

# 模拟集成电路设计原理

## (Principle of Analog Integrated Circuit Design, INF0130025.02)

2014年11月13日

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<http://rfic.fudan.edu.cn/Courses.htm>

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# 运算放大器的稳定性

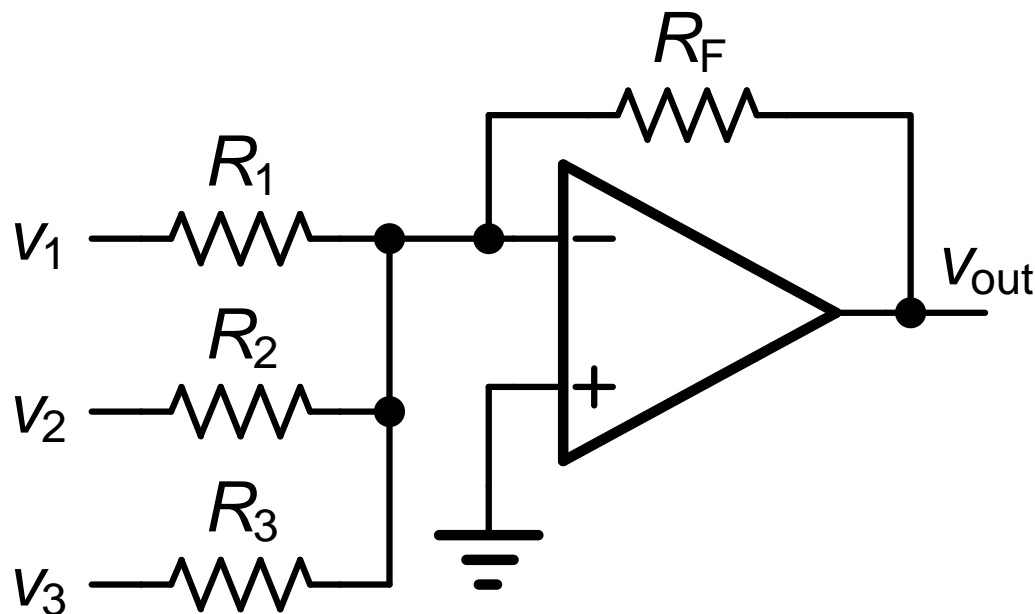
# 目录

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- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

