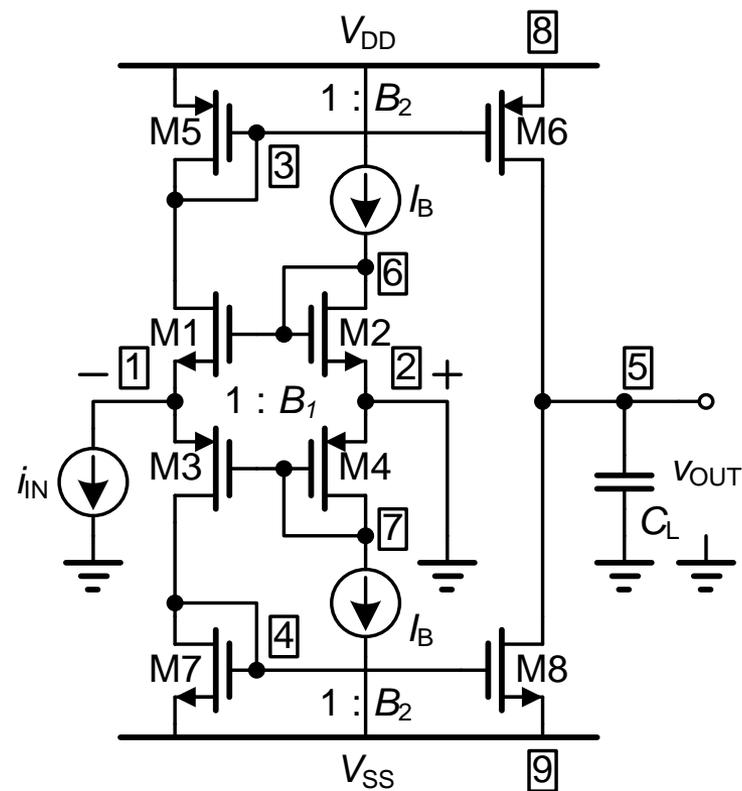
单端？全差分？



电压输入？ 电流输入？



电压输入
电流输出

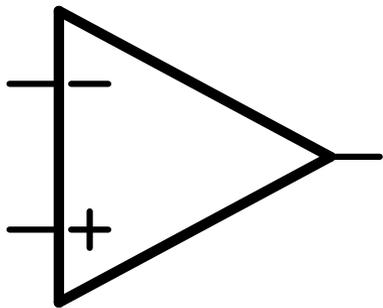


电流输入
电流输出

分类

Opamp

电压
放大器



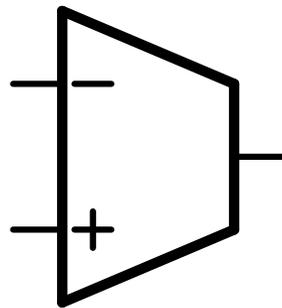
$$A_V = \frac{V_{out}}{V_{in}}$$

$$A_V =$$

GBW

OTA

跨导
放大器

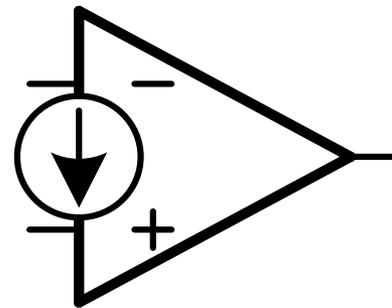


$$A_G = \frac{i_{out}}{V_{in}}$$

$$A_V = A_G R_L$$

OCA

电流
放大器

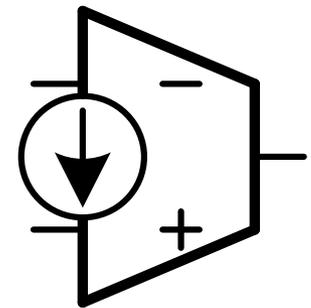


$$A_I = \frac{i_{out}}{i_{in}}$$

$$A_V = A_I \frac{R_L}{R_S}$$

CM amp

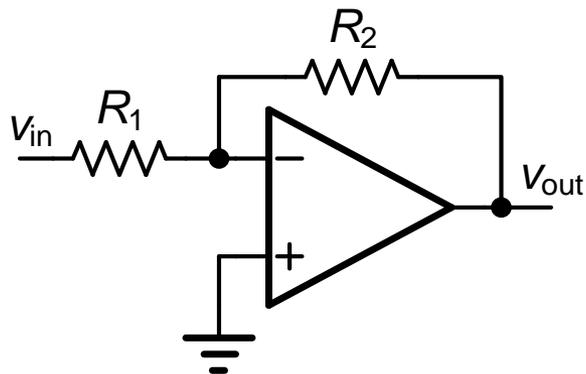
跨阻
放大器



$$A_R = \frac{V_{out}}{i_{in}}$$

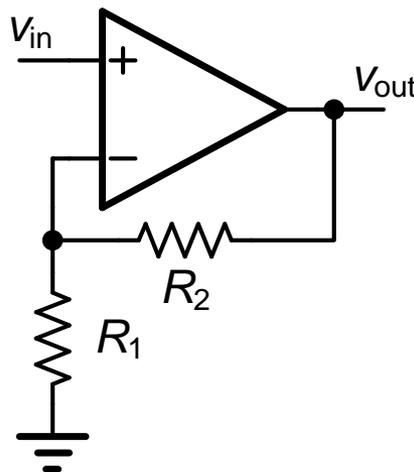
$$A_V = A_R \frac{1}{R_S}$$

反馈结构



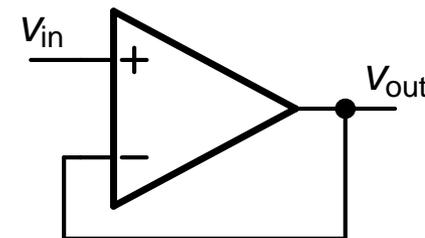
$$A_V = -\frac{R_2}{R_1}$$

$$R_{IN} = R_1$$



$$A_V = 1 + \frac{R_2}{R_1}$$

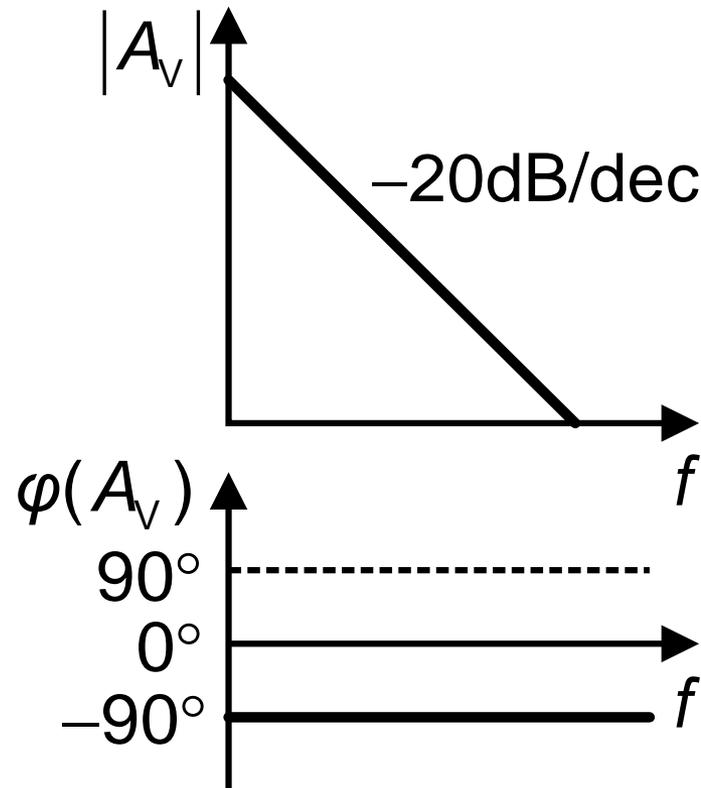
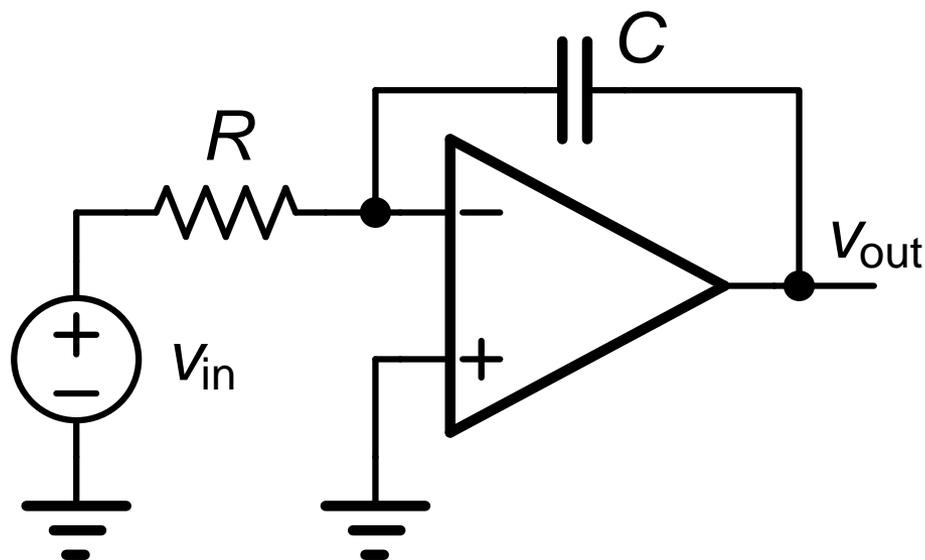
$$R_{IN} = \infty$$



$$A_V = 1$$

$$R_{IN} = \infty$$

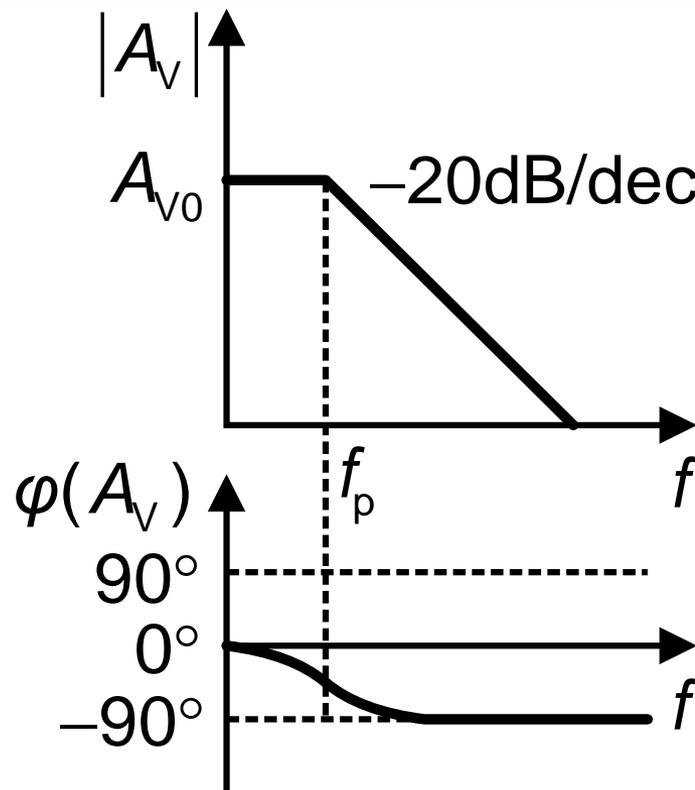
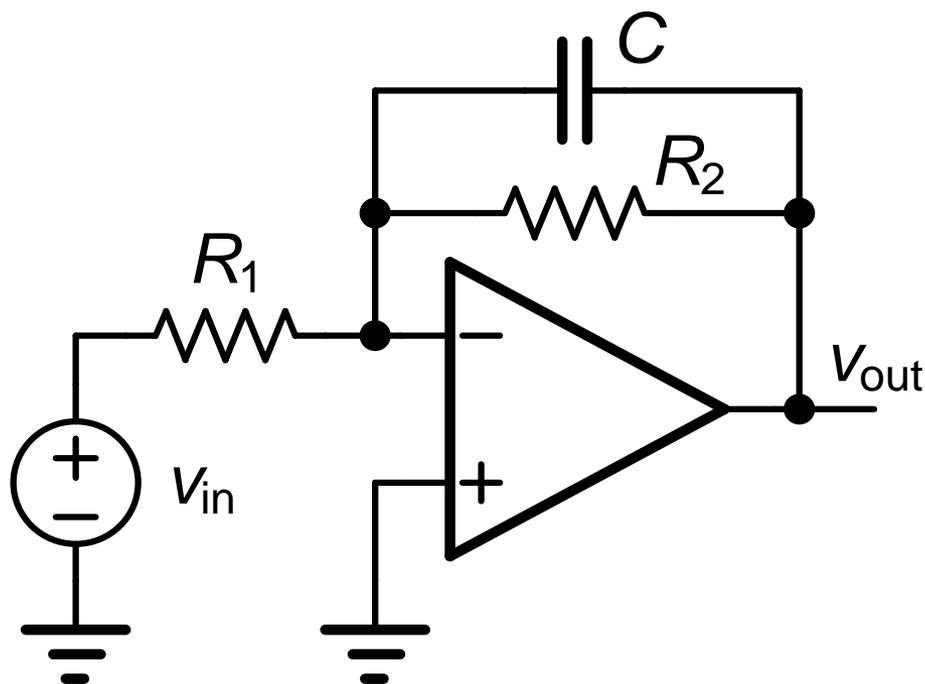
积分器



$$A_v = -\frac{1}{j \frac{f}{f_p}}$$

$$f_p = \frac{1}{2\pi RC}$$

有损积分器：低通滤波器

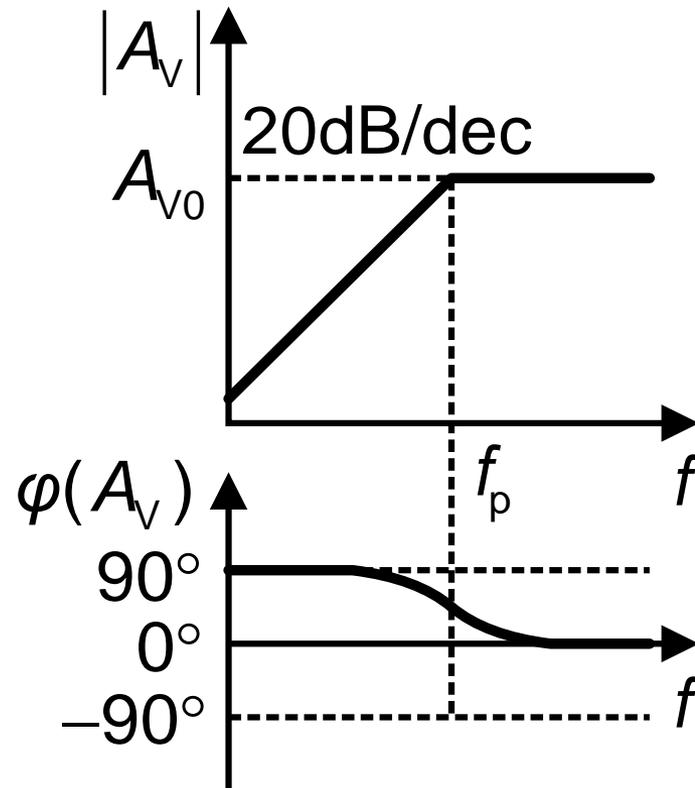
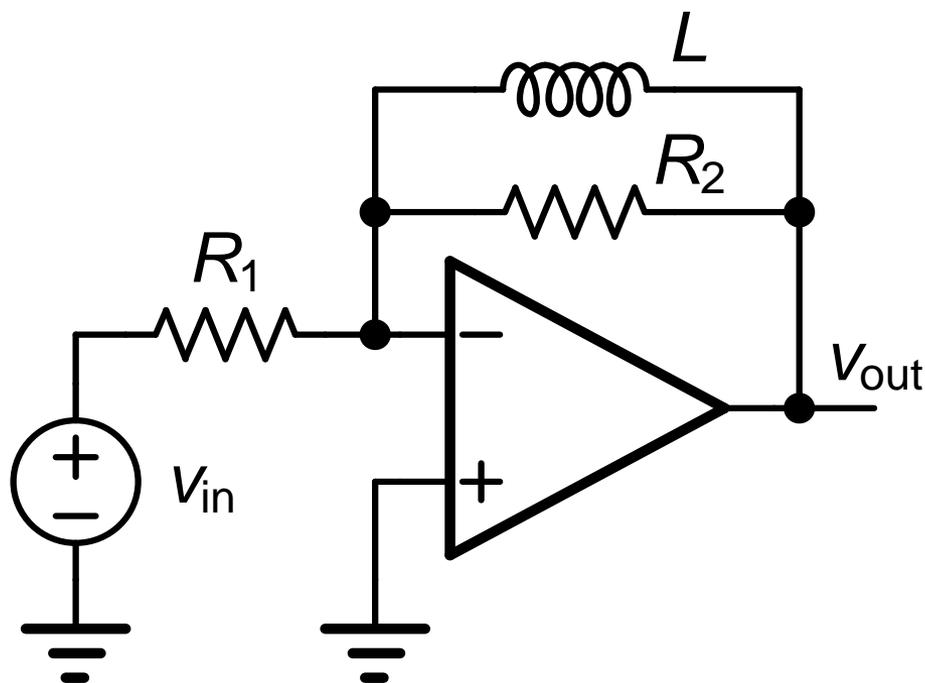


$$A_{V0} = -\frac{R_2}{R_1}$$

$$A_V = A_{V0} \frac{1}{1 + j \frac{f}{f_p}}$$

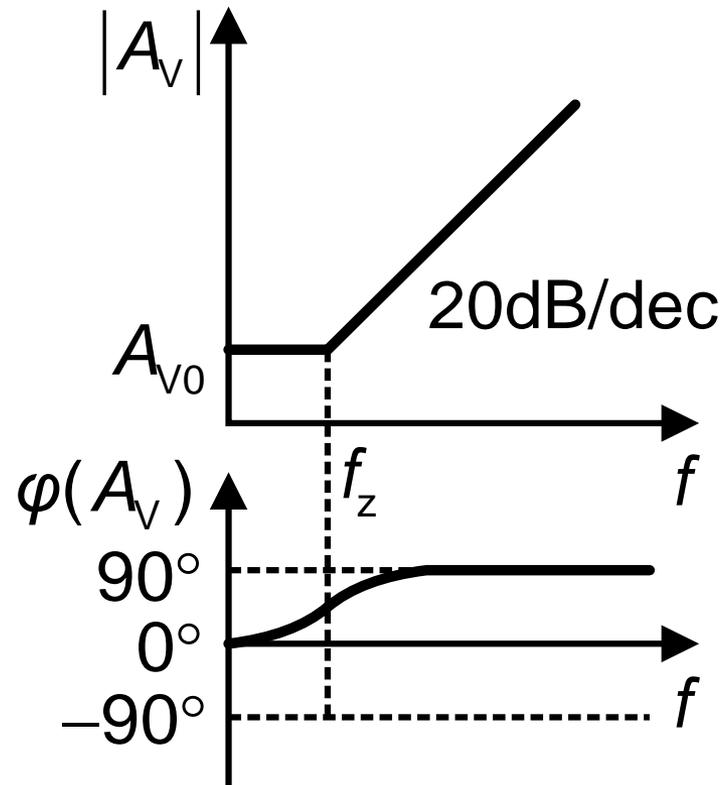
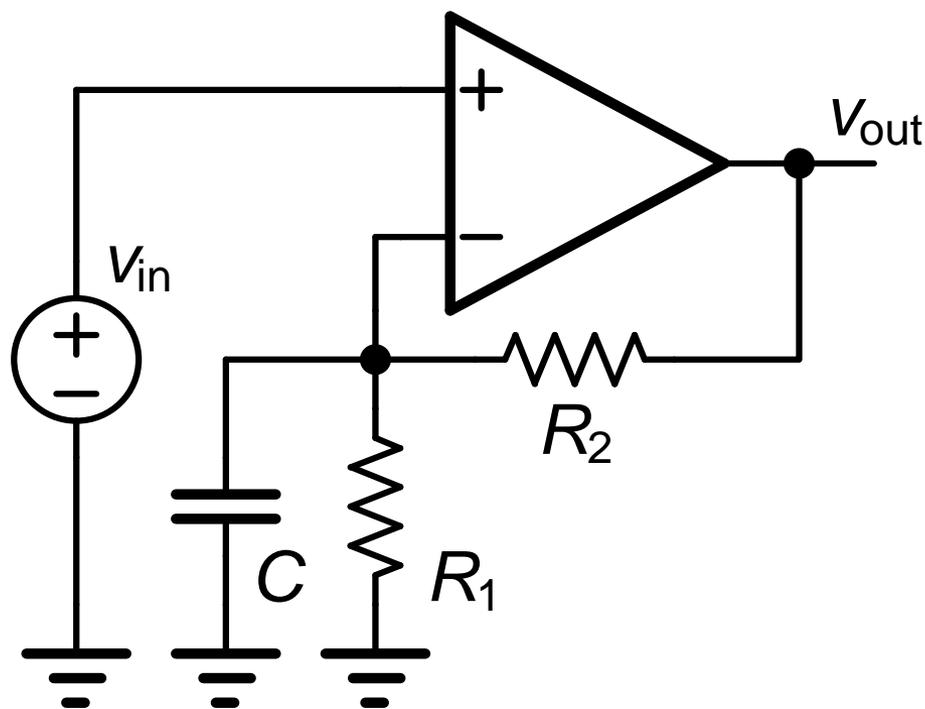
$$f_p = \frac{1}{2\pi R_2 C}$$