# 运算放大器的运算功能

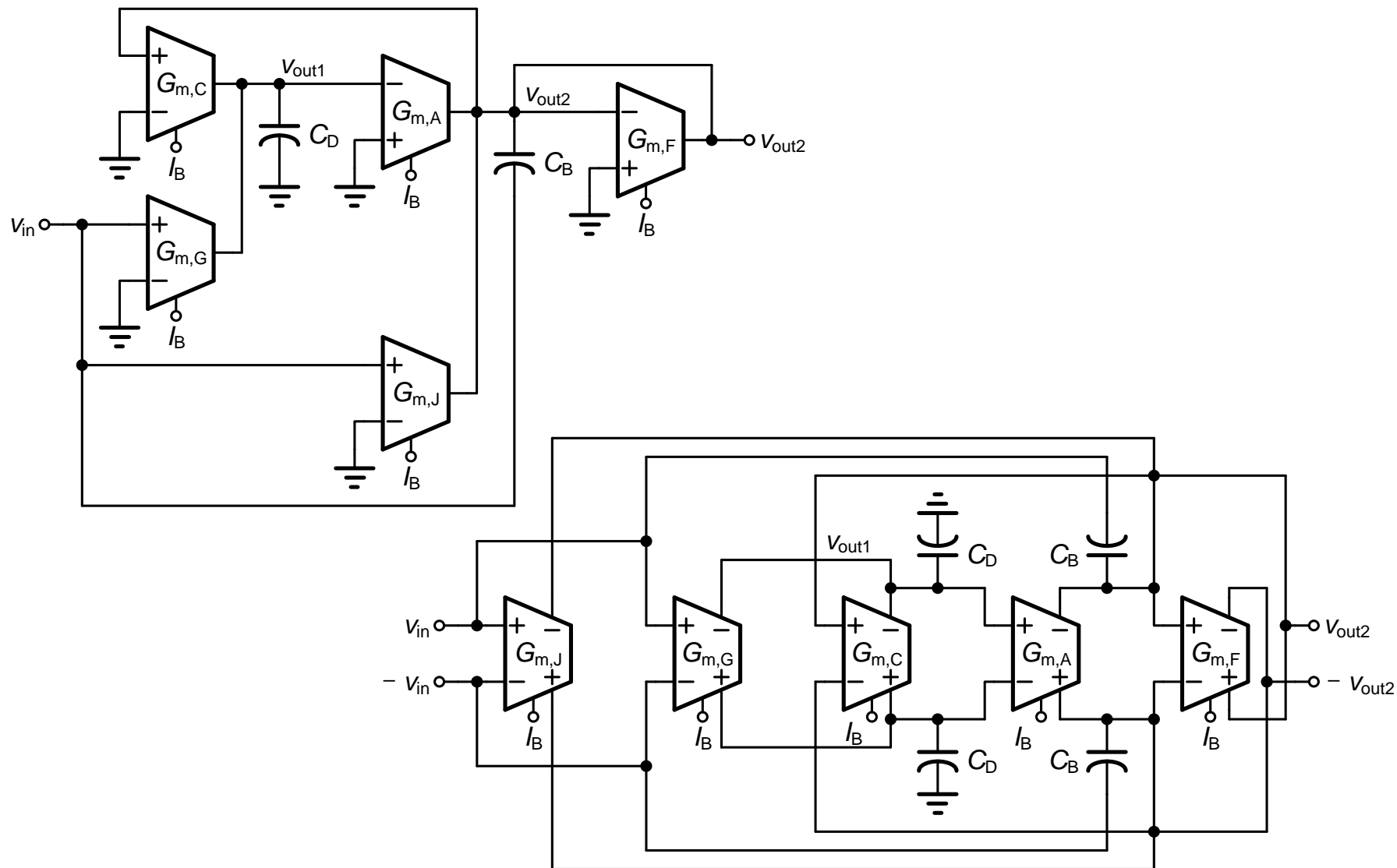


$$-\frac{V_{out}}{R_F} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

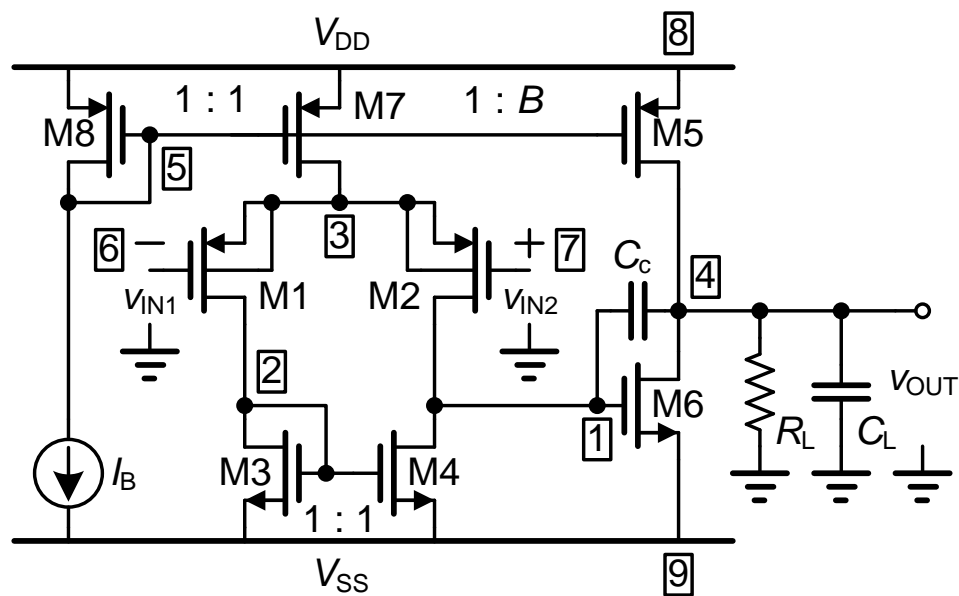
要求：高增益  
高速  
低噪声  
低功耗

运放性能指标：  
 高电压增益  
 差分输入电压  $\approx 0$   
 输入电流 = 0  
 高带宽  
 增益带宽乘积 **GBW** 非常，非常高！

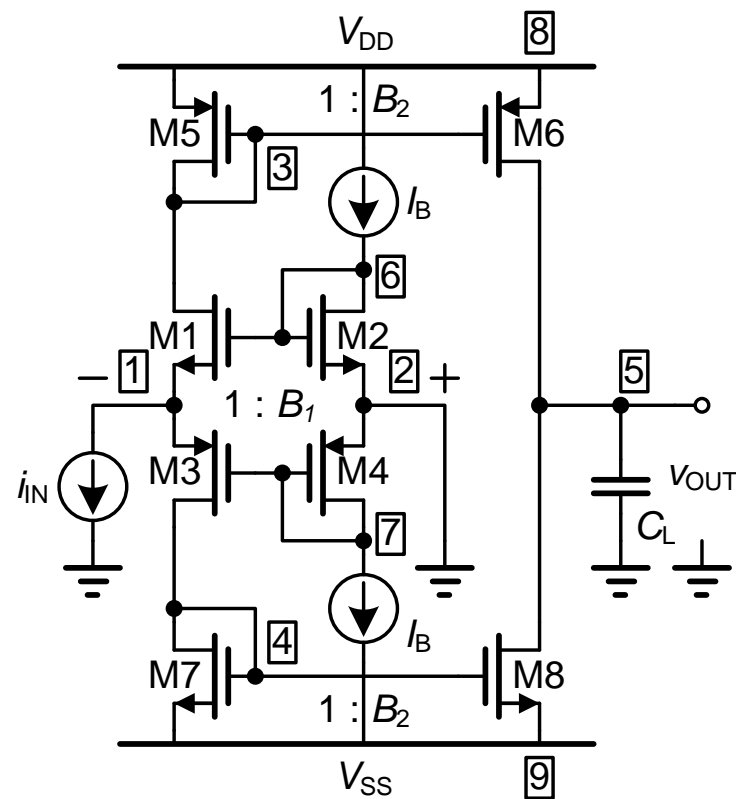
# 单端？全差分？



# 电压输入？ 电流输入？



电压输入  
电流输出

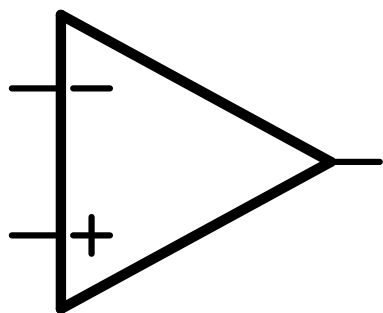


电流输入  
电流输出

# 分类

Opamp

电压  
放大器



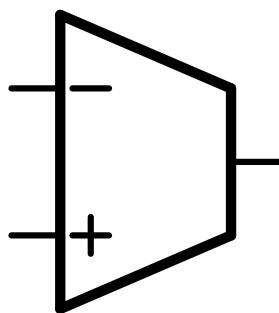
$$A_V = \frac{V_{out}}{V_{in}}$$

$$A_V =$$

*GBW*

OTA

跨导  
放大器

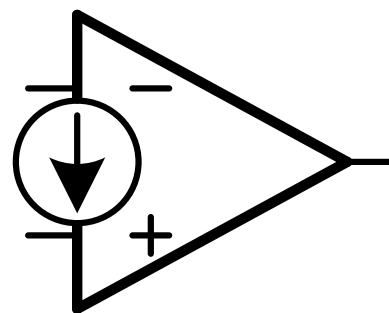


$$A_G = \frac{i_{out}}{V_{in}}$$

$$A_V = A_G R_L$$

OCA

电流  
放大器

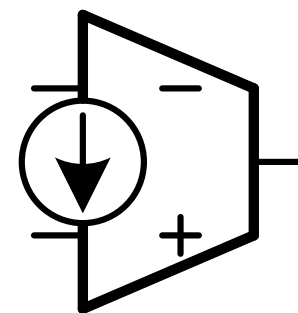


$$A_I = \frac{i_{out}}{i_{in}}$$

$$A_V = A_I \frac{R_L}{R_S}$$

CM amp