高通滤波器 1



$$A_{V0} = -\frac{R_2}{R_1} \quad A_V = A_{V0} \frac{j \frac{f}{f_p}}{1 + j \frac{f}{f_p}} \quad f_p = \frac{R_2}{2\pi L}$$

高通滤波器 2

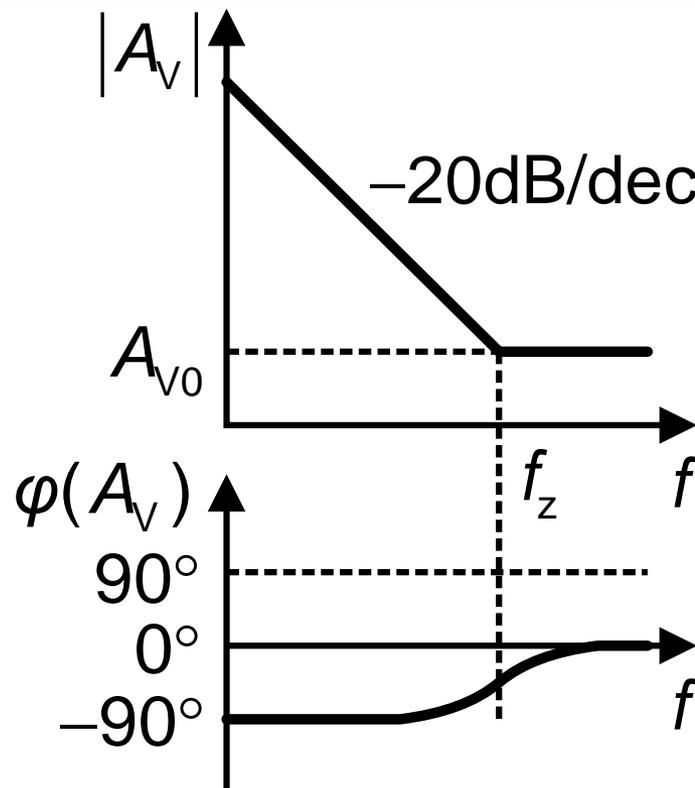
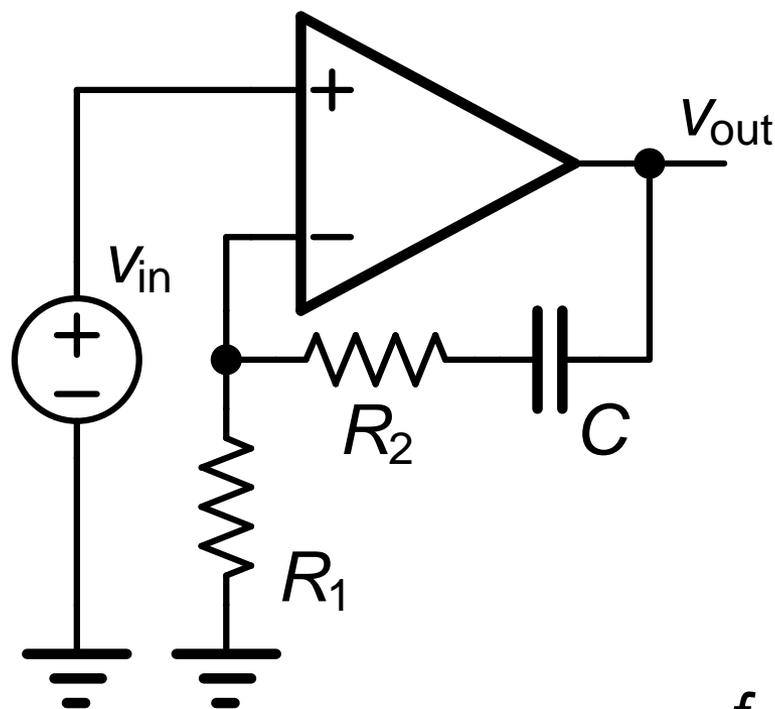


$$A_{V0} = 1 + \frac{R_2}{R_1} \quad A_V = A_{V0} \left(1 + j \frac{f}{f_z} \right)$$

$$f_z = \frac{1}{2\pi RC}$$

$$R = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

有限衰减的低通滤波器



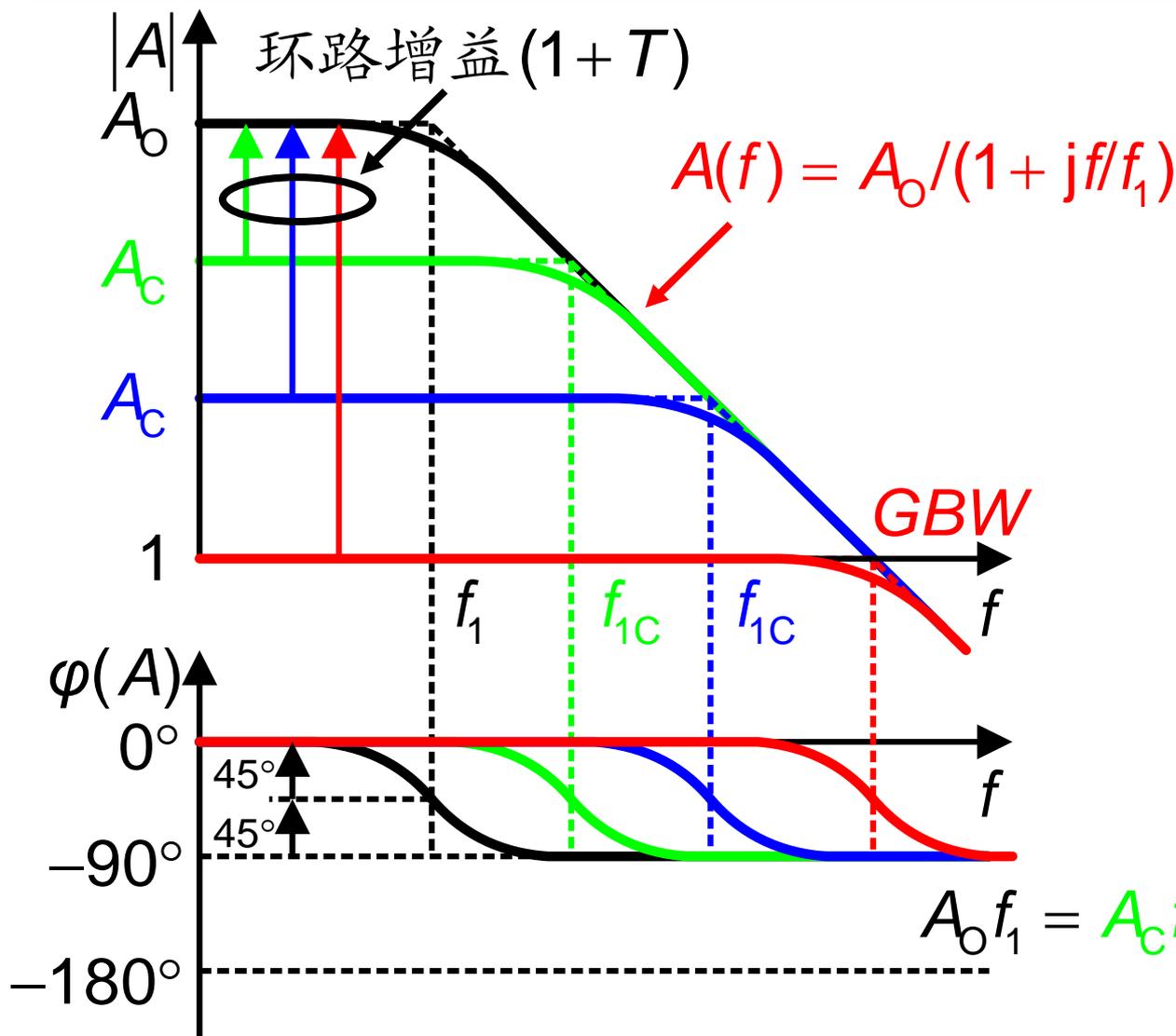
$$A_{V0} = 1 + \frac{R_2}{R_1}$$

$$A_V = A_{V0} \frac{1 + j \frac{f}{f_z}}{j \frac{f}{f_z}}$$

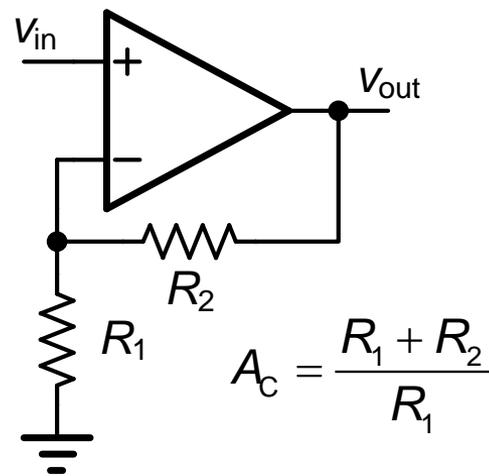
$$f_z = \frac{1}{2\pi RC}$$

$$R = R_1 + R_2$$

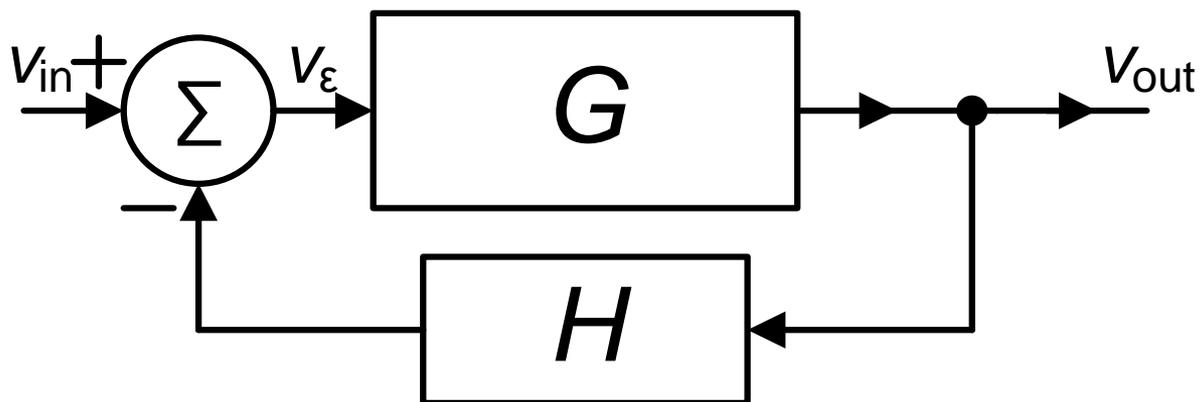
增益和带宽之间的交换



A_o 开环增益
 A_c 闭环增益



开环增益和闭环增益

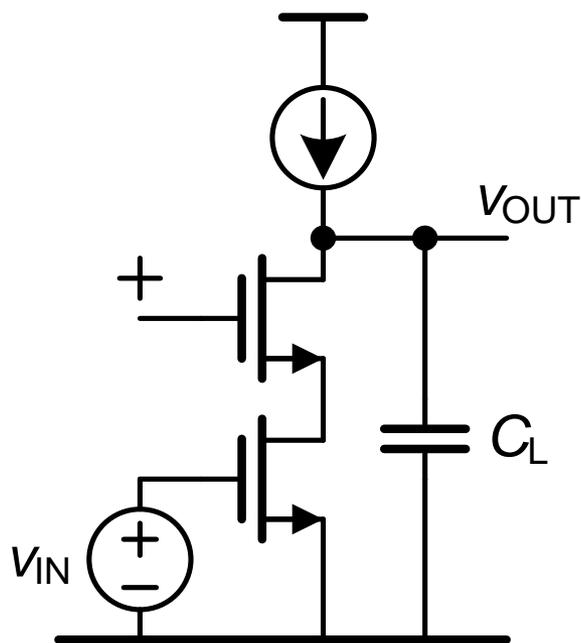


$$\left. \begin{aligned} V_{\varepsilon} &= V_{in} - HV_{out} \\ V_{out} &= GV_{\varepsilon} \end{aligned} \right\} \Rightarrow A_c = \frac{V_{out}}{V_{in}} = \frac{G}{1 + GH} \approx \frac{1}{H}$$

如果环路增益 $GH = T \gg 1$

Ref.: P. Gray, P.Hurst, S.Lewis, R. Meyer: Design of analog integrated circuits,
4th ed., Wiley 2001