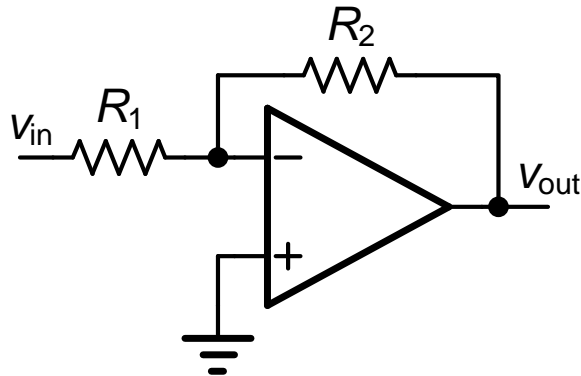
跨阻  
放大器



$$A_R = \frac{V_{out}}{i_{in}}$$

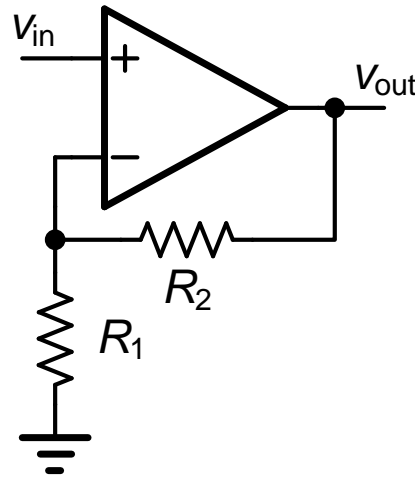
$$A_V = A_R \frac{1}{R_S}$$

# 反馈结构



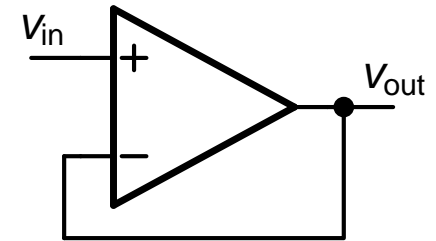
$$A_V = -\frac{R_2}{R_1}$$

$$R_{IN} = R_1$$



$$A_V = 1 + \frac{R_2}{R_1}$$

$$R_{IN} = \infty$$

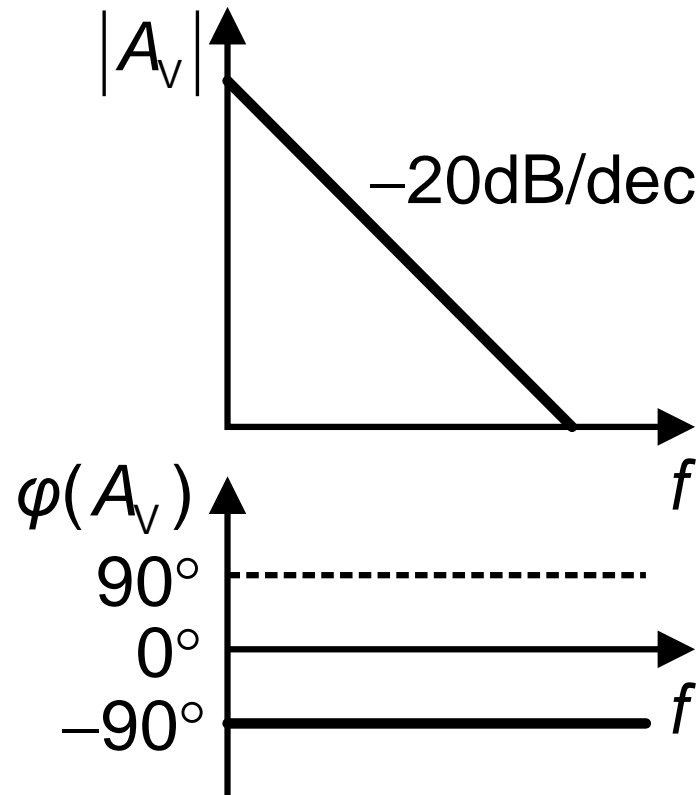
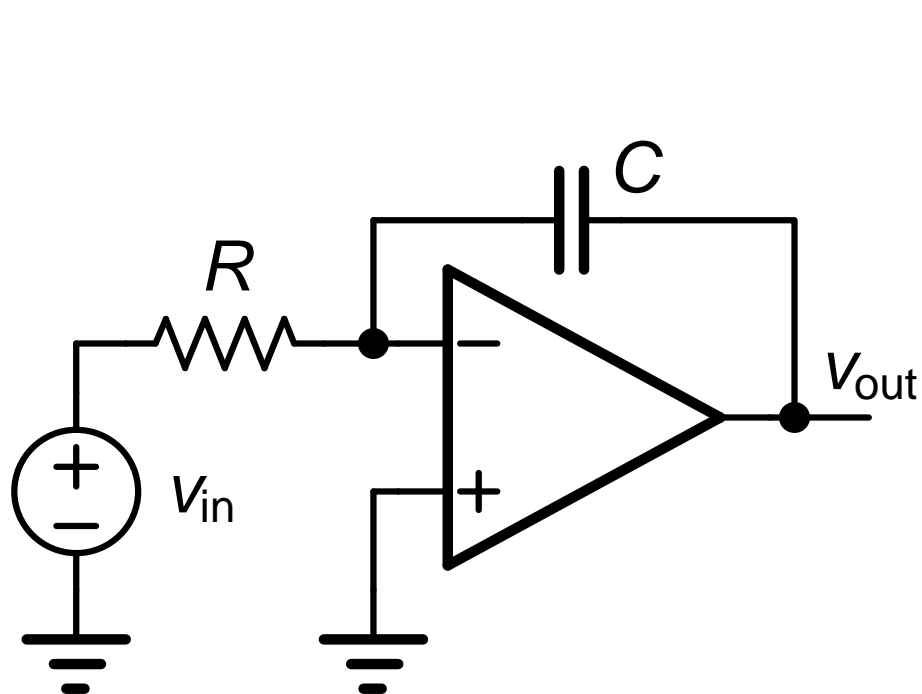


$$A_V = 1$$

$$R_{IN} = \infty$$



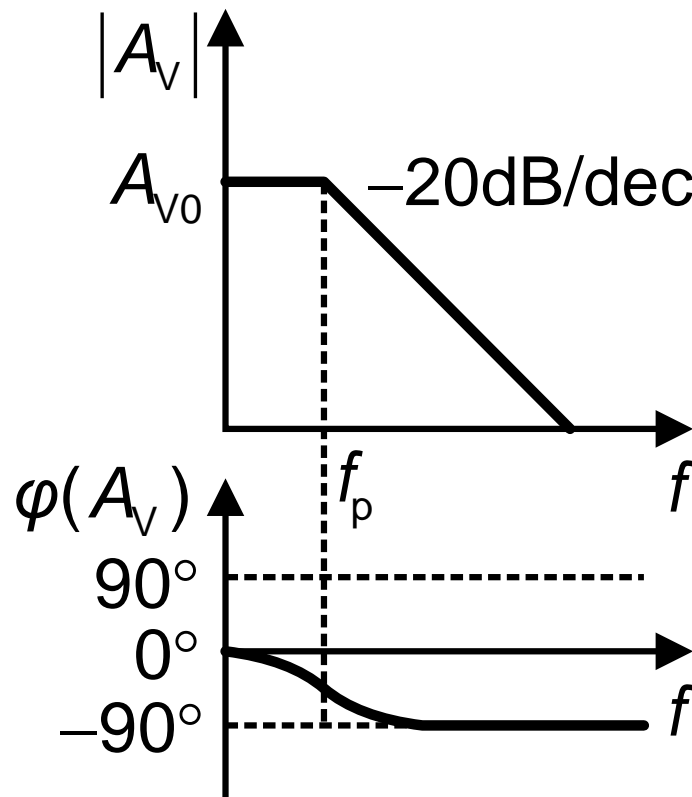
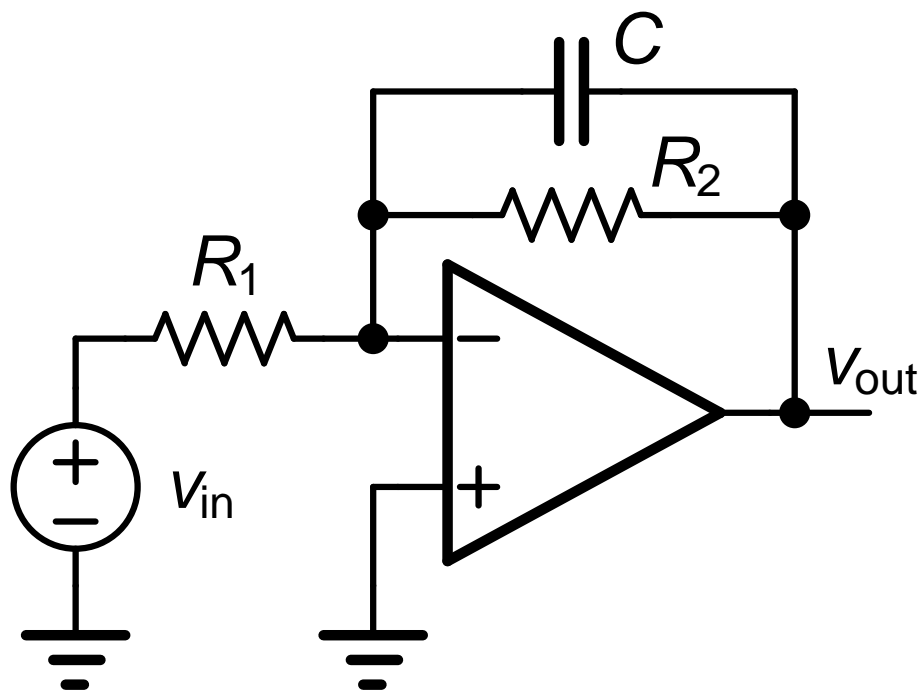
# 积分器



$$A_v = -\frac{1}{j \frac{f}{f_p}}$$

$$f_p = \frac{1}{2\pi RC}$$

# 有损积分器：低通滤波器

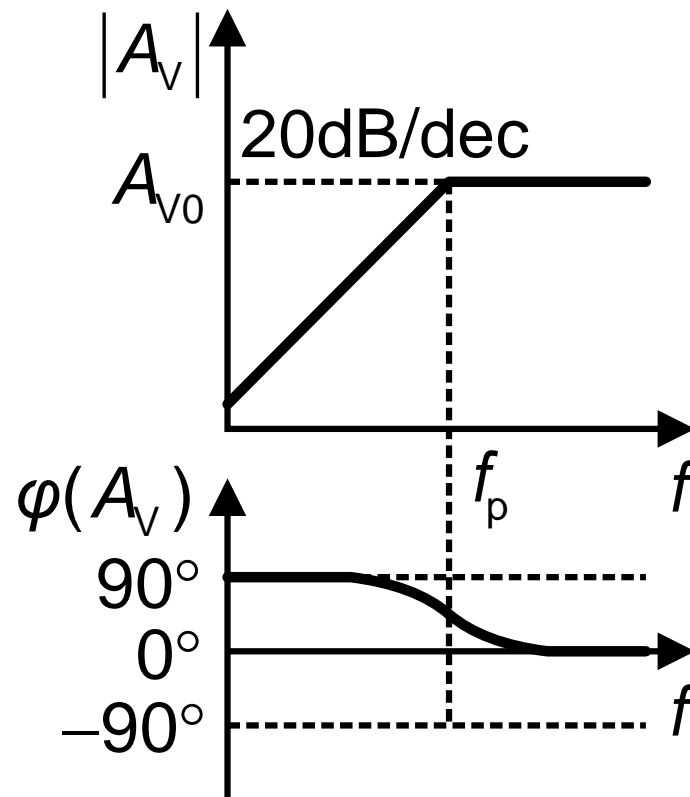
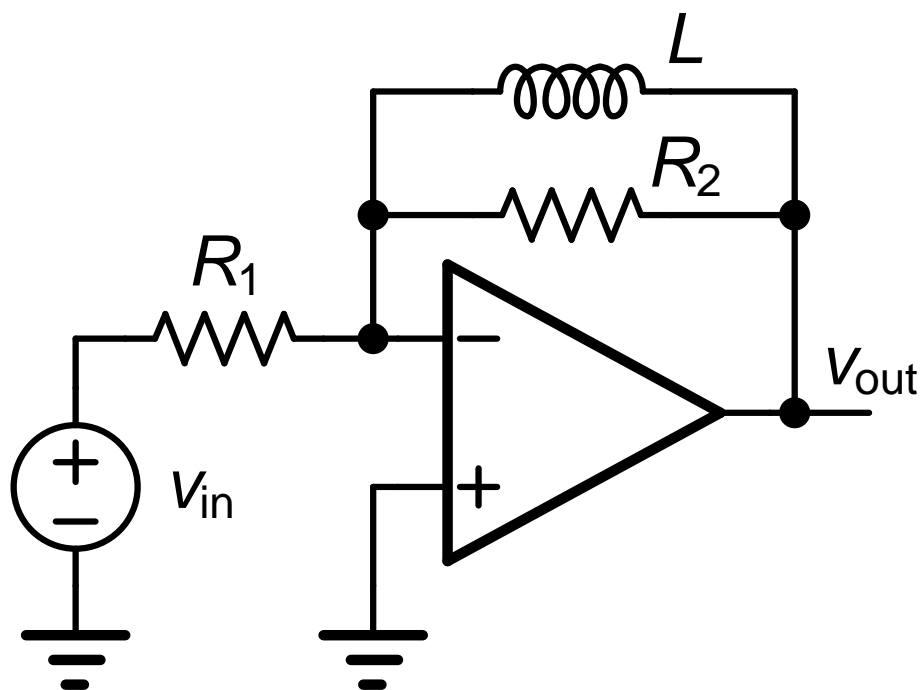