运放成为运放的原因？



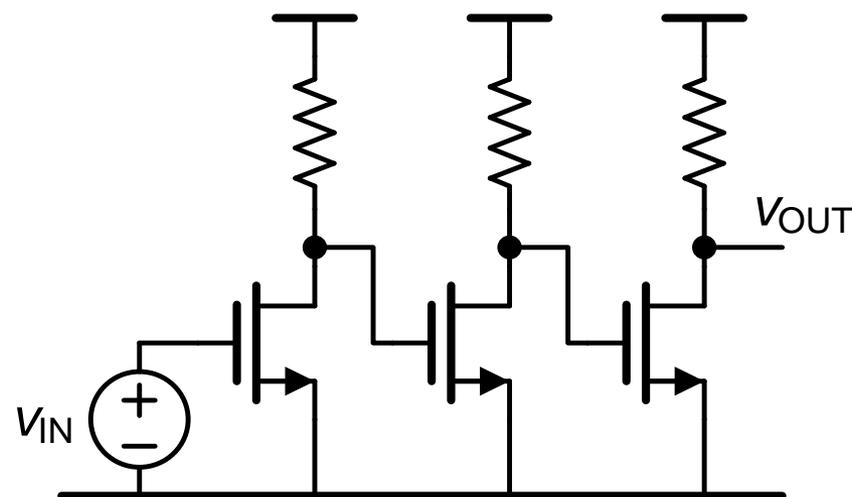
运算放大器

单级点放大器

高阻抗 = 高增益

增益与带宽交换

任何增益下都稳定



宽带放大器

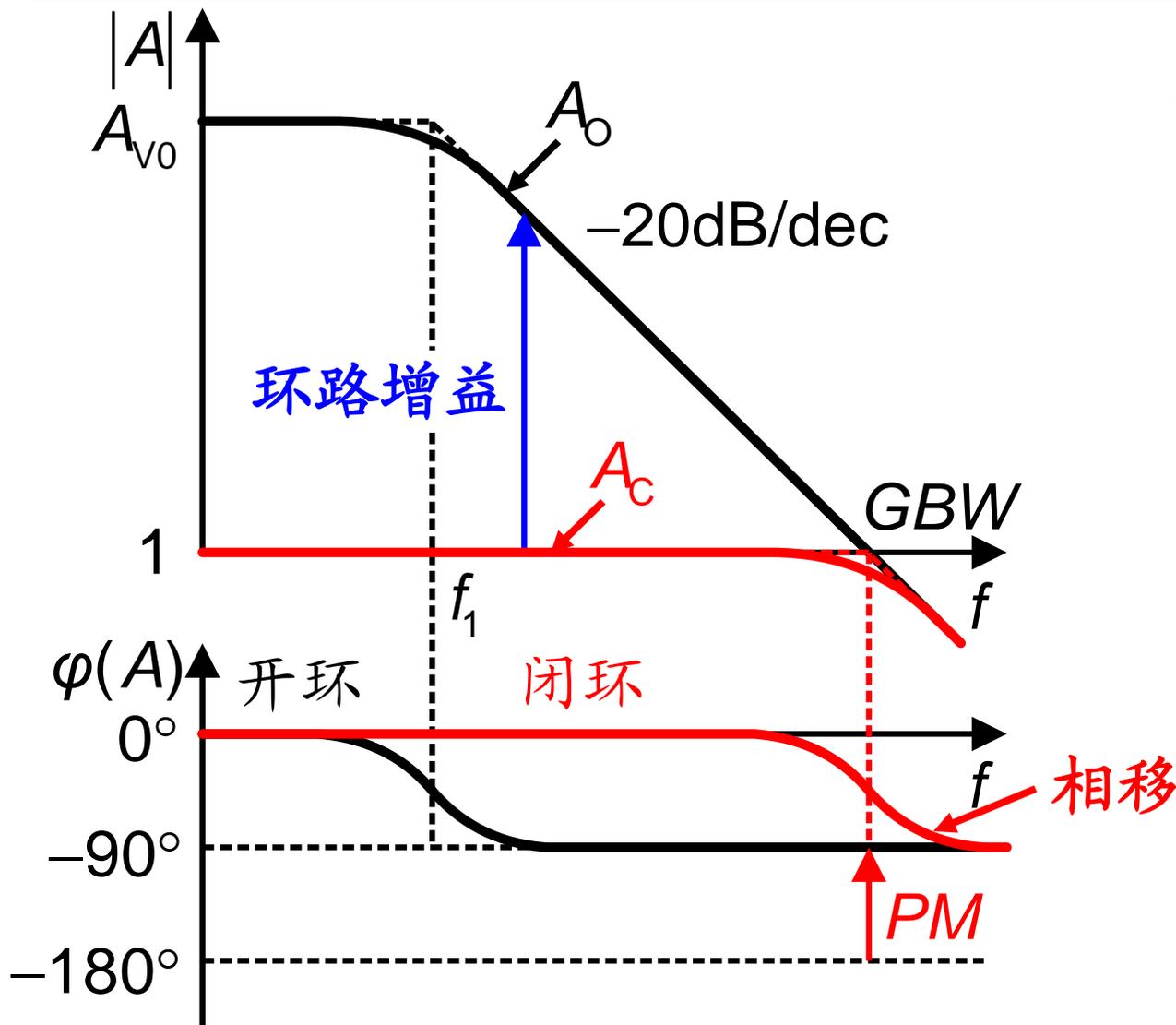
多级点放大器

低阻抗节点

高带宽

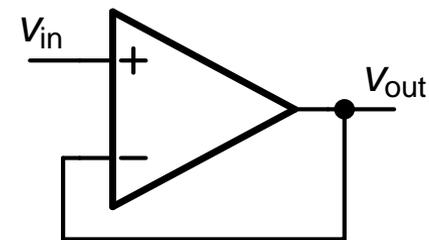
只在某些增益下稳定

单极点系统



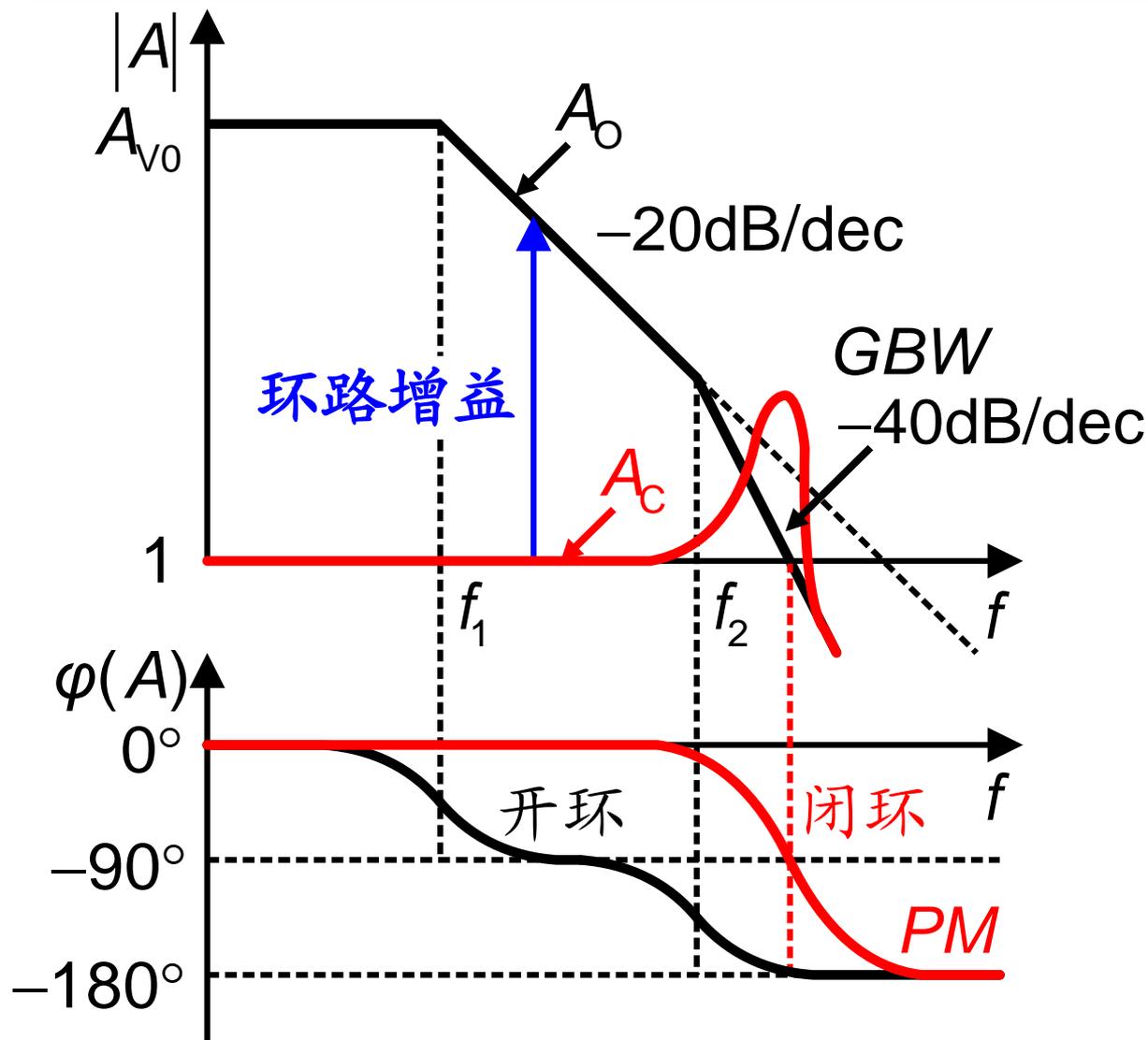
A_o 开环增益

A_c 闭环增益



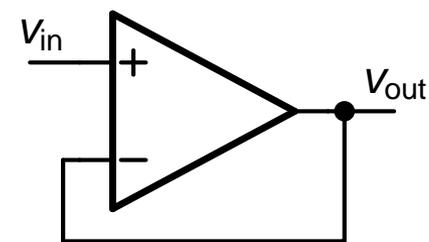
PM 相位裕度

双极点系统



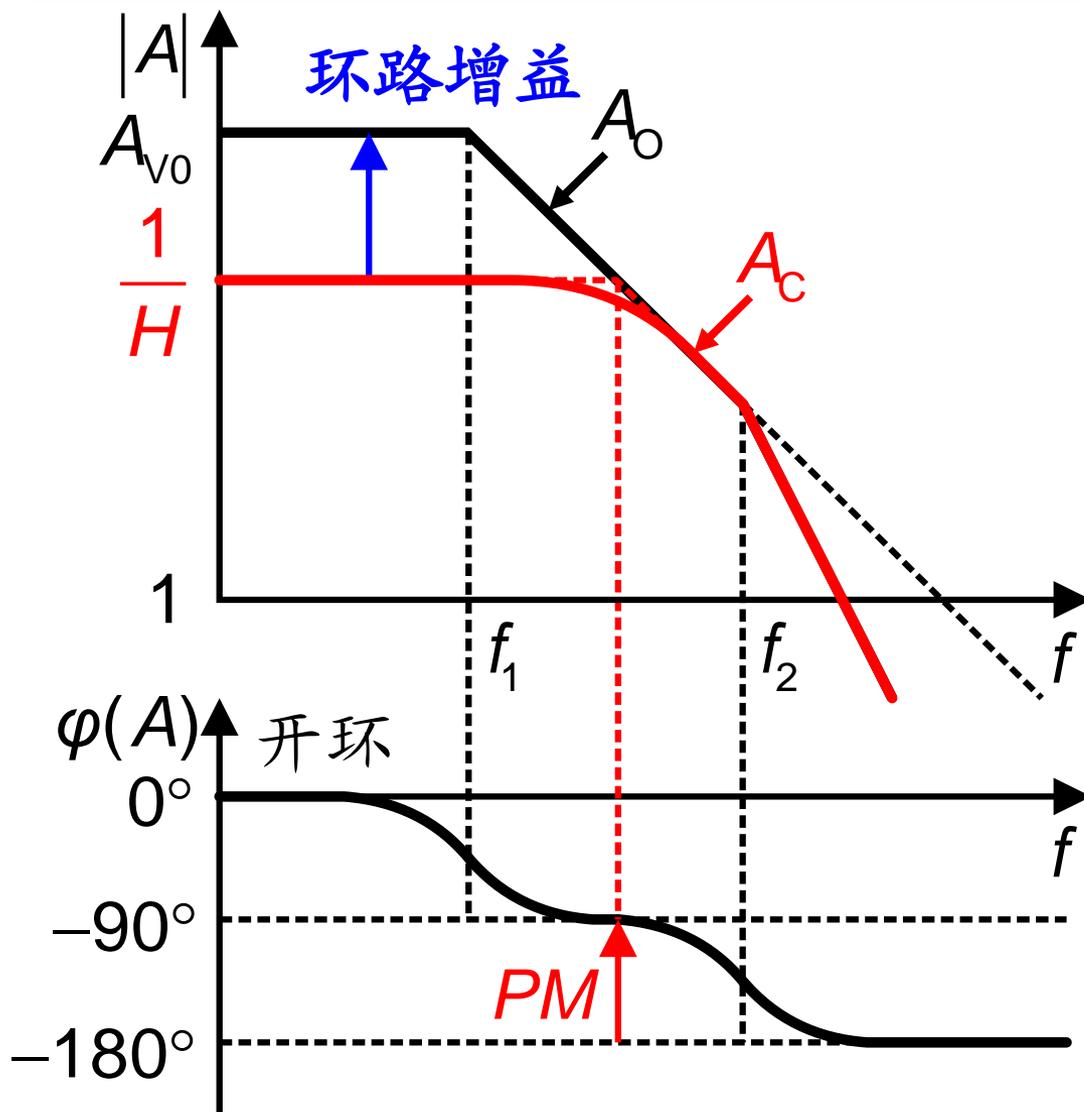
A_o 开环增益

A_c 闭环增益



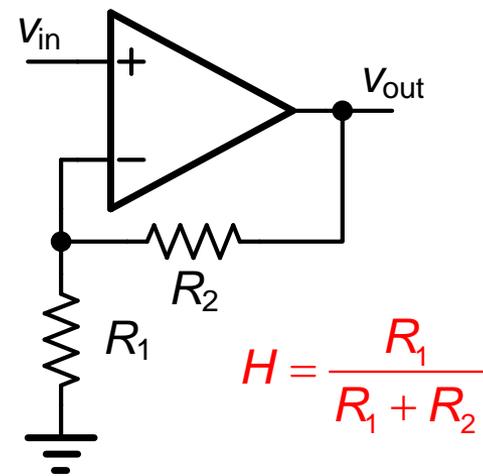
PM相位裕度

环路增益与相位裕度的关系 1



A_o 开环增益

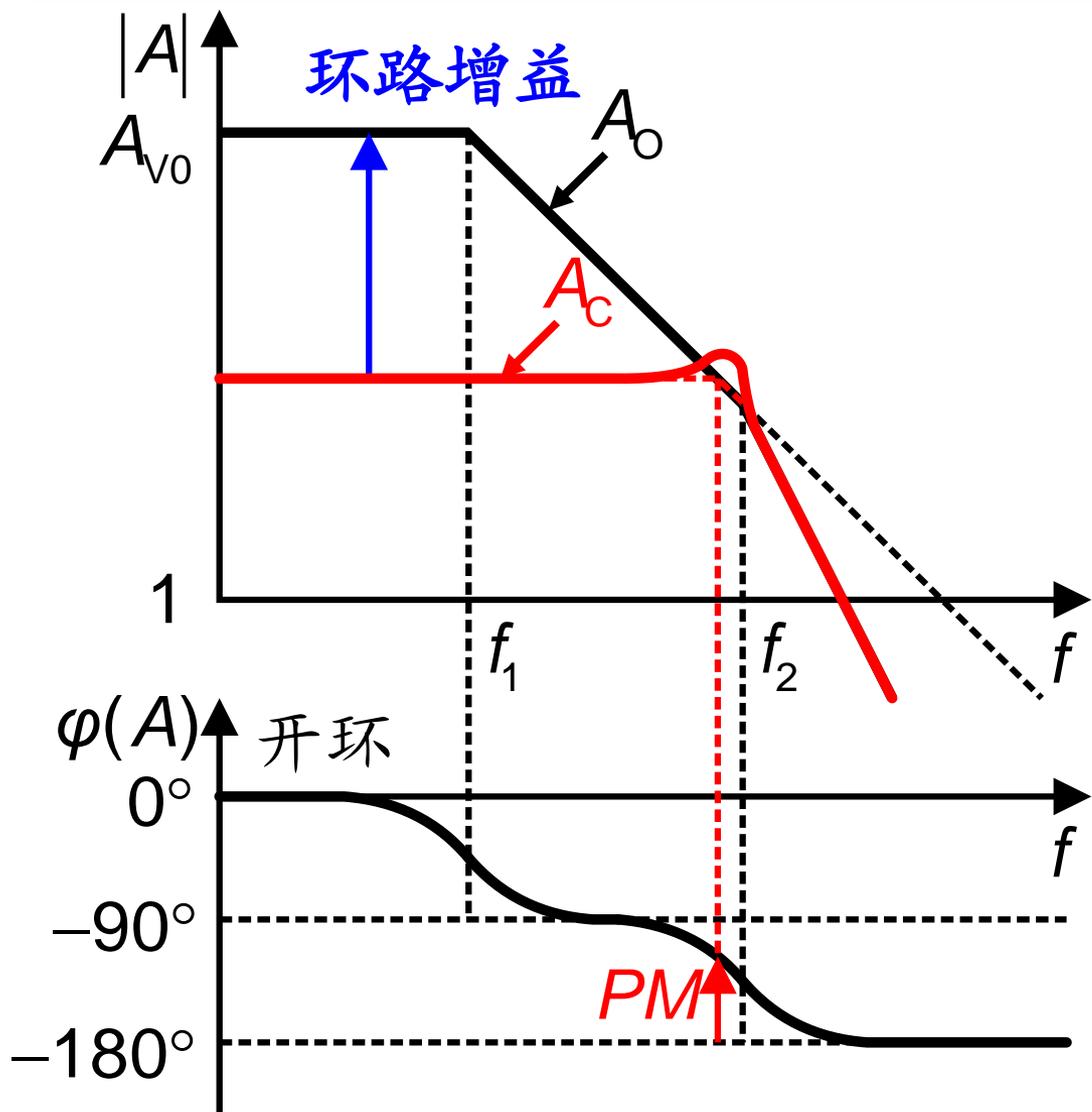
A_c 闭环增益



$$H = \frac{R_1}{R_1 + R_2}$$

PM相位裕度

环路增益与相位裕度的关系 2

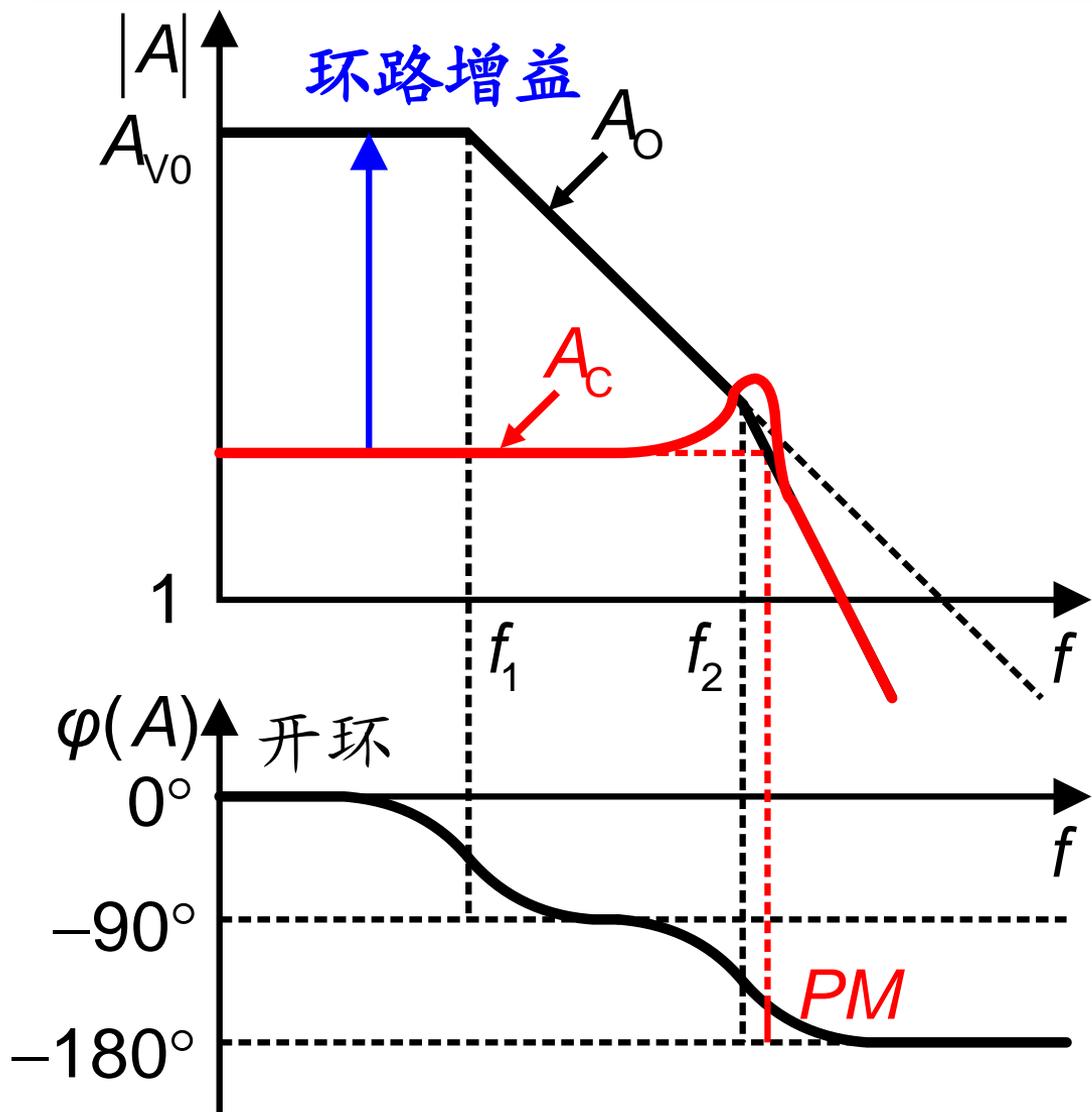


A_o 开环增益

A_c 闭环增益

PM 相位裕度

环路增益与相位裕度的关系 3

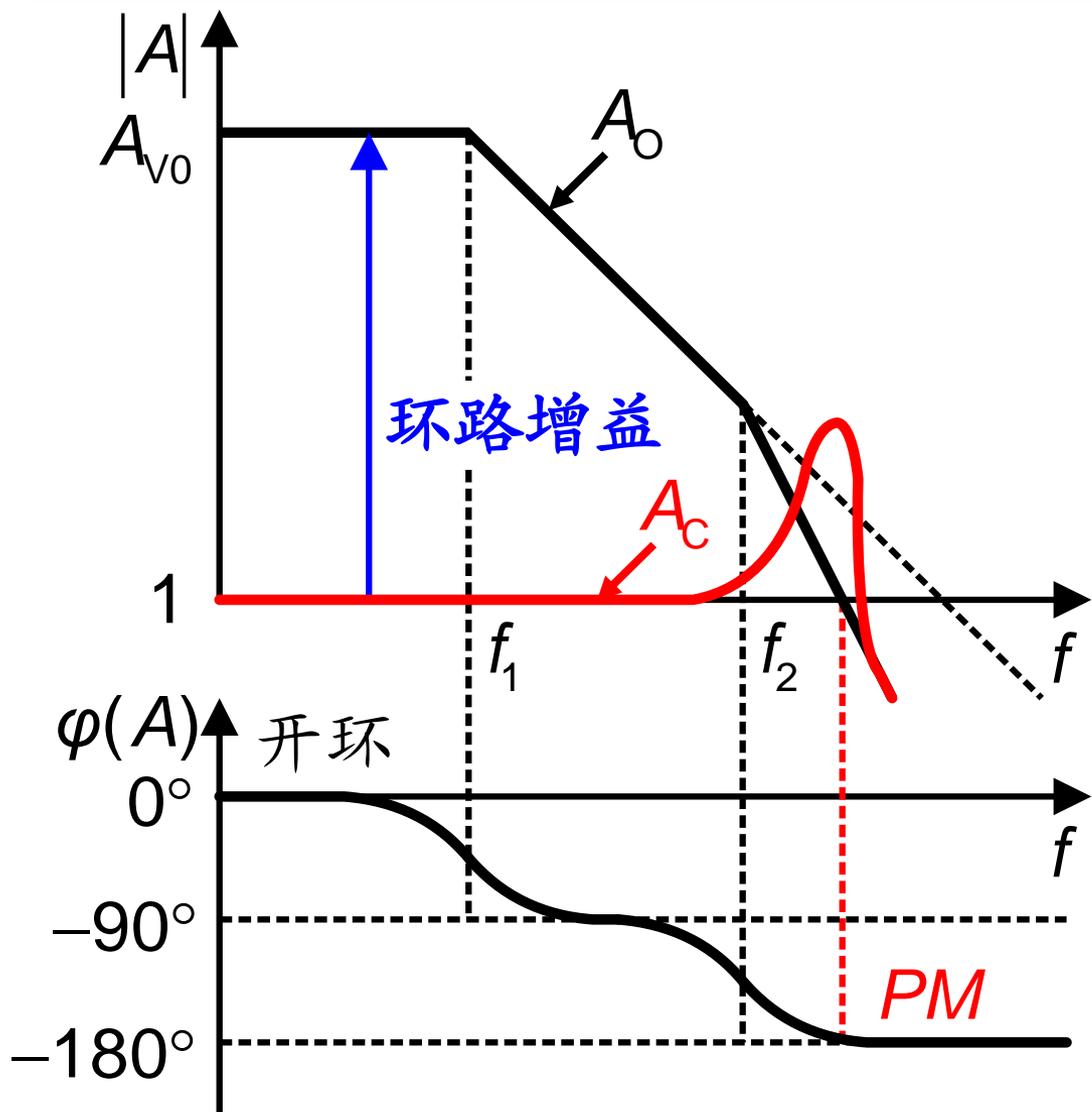


A_o 开环增益

A_c 闭环增益

PM 相位裕度

环路增益与相位裕度的关系 4