$$A_{V0} = -\frac{R_2}{R_1}$$

$$A_V = A_{V0} \frac{1}{1 + j \frac{f}{f_p}}$$

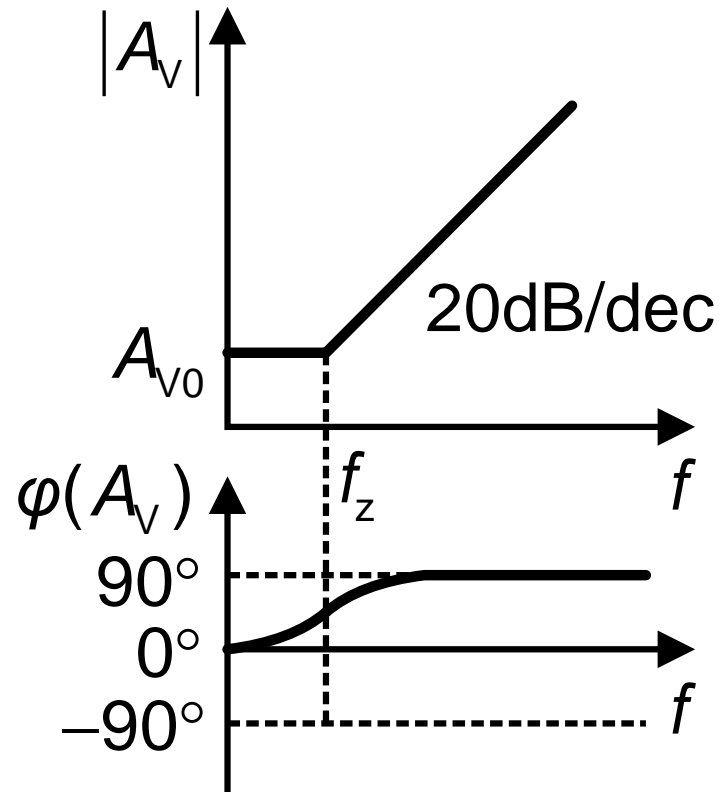
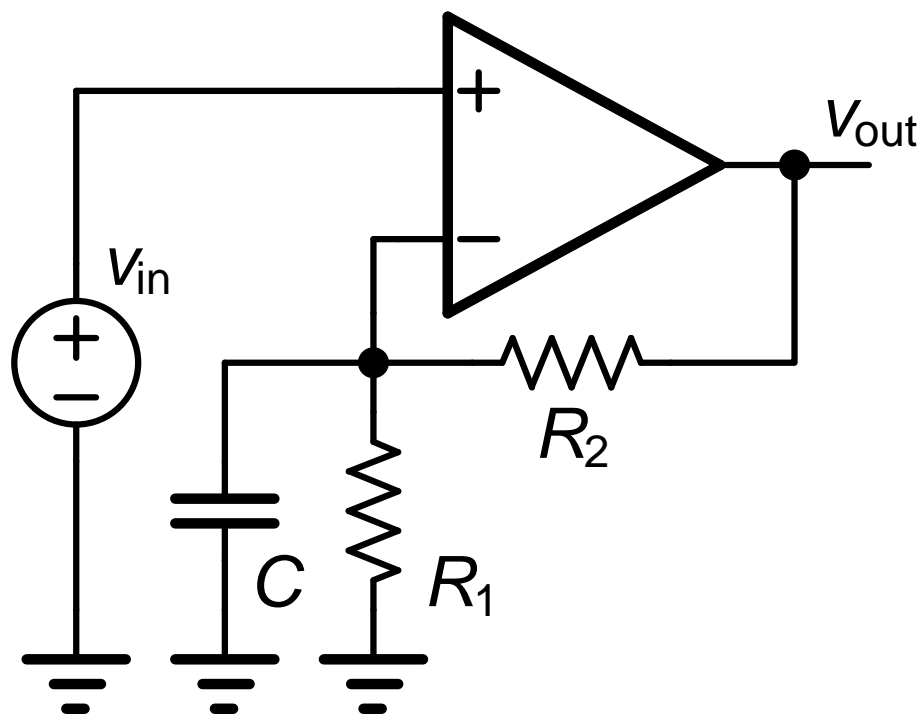
$$f_p = \frac{1}{2\pi R_2 C}$$

# 高通滤波器 1



$$A_{V0} = -\frac{R_2}{R_1} \quad A_V = A_{V0} \frac{j \frac{f}{f_p}}{1 + j \frac{f}{f_p}} \quad f_p = \frac{R_2}{2\pi L}$$

# 高通滤波器 2

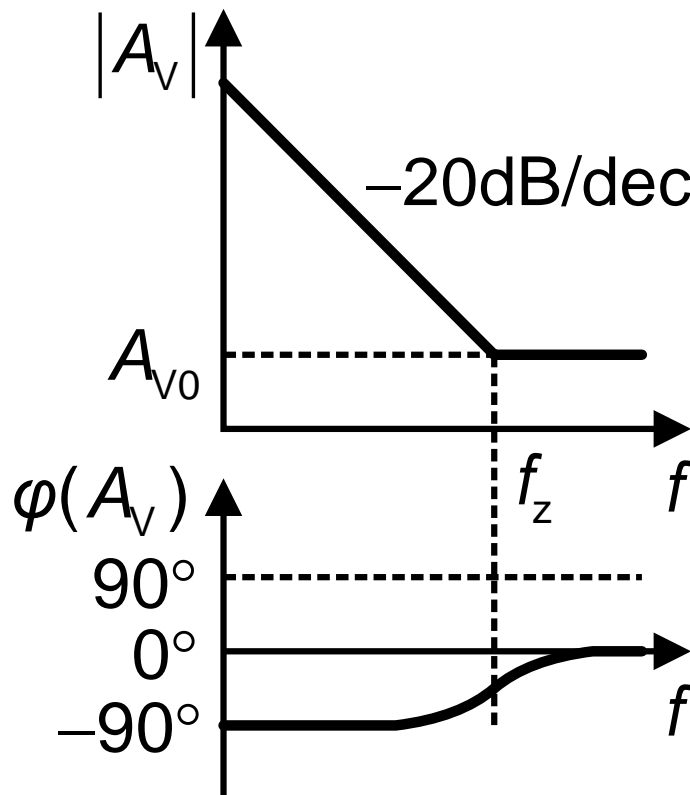
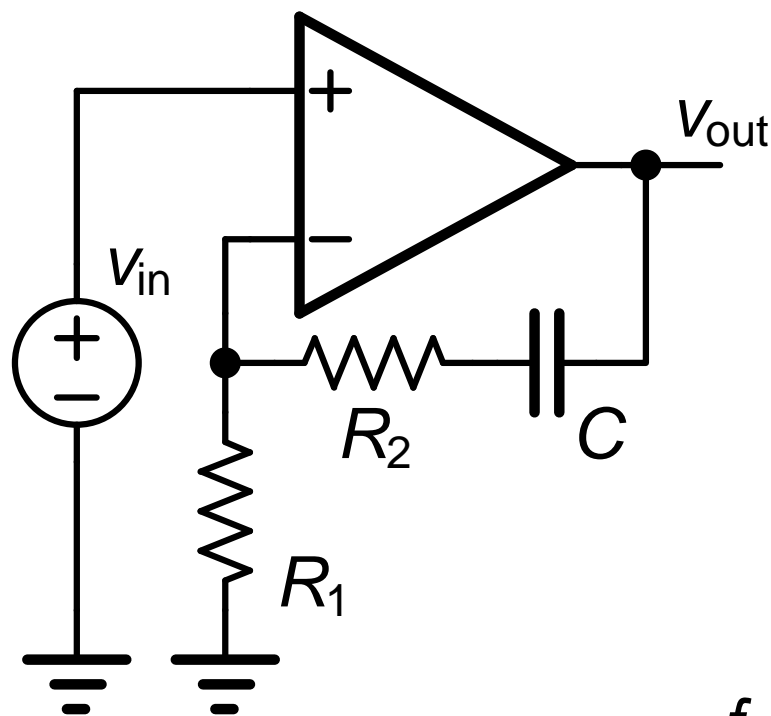


$$A_{V0} = 1 + \frac{R_2}{R_1} \quad A_V = A_{V0} \left( 1 + j \frac{f}{f_z} \right)$$