A_o 开环增益

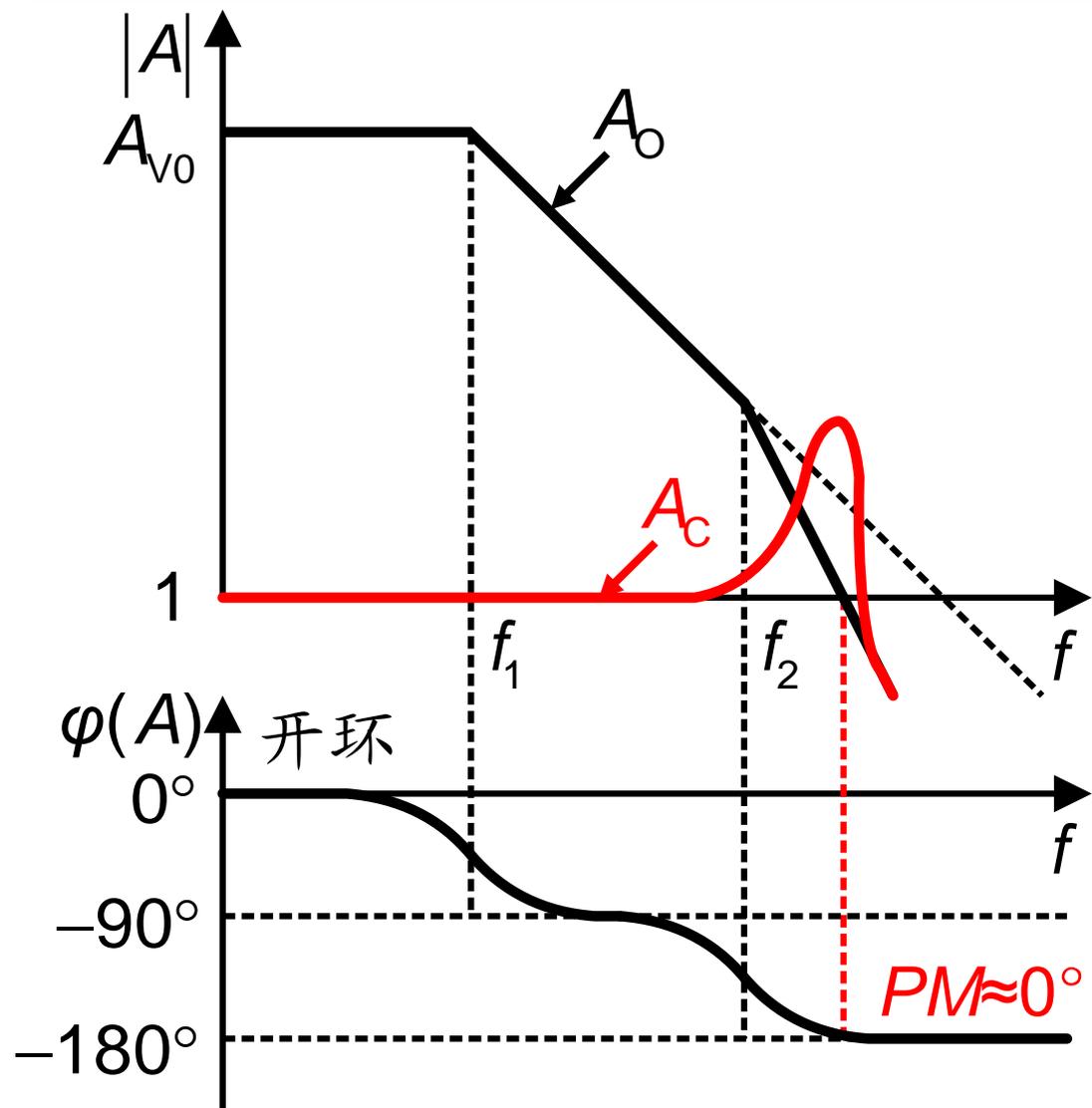
A_c 闭环增益

最坏情况

$$A_c = 1$$

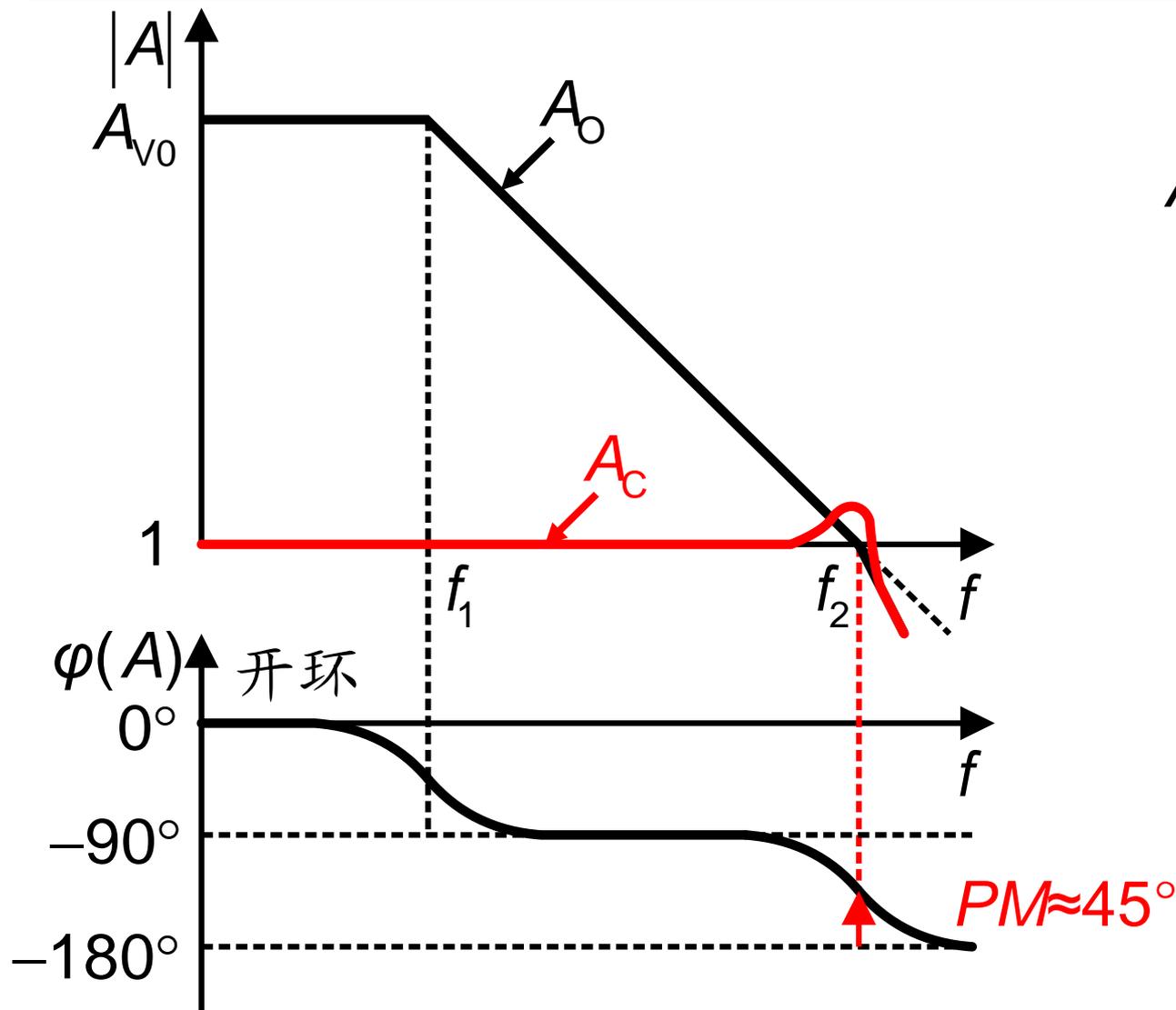
PM 相位裕度

当 f_2 频率较低时



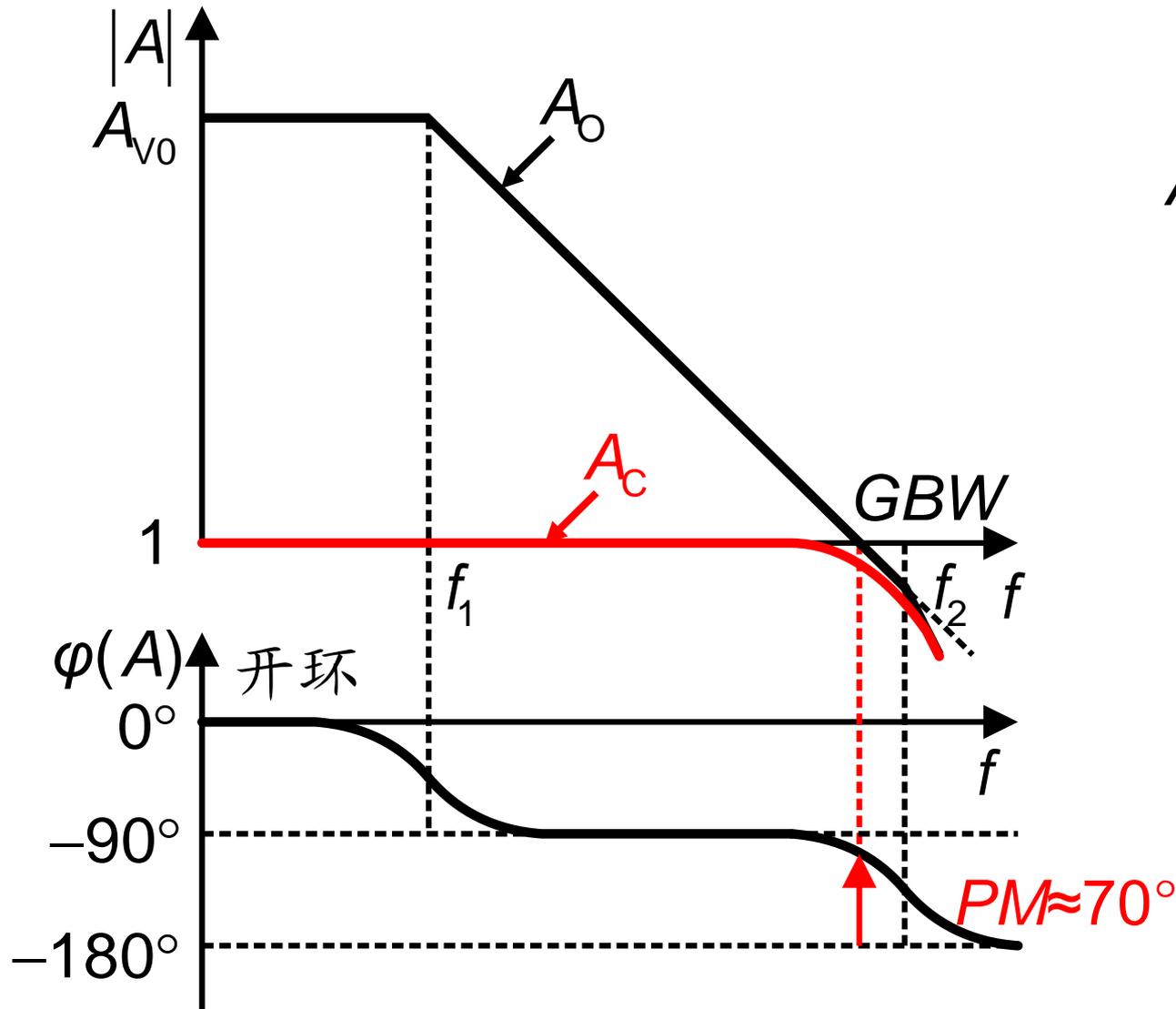
A_c 闭环增益

通过增加 f_2 , 提高 PM 。($f_2=GBW$)



A_c 闭环增益

通过增加 f_2 , 提高 PM 。($f_2 \approx 3GBW$)

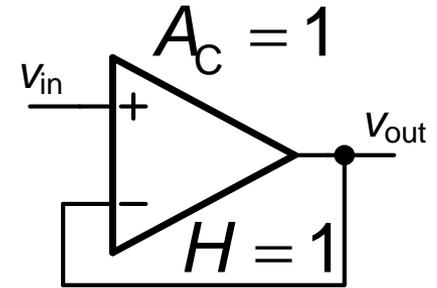


A_c 闭环增益

$f_2 \approx 3GBW$

当 $f_2 \approx 3GBW$ 时，计算 PM

开环增益 $A_o = \frac{A_{V0}}{(1 + j\frac{f}{f_1})(1 + j\frac{f}{f_2})}$



闭环增益 $A_c = \frac{A_o}{1 + A_o} \approx \frac{A_{V0}}{1 + A_{V0}} \frac{1}{1 + j\frac{f}{GBW} + j^2\frac{f^2}{GBWf_2}}$

$$\approx \frac{1}{1 + j2\zeta\frac{f}{f_r} + j^2\frac{f^2}{f_r^2}}$$

$f_r = \sqrt{GBWf_2}$ 自激振荡频率

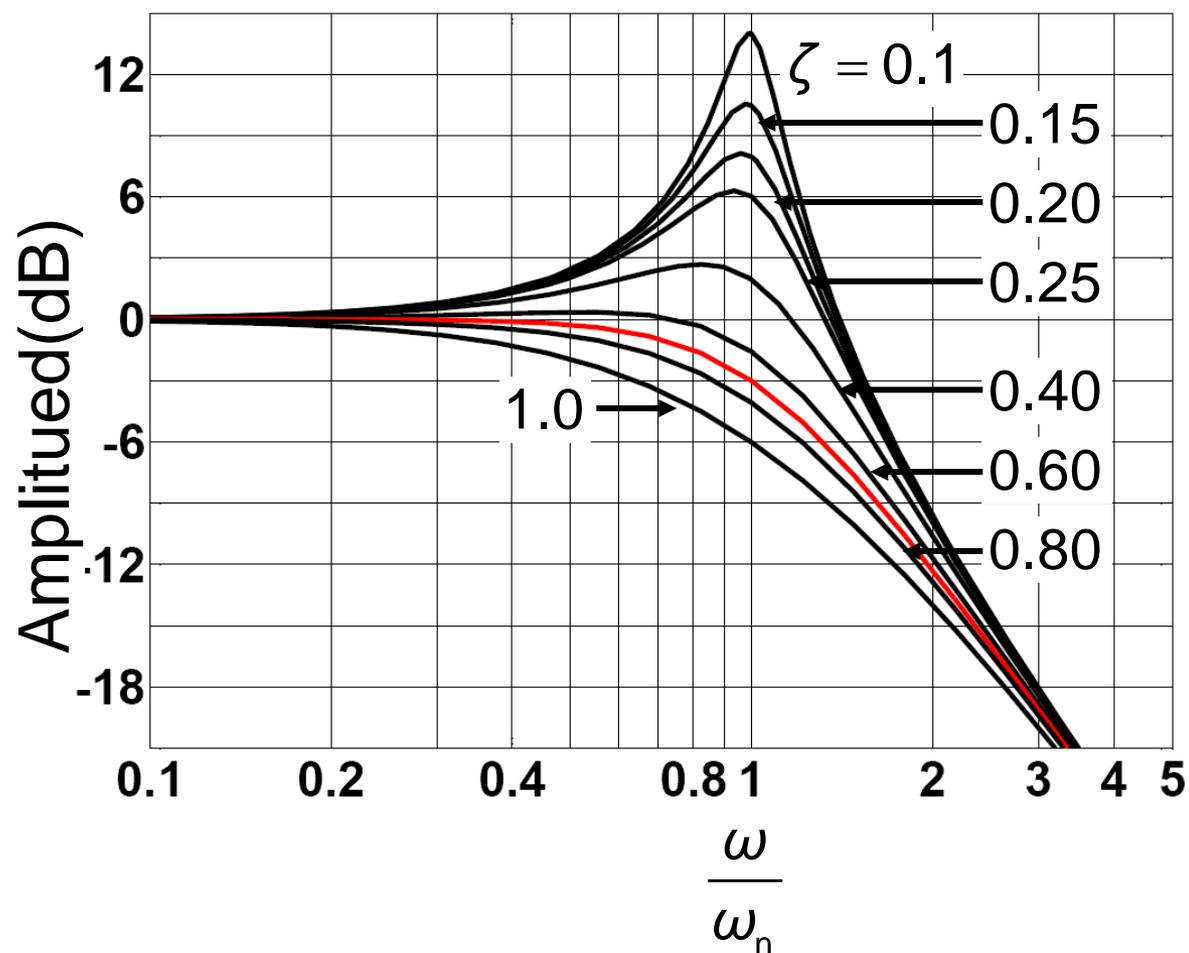
$\zeta = \frac{1}{2}\sqrt{\frac{f_2}{GBW}}$ 阻尼系数(= 1/2Q)

PM , ζ , P_f 和 P_t

$$f_r = \sqrt{GBW f_2} \quad PM(^{\circ}) = 90^{\circ} - \arctan \frac{GBW}{f_2} = \arctan \frac{f_2}{GBW}$$

$\frac{f_2}{GBW}$	$PM(^{\circ})$	$\zeta = \frac{1}{2} \sqrt{\frac{f_2}{GBW}}$	$P_f(\text{dB})$	$P_t(\text{dB})$
0.5	27	0.35	3.6	2.3
1	45	0.5	1.25	1.3
1.5	56	0.61	0.28	0.73
2	63	$\sqrt{2}/2$	0	0.37
3	72	0.87	0	0.04
4	76	1		
5	79			

闭环幅度频率响应

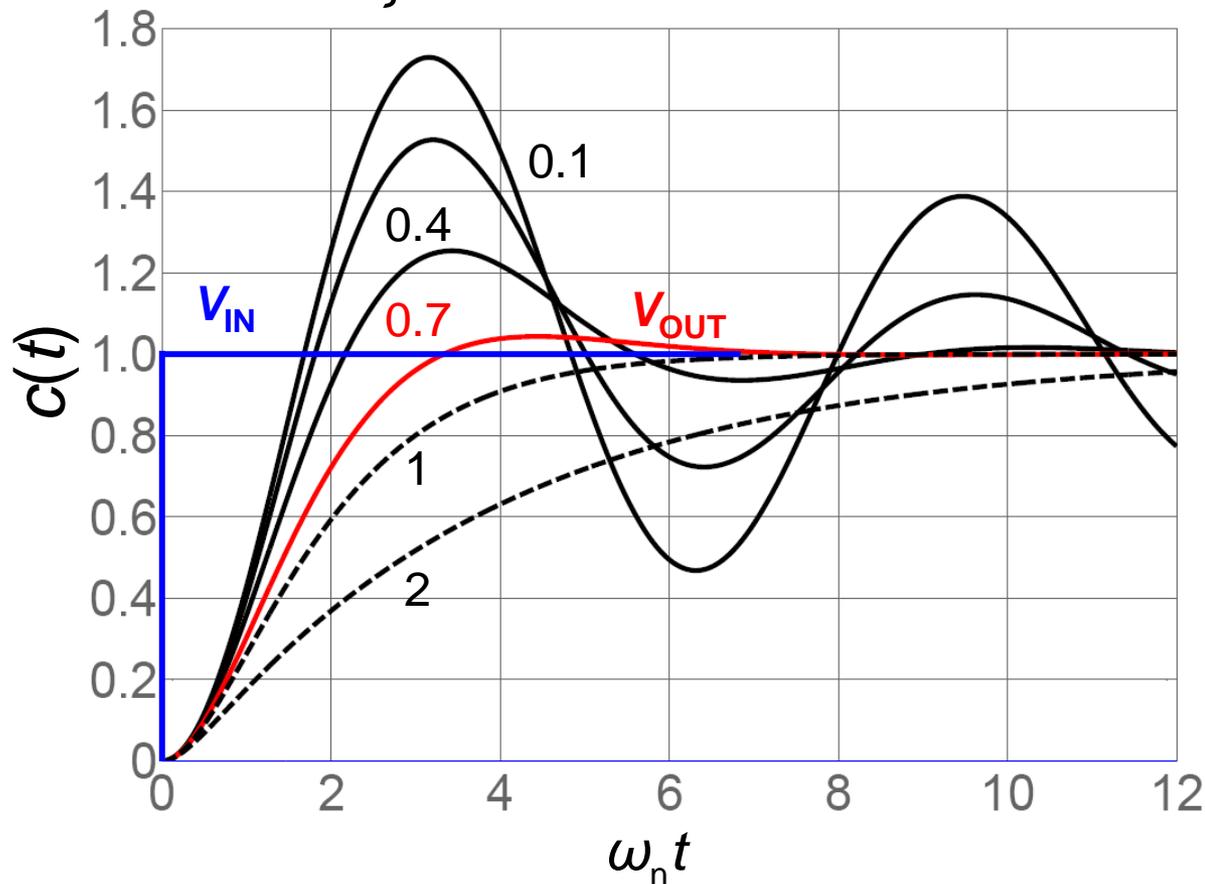


$$\zeta = Q = \frac{\sqrt{2}}{2}$$

$$P_f = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

闭环阶跃冲击响应

$\zeta = 0.1 \ 0.2 \ 0.4 \ 0.7 \ 1 \ 2$



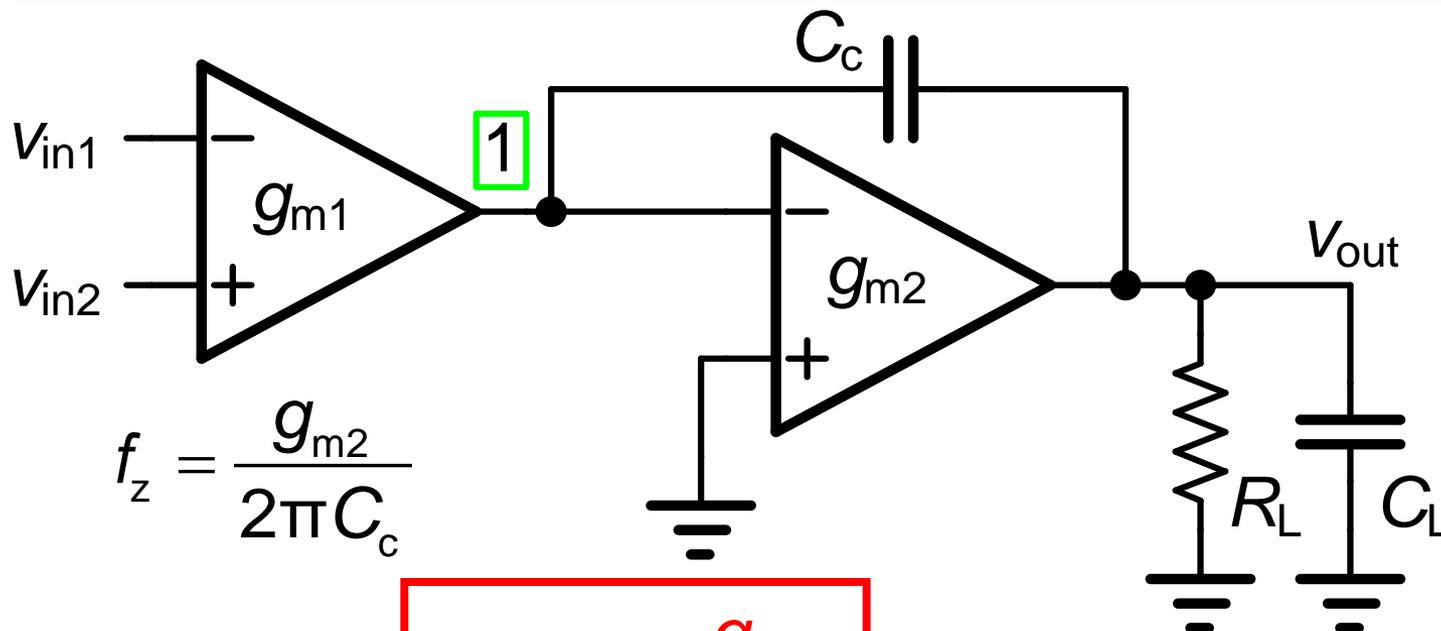
$$\zeta = Q = \frac{\sqrt{2}}{2}$$

$$P_t = 1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

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通用两级放大器 1

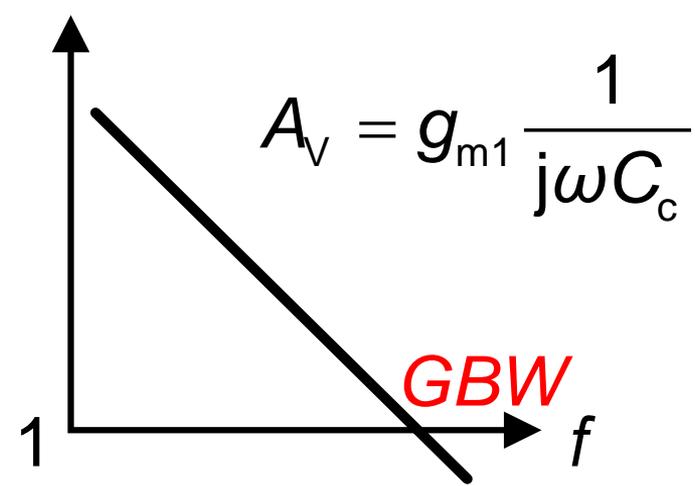


$$f_z = \frac{g_{m2}}{2\pi C_c}$$

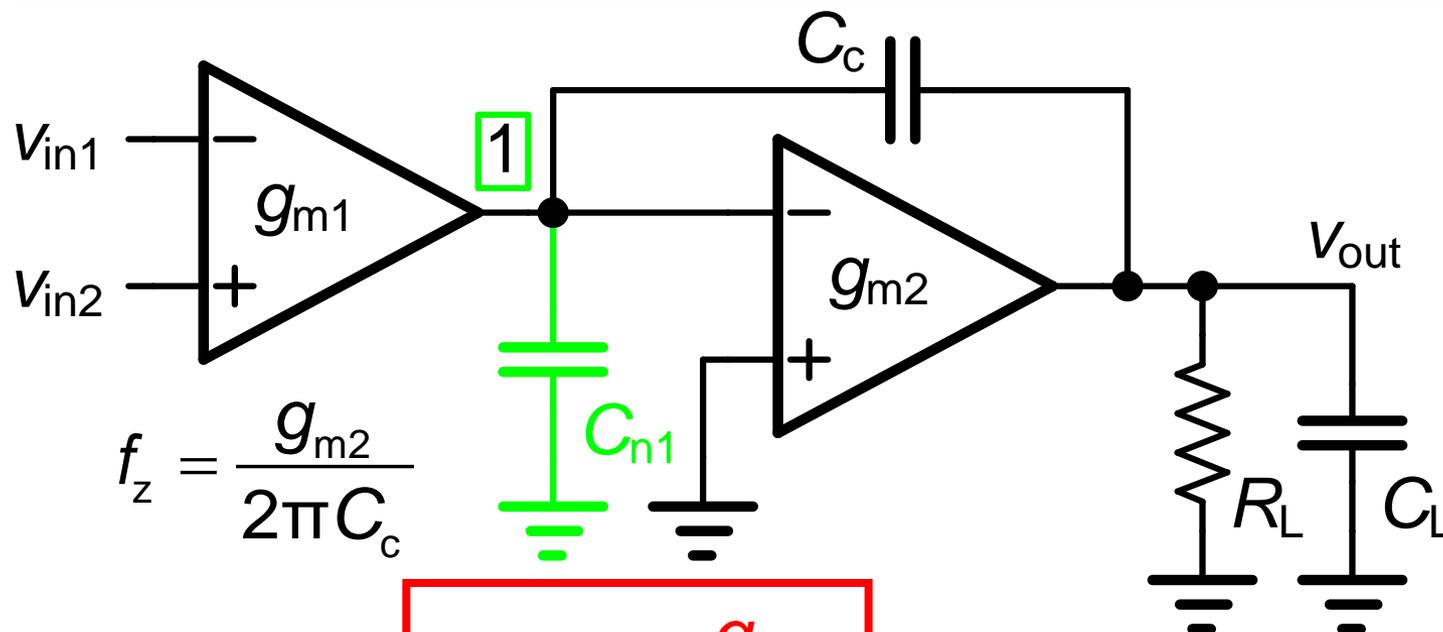
$$|A_V| = 1 \Rightarrow$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd} = \frac{g_{m2}}{2\pi C_L}$$



通用两级放大器 2

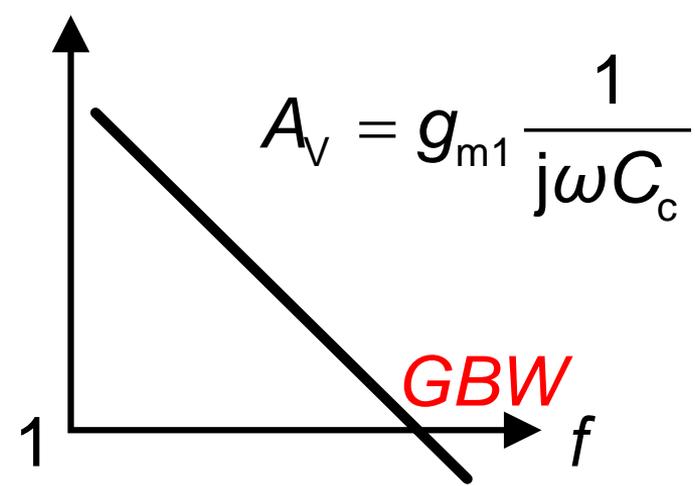


$$f_z = \frac{g_{m2}}{2\pi C_c}$$

$$|A_V| = 1 \Rightarrow$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd} = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \frac{C_{n1}}{C_c}}$$



初步设计两级运放

$$GBW = \frac{g_{m1}}{2\pi C_c} \quad f_{nd} = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \underbrace{\frac{C_{n1}}{C_c}}_{\approx 0.3}}$$

$$\left. \begin{array}{l} f_{nd} = 3GBW \\ \frac{C_{n1}}{C_c} \approx 0.3 \end{array} \right\} \Rightarrow \boxed{\frac{g_{m2}}{g_{m1}} \approx 4 \frac{C_L}{C_c}} \text{ 第二级需要很大的电流!}$$

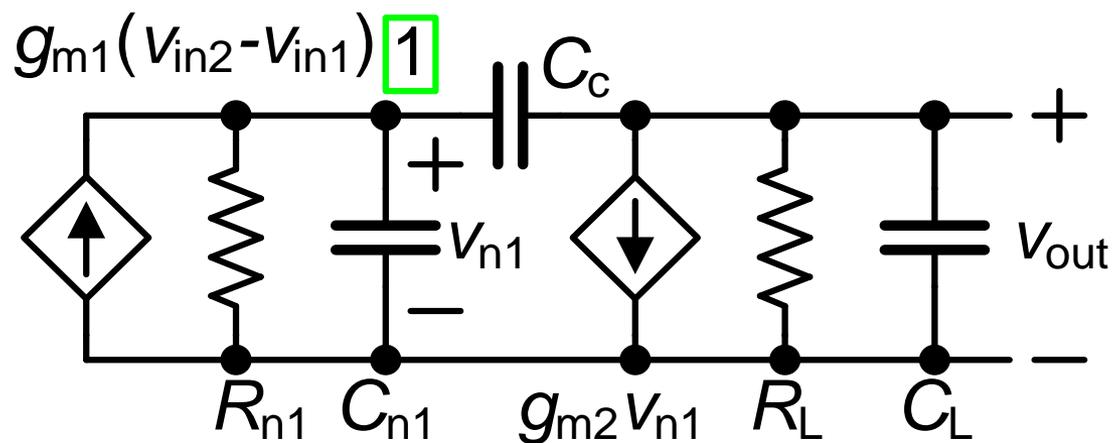
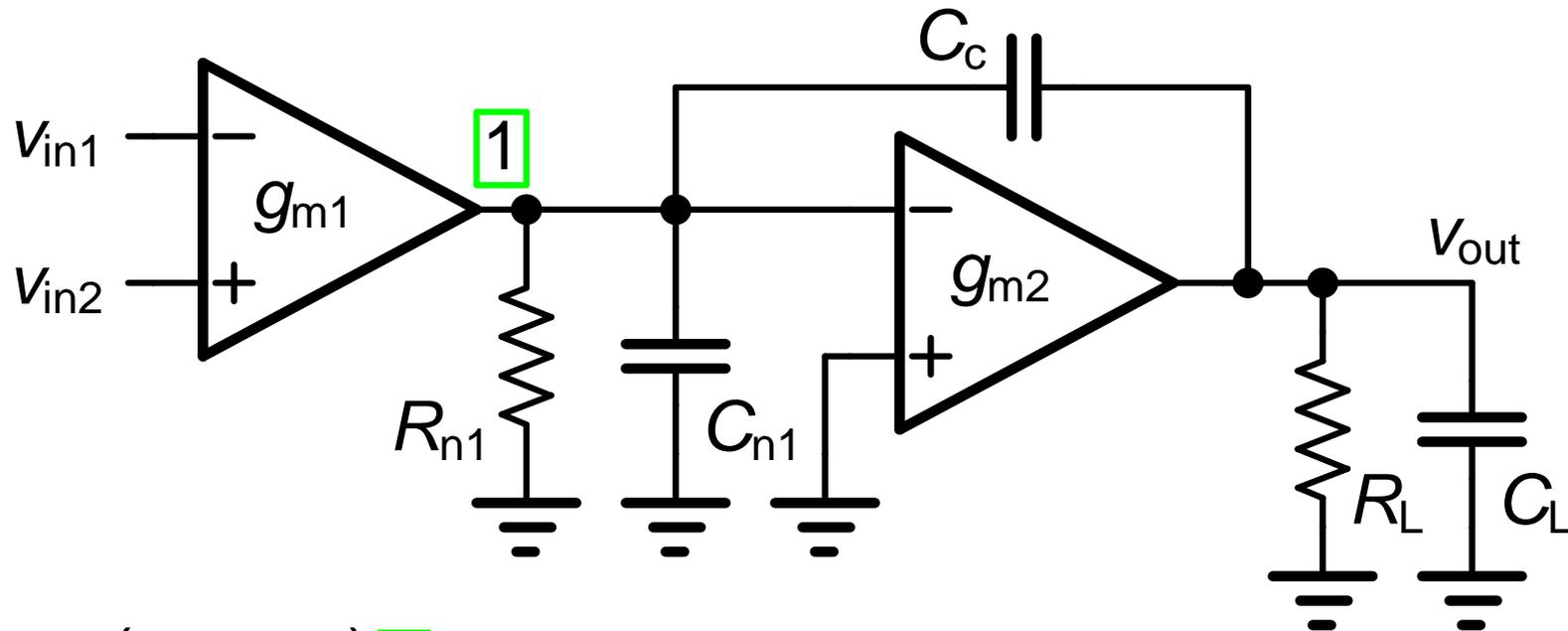
当 $GBW = 100 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$ 时

解：选择 $C_c = 1 \text{ pF}$

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通用两级运放：密勒OTA

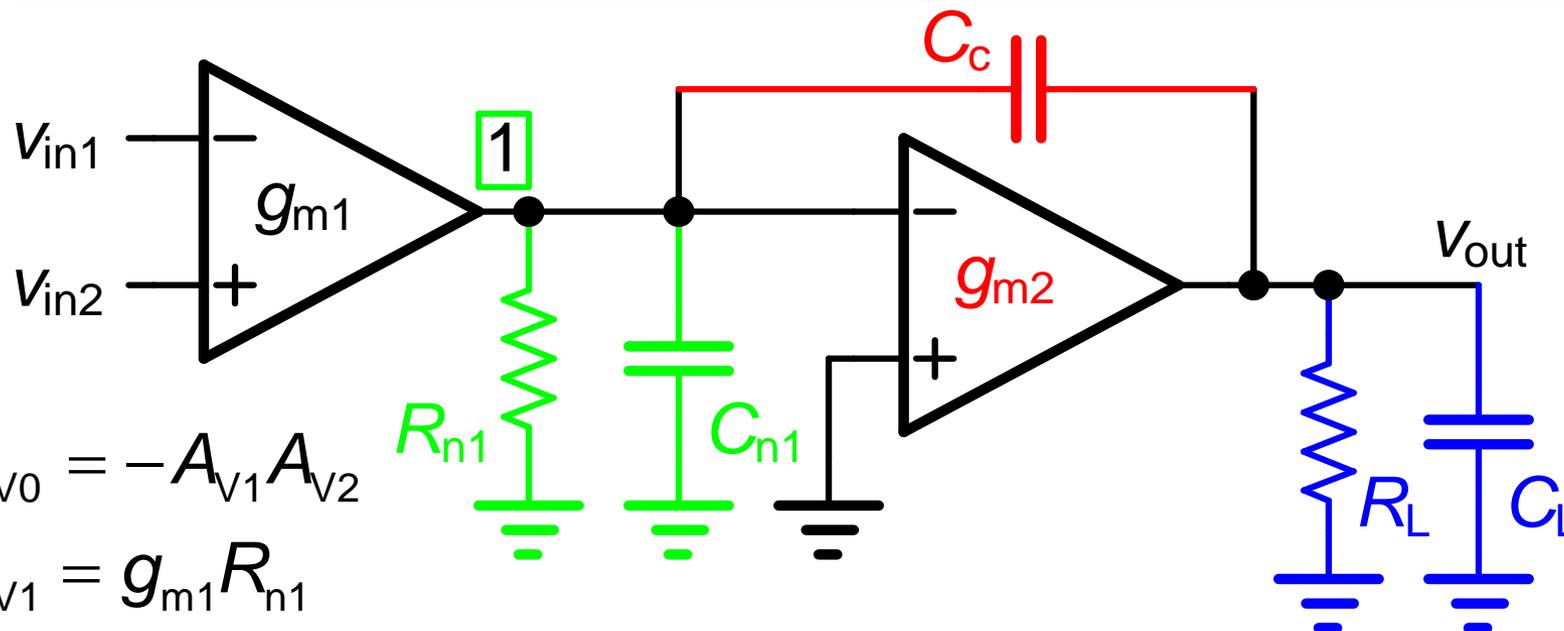


$$A_{V0} = -A_{V1}A_{V2}$$

$$A_{V1} = g_{m1}R_{n1}$$

$$A_{V2} = g_{m2}R_L$$

通用两级运放



$$A_{V0} = -A_{V1}A_{V2}$$

$$A_{V1} = g_{m1}R_{n1}$$

$$A_{V2} = g_{m2}R_L$$

$$1 - \frac{C_c}{g_{m2}}s$$

$$A_V = A_{V0} \frac{1 - \frac{C_c}{g_{m2}}s}{1 + (R_{n1}C_{n1} + R_{n1}C_c + A_{V2}R_{n1}C_c + R_L C_L + R_L C_c)s + R_{n1}R_L C C s^2}$$

$$CC = C_{n1}C_c + C_{n1}C_L + C_c C_L$$

零极点近似

$$A = A_0 \frac{1 - cs}{1 + as + bs^2}$$

零点 $z = \frac{1}{c}$

极点 $s_1 = -\frac{1}{a}$ $s_2 = -\frac{a}{b}$

$$a \uparrow \Rightarrow s_1 \downarrow \text{ and } s_2 \uparrow$$

如果 $s_2 \gg s_1$:

s 很小

$$1 + as + bs^2 = 0$$

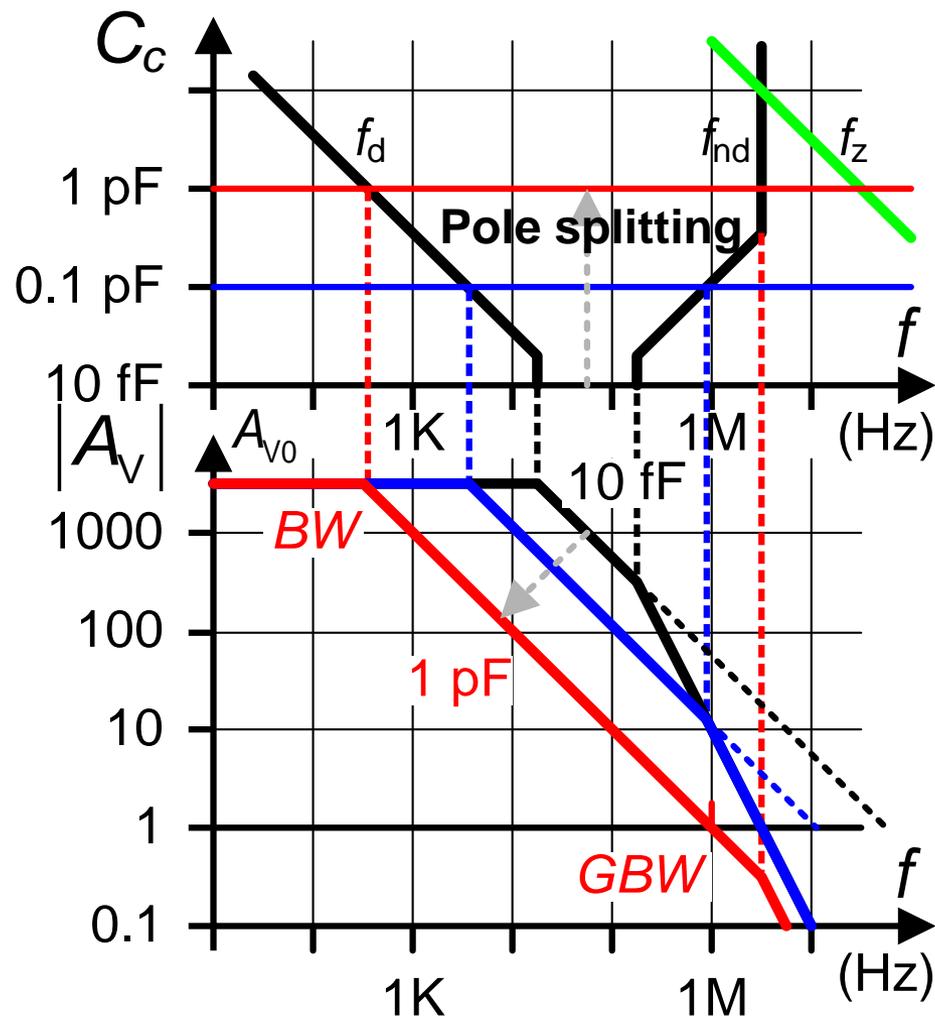
$s_1 = -\frac{1}{a}$

s 很大

$$1 + as + bs^2 = 0$$

$s_2 = -\frac{a}{b}$

密勒OTA: 用 C_c 进行极点分离



C_c 取较大值，
进行极点分离：

$$f_d = \frac{1}{2\pi A_{V2} R_{n1} C_c}$$

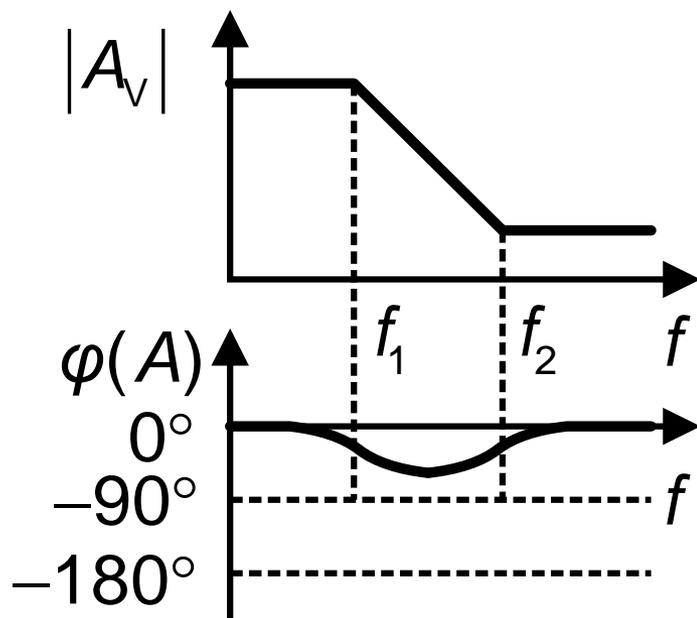
$$f_z = \frac{g_{m2}}{2\pi C_c} \text{ 为正零点!}$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

正零点的作用

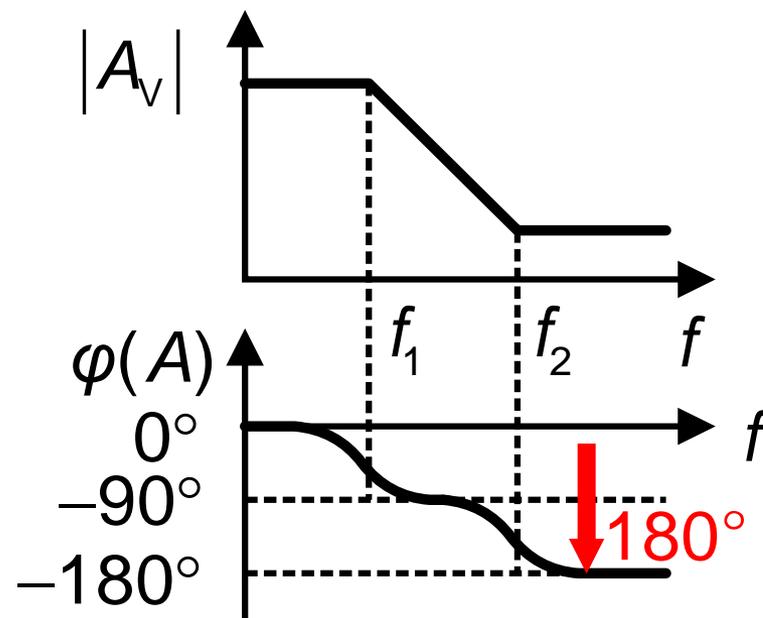
负零点

$$A_V = A_{V0} \frac{1 + jf/f_2}{1 + jf/f_1}$$



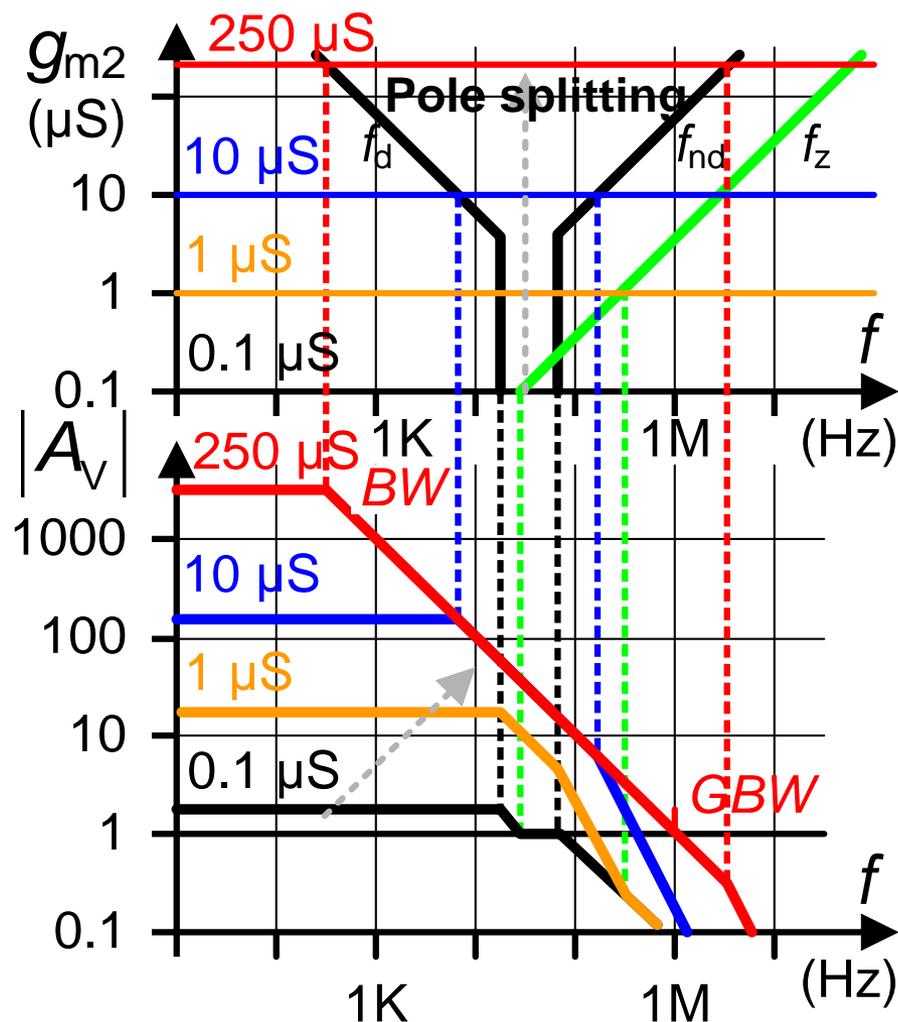
正零点

$$A_V = A_{V0} \frac{1 - jf/f_2}{1 + jf/f_1}$$



对于相位，正零点像一个负极点!!!

密勒OTA: 用 g_{m2} 进行极点分离



g_{m2} 取较大值,
进行极点分离:

$$f_d = \frac{1}{2\pi A_{V2} R_{n1} C_c}$$

$$f_z = \frac{g_{m2}}{2\pi C_c} \text{ 为正零点!}$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

极点分离的方式

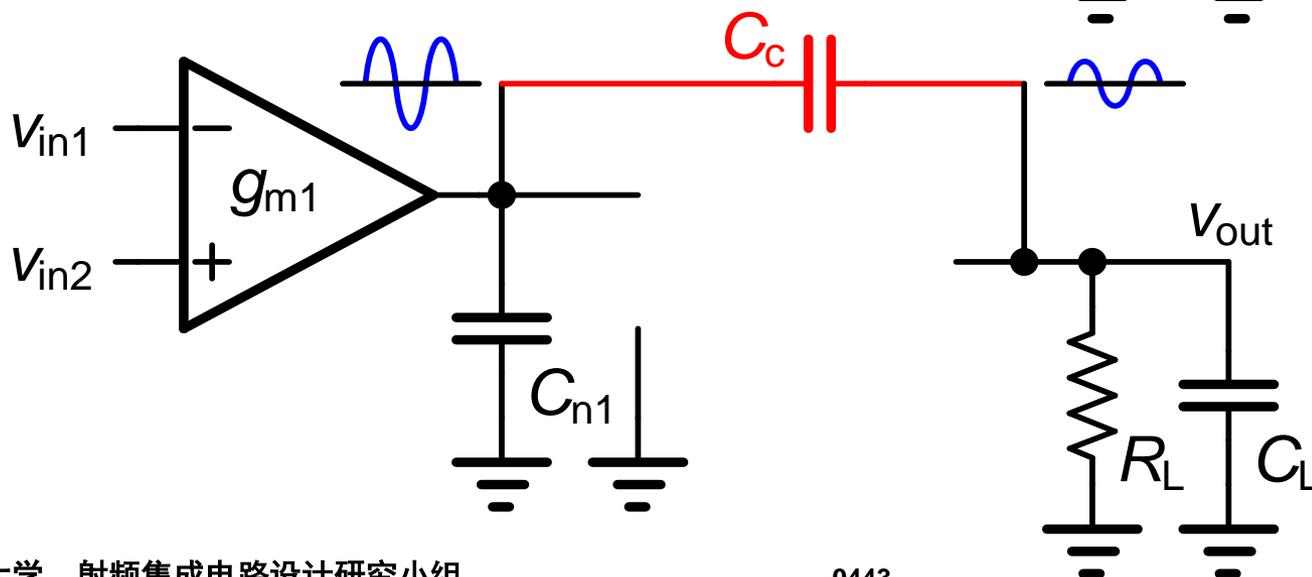
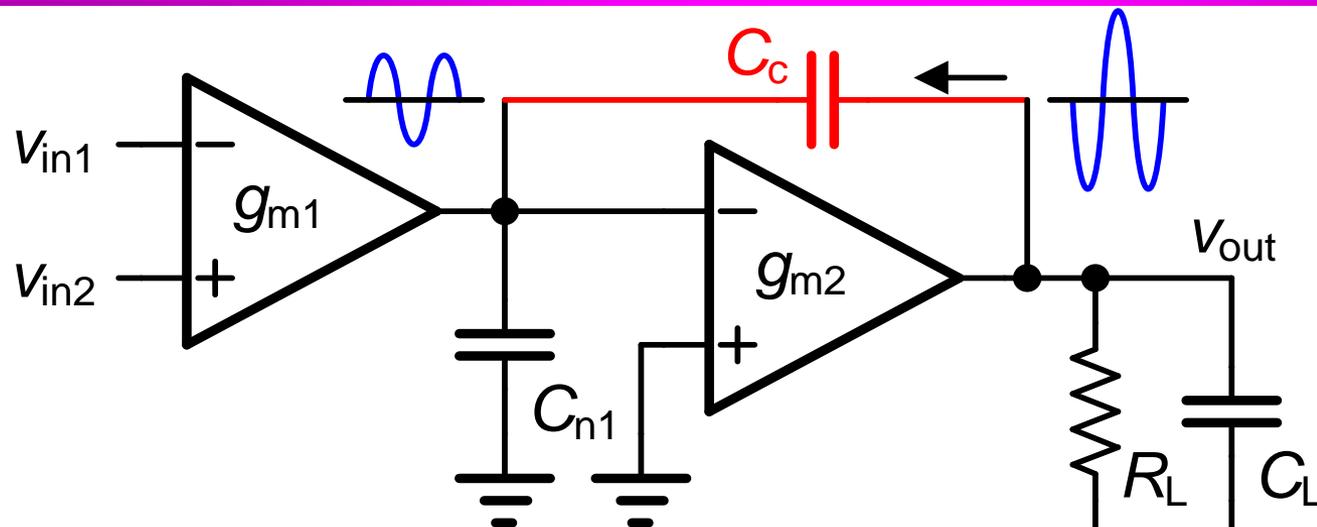
$$\frac{g_{m2}}{g_{m1}} \approx 4 \frac{C_L}{C_c} \quad \text{或表示为} \quad g_{m2} C_c \approx 4 g_{m1} C_L$$

g_{m2} 和 C_c 二者均可

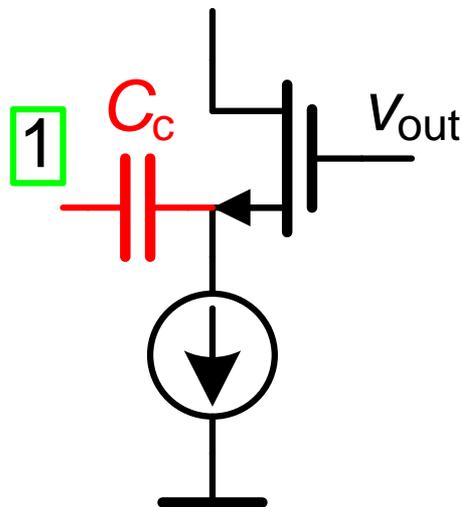
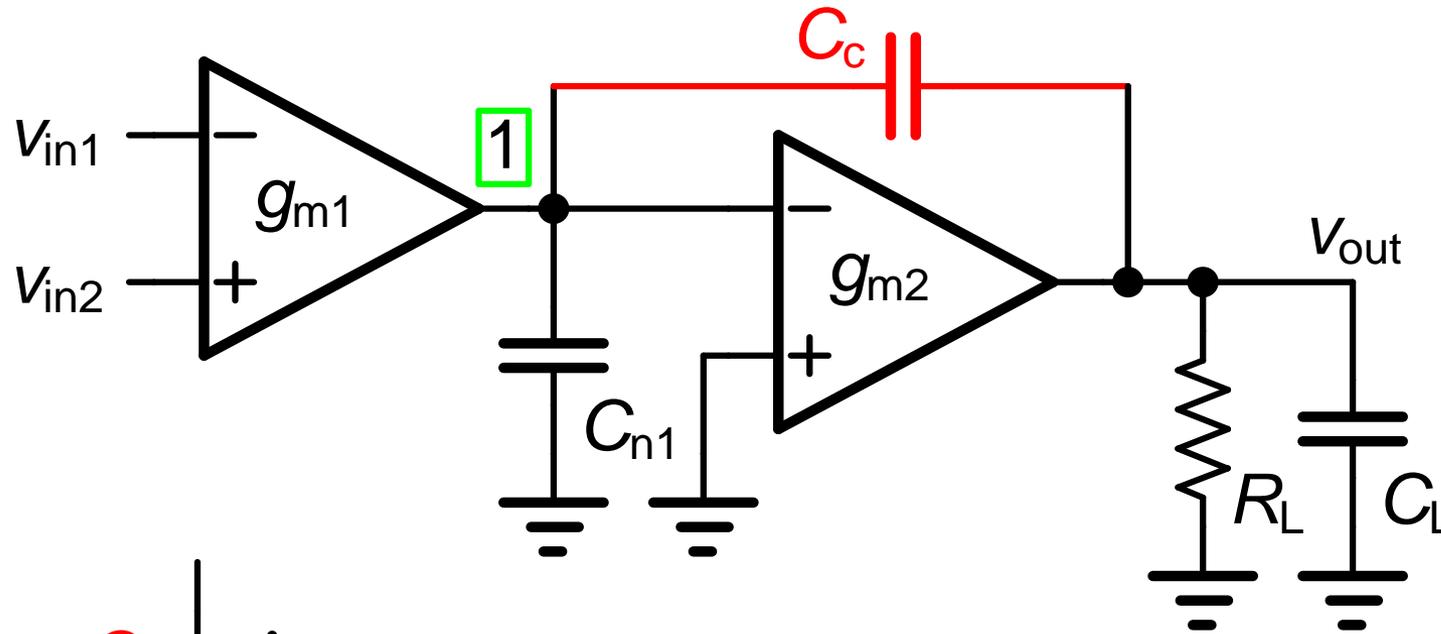
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- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

密勒效应的正反馈特性产生正零点



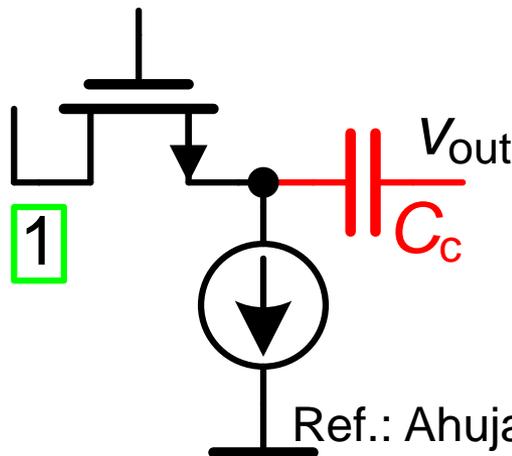
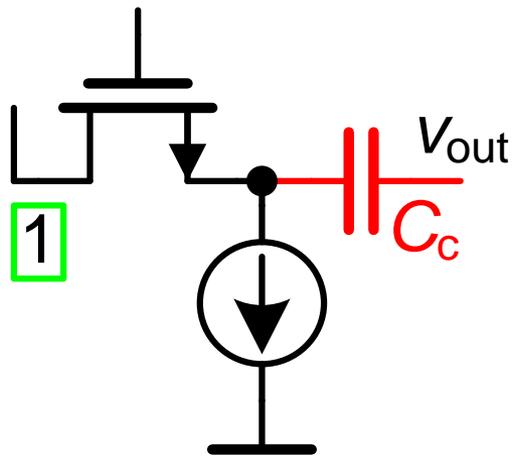
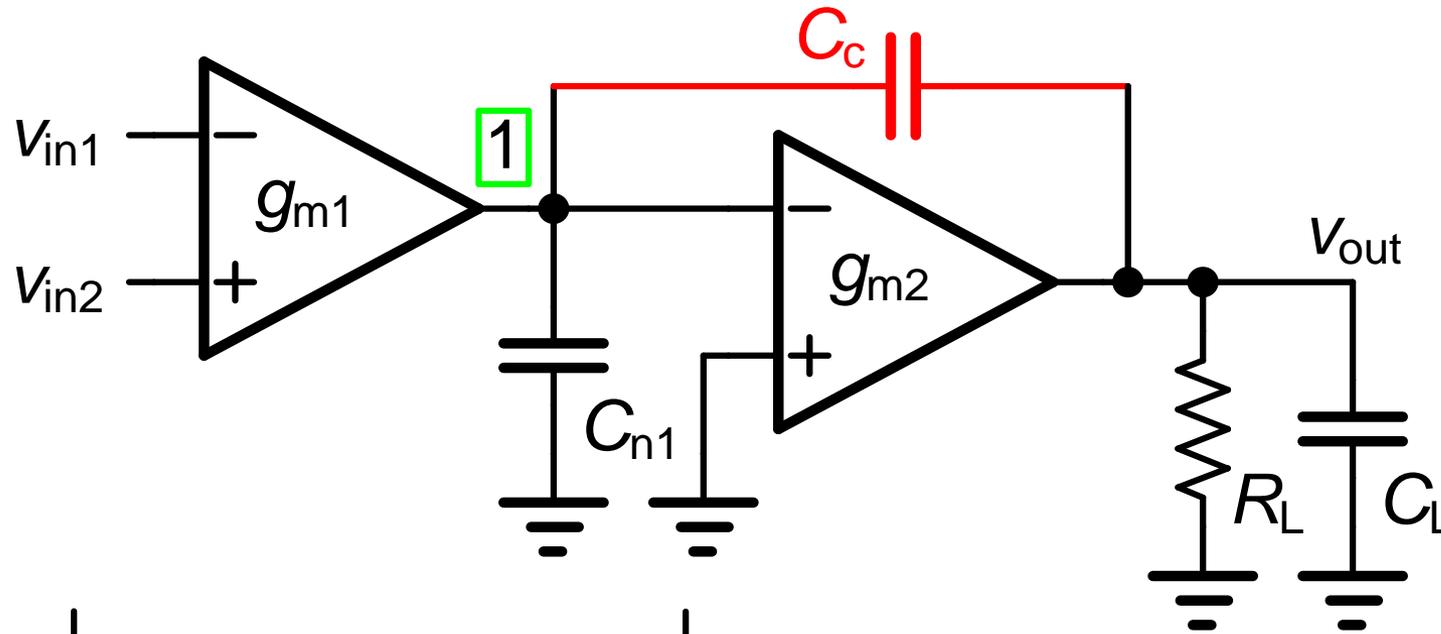
截断正馈通路 1



电压缓冲源极跟随器

Ref.: Tsividis, JSSC Dec.76, 748-753

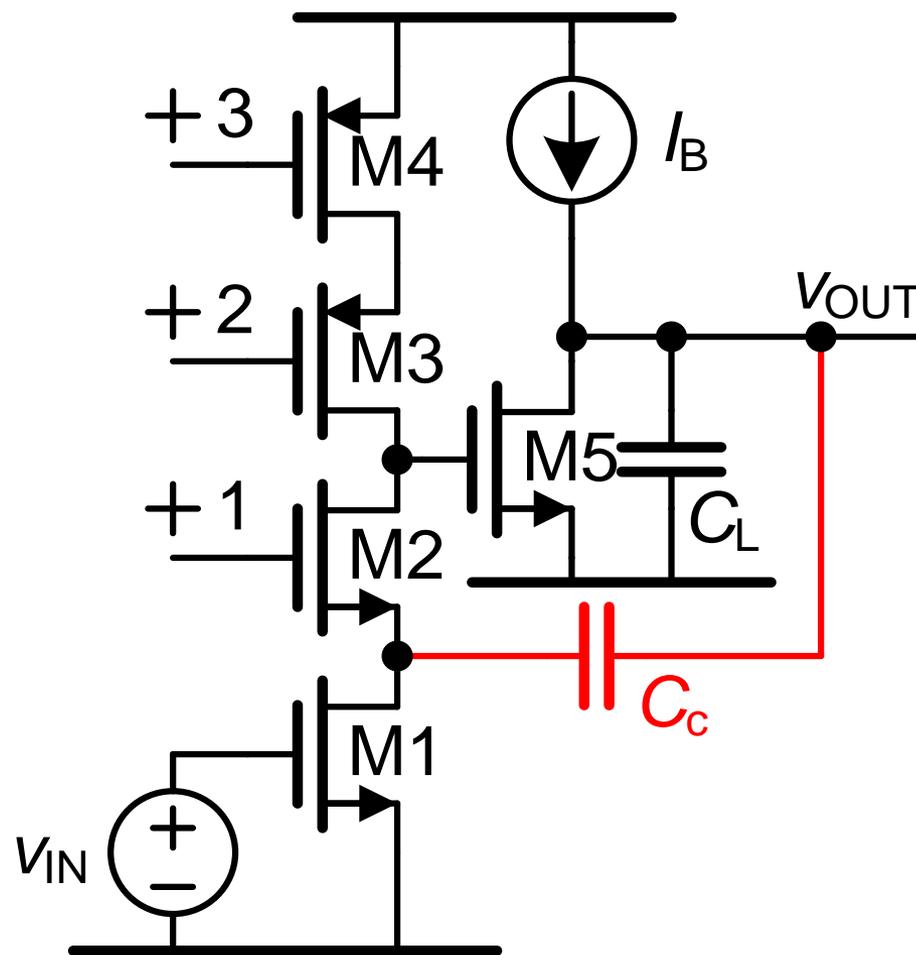
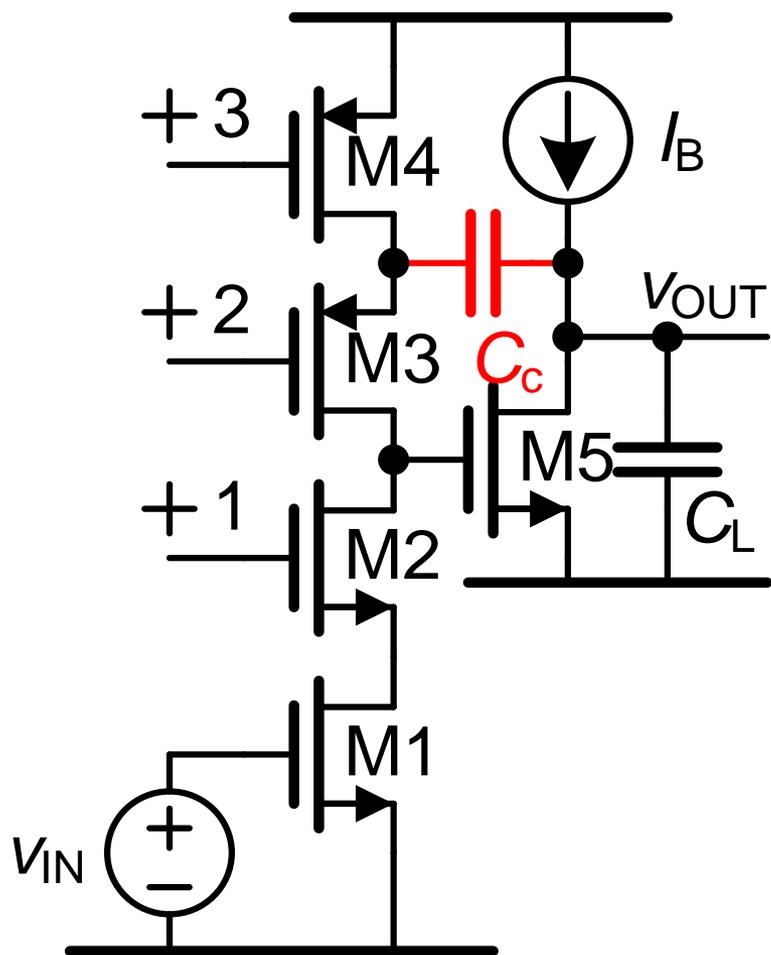
截断正馈通路 2



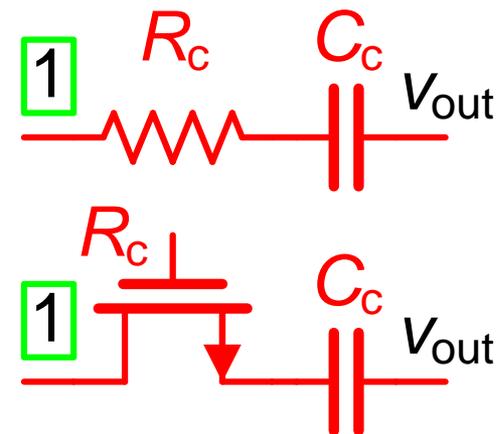
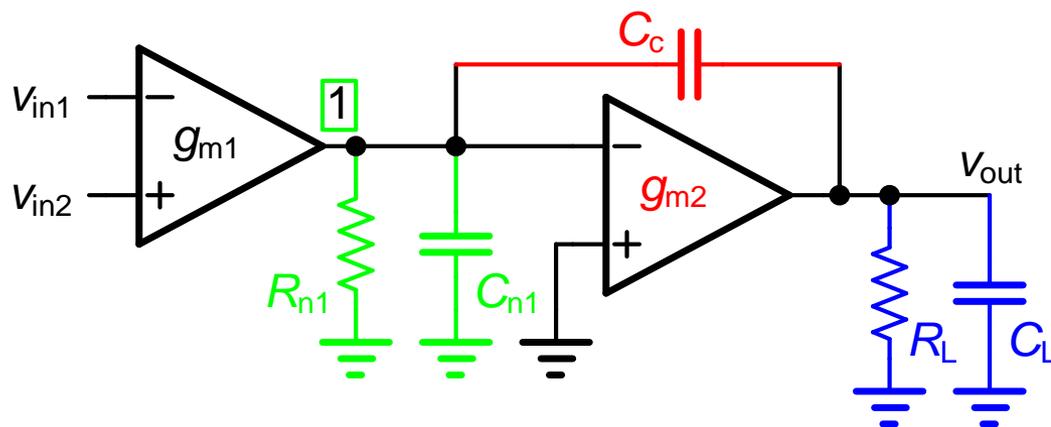
电流缓冲共源共栅

Ref.: Ahuja, JSSC Dec 83, 629-633

共源共栅密勒补偿



截断正馈通路 3



$$f_z = \frac{1}{2\pi C_c (1/g_{m2} - R_c)}$$

$R_c = 1/g_{m2}$ 无穷远处零点

$R_c > 1/g_{m2}$ 负零点

Ref.: Senderovics, JSSC Dec 78, 760-766

负零点补偿

$$R_c \gg 1/g_{m2} \Rightarrow f_z = -\frac{1}{2\pi C_c R_c}$$

$$f_z = 3GBW \Rightarrow R_c = \frac{1}{3g_{m1}}$$

选择

$$\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}}$$

练习：两级运放

已知： $GBW = 50 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$ 和 $V_{GS1} - V_T = 0.2 \text{ V}$

求： I_{DS1} 、 I_{DS2} 、 C_c 和 R_c

选择 $C_c = 1 \text{ pF} \Rightarrow g_{m1} = 2\pi C_c GBW = 315 \mu\text{S}$

$$I_{DS1} = 31.5 \mu\text{A} \quad 1/g_{m1} \approx 3.2 \text{ k}\Omega$$

$f_{nd} = 150 \text{ MHz} \Rightarrow g_{m2} = 2\pi C_L 4GBW = 8g_{m1} = 2520 \mu\text{S}$

$$I_{DS2} = 252 \mu\text{A} \quad 1/g_{m2} \approx 400 \Omega$$

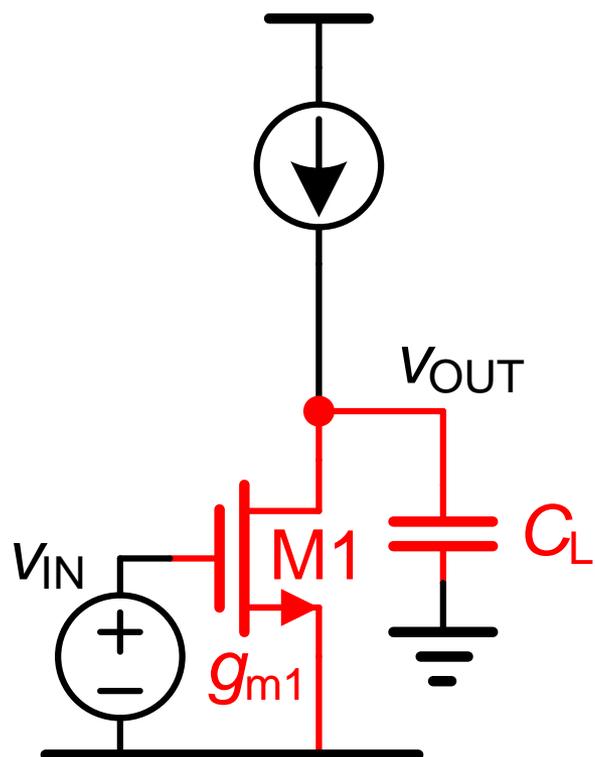
$$\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}} \Rightarrow 400 \Omega < R_c < 1 \text{ k}\Omega$$

$$R_c \approx 400\sqrt{2.5} \approx 640 \Omega \pm 60\%$$

目录

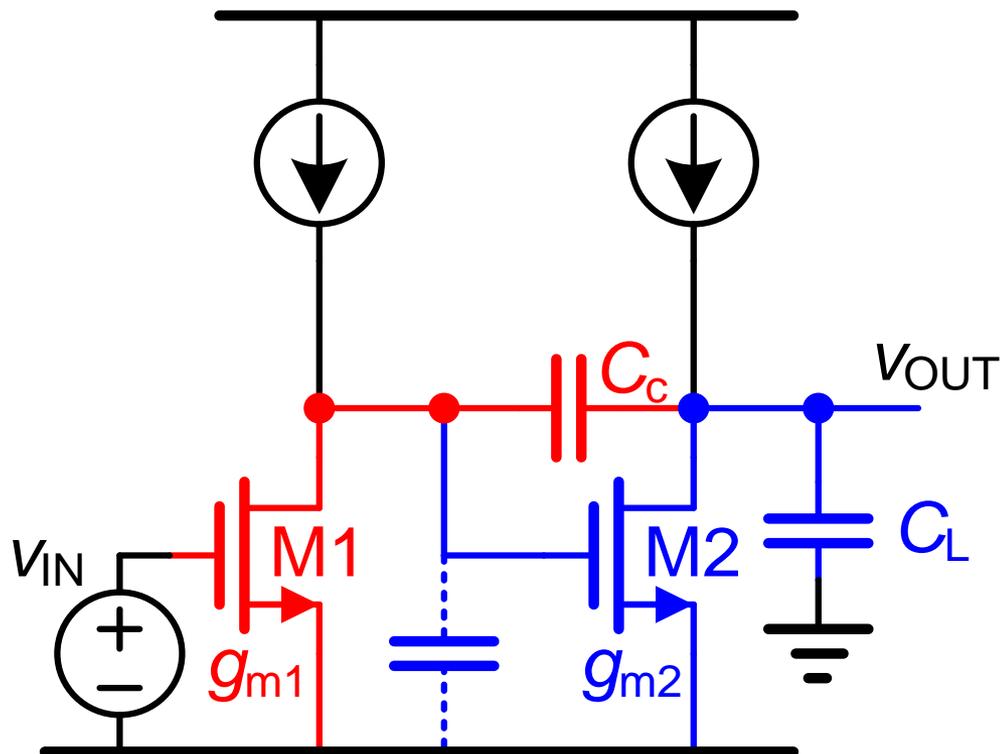
- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

一级CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi C_L}$$

两级密勒CMOS OTA

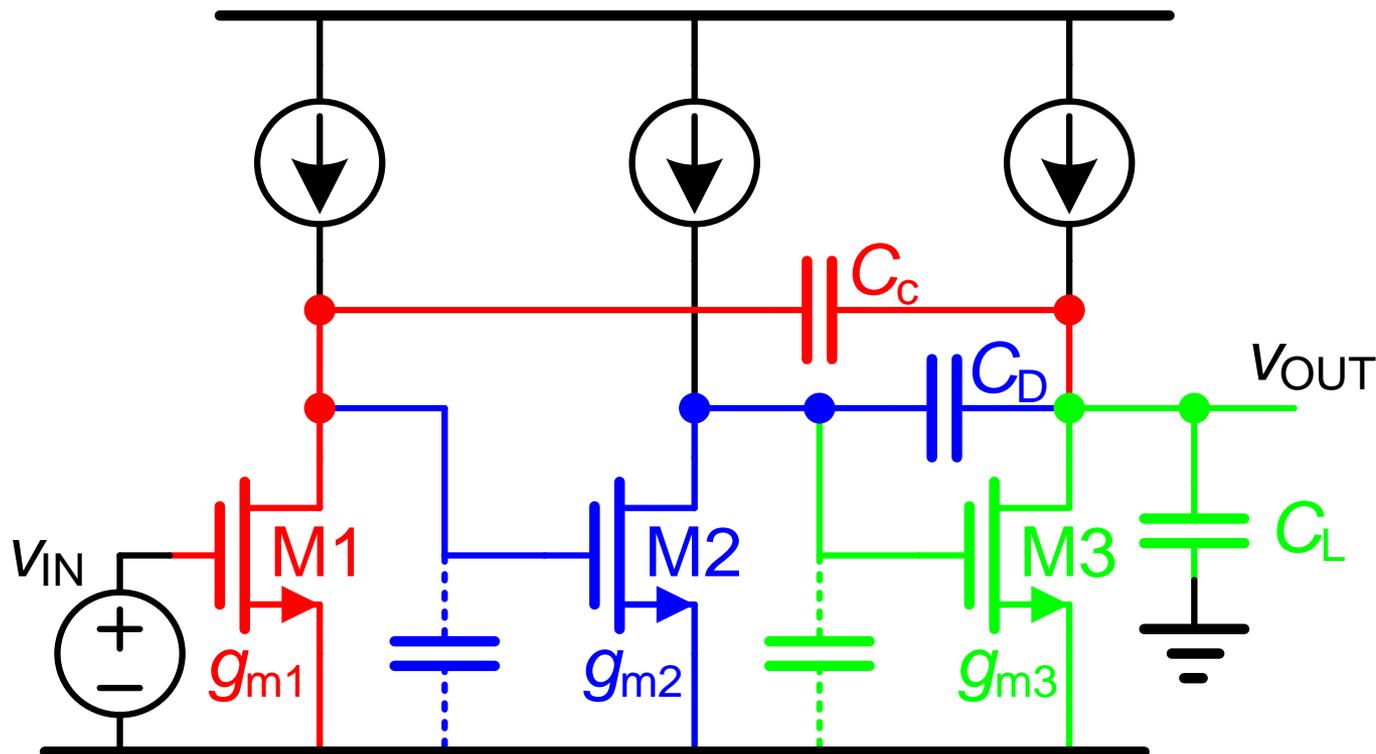


$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

三级嵌套密勒CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi C_c}$$

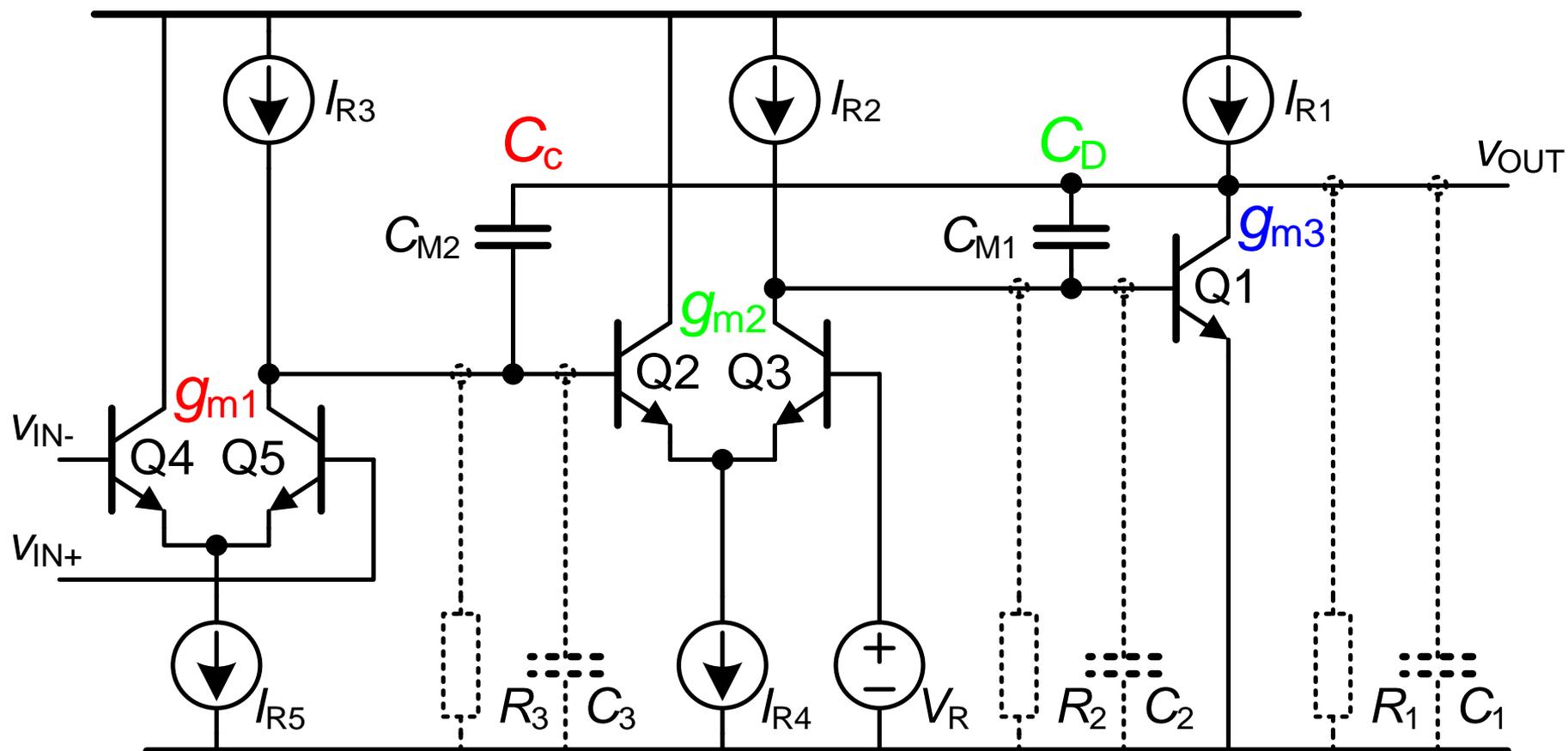
$$f_{nd1} = \frac{g_{m2}}{2\pi C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

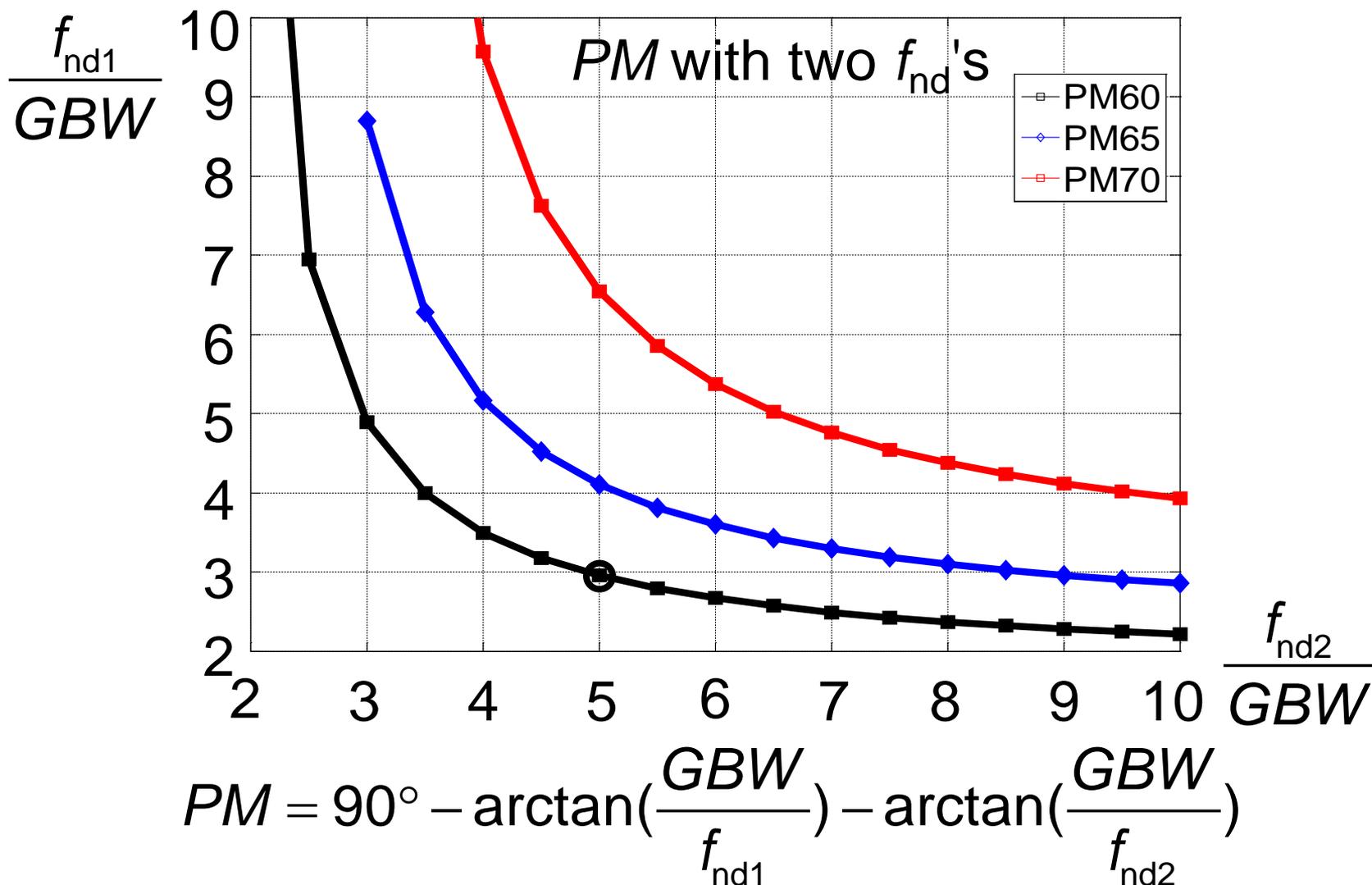
$$f_{nd2} = 5GBW$$

差分对构建嵌套密勒OTA

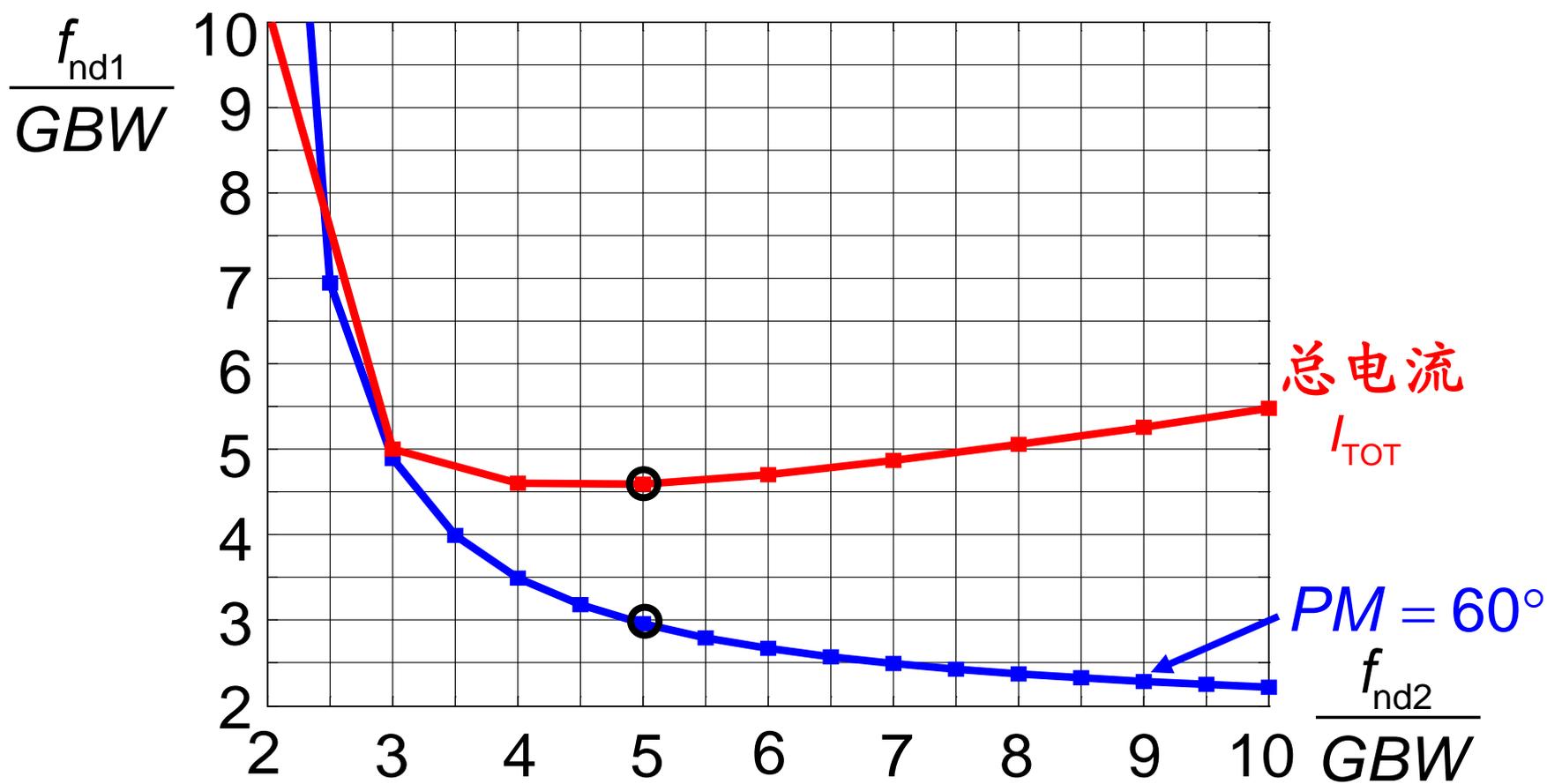


Ref.: Huijsing, JSSC Dec.85, pp.1144-1150

PM与两个非主极点的关系



功耗与两个非主极点的关系



$$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3}$$

三级运放的初步设计

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

$$f_{nd2} = 5GBW$$

选择 $C_D \approx C_c \Rightarrow$

$$\frac{g_{m2}}{g_{m1}} \approx 3 \qquad \frac{g_{m3}}{g_{m1}} \approx 5 \frac{C_L}{C_c}$$

输出级需要大电流!

练习：三级运放设计

已知： $GBW = 50 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$ 和 $V_{GS} - V_T = 0.2 \text{ V}$

求： I_{DS1} 、 I_{DS2} 、 I_{DS3} 、 C_c 和 C_D

$$\text{选择 } C_c = C_D = 1 \text{ pF} \Rightarrow g_{m1} = 2\pi C_c GBW = 315 \mu\text{S}$$

$$I_{DS1} = 31.5 \mu\text{A}$$

$$f_{nd1} = 150 \text{ MHz} \Rightarrow g_{m2} = 2\pi C_D 3GBW = 3g_{m1} = 945 \mu\text{S}$$

$$I_{DS2} = 94.5 \mu\text{A}$$

$$f_{nd2} = 250 \text{ MHz} \Rightarrow g_{m3} = 2\pi C_L 5GBW = 10g_{m1} = 3150 \mu\text{S}$$

$$I_{DS3} = 315 \mu\text{A}$$

一/两/三级运放的比较

$$GBW = 50 \text{ MHz} \quad C_L = 2 \text{ pF}$$

单级: $I_{DS1} = 31.5 \text{ } \mu\text{A}$

$$I_{TOT} = 2I_{DS1} = 63 \text{ } \mu\text{A}$$

两级: 选择 $C_c = 1 \text{ pF}$ $I_{DS1} = 31.5 \text{ } \mu\text{A}$ $I_{DS2} = 252 \text{ } \mu\text{A}$

$$I_{TOT} = 2I_{DS1} + I_{DS2} = 315 \text{ } \mu\text{A}$$

三级: 选择 $C_c = C_D = 1 \text{ pF}$

$$I_{DS1} = 31.5 \text{ } \mu\text{A} \quad I_{DS2} = 94.5 \text{ } \mu\text{A} \quad I_{DS3} = 315 \text{ } \mu\text{A}$$

$$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3} = 576 \text{ } \mu\text{A}$$