$$f_z = \frac{1}{2\pi RC}$$

$$R = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

# 有限衰减的低通滤波器



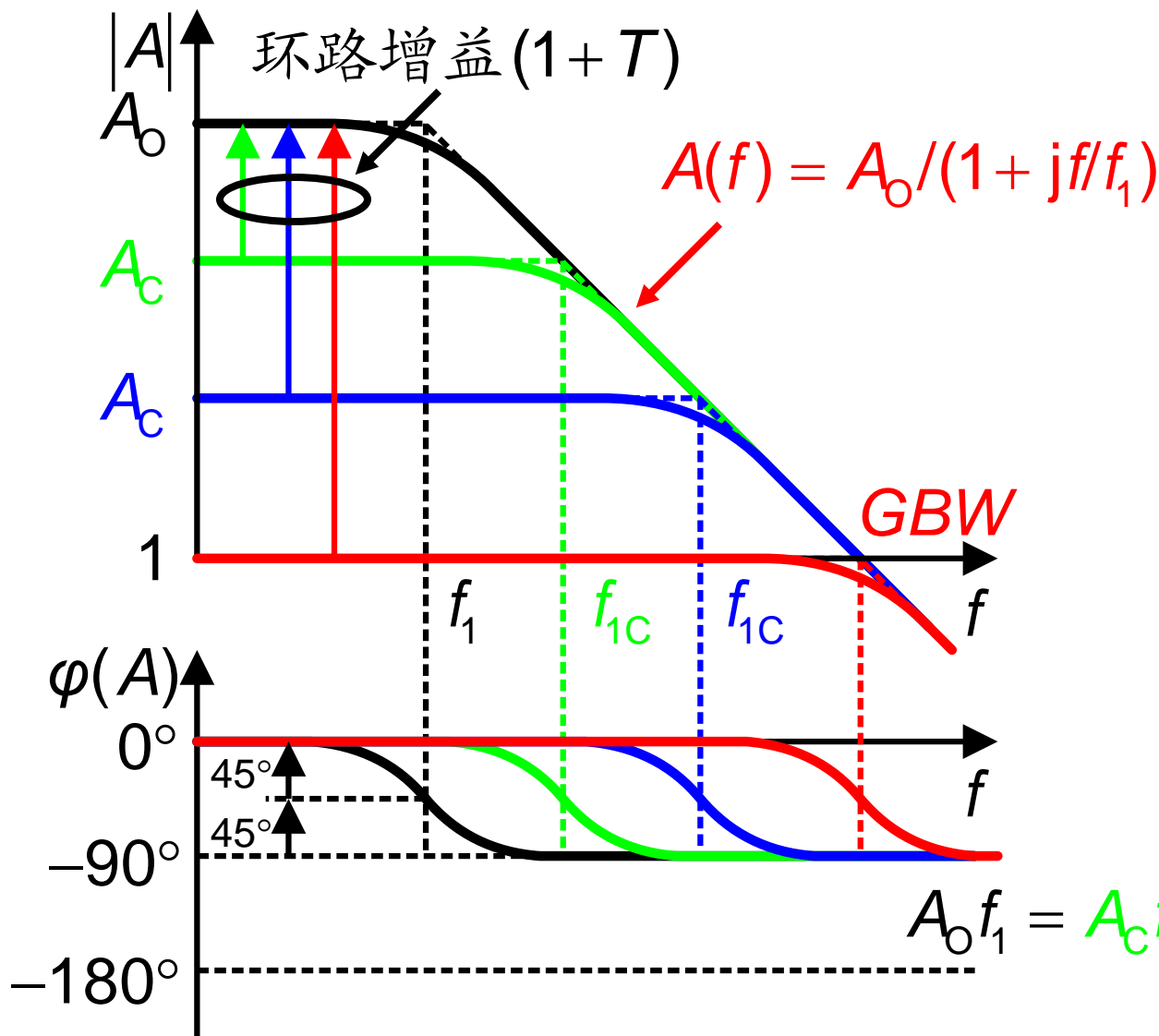
$$A_{V0} = 1 + \frac{R_2}{R_1}$$

$$A_V = A_{V0} \frac{1 + j \frac{f}{f_z}}{j \frac{f}{f_z}}$$

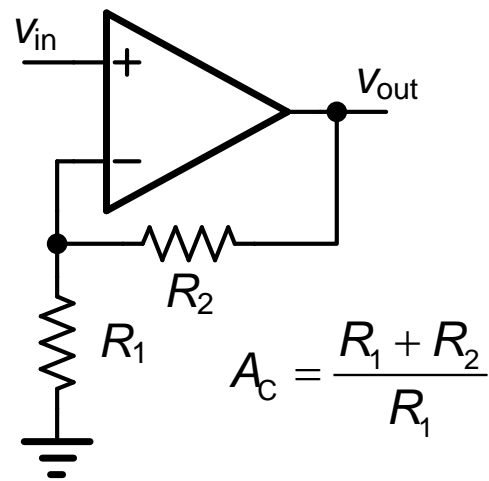
$$f_z = \frac{1}{2\pi RC}$$

$$R = R_1 + R_2$$

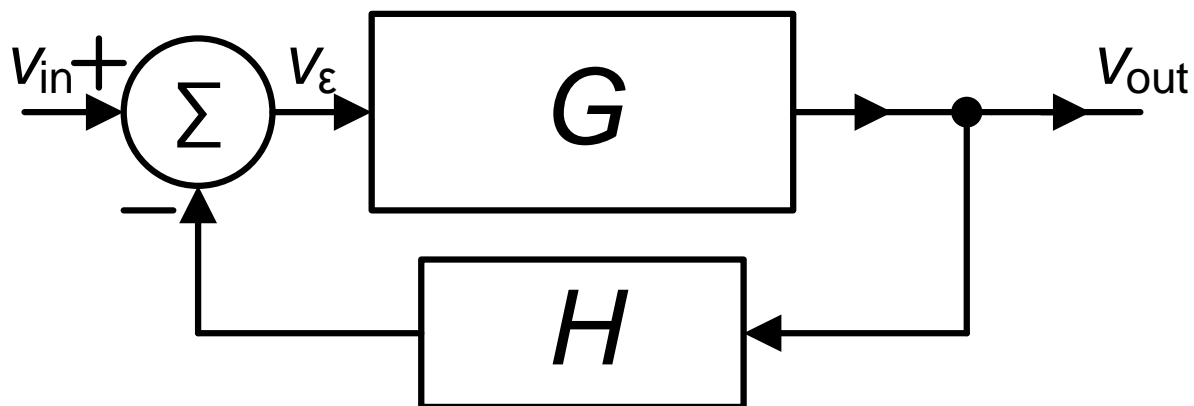
# 增益和带宽之间的交换



$A_o$  开环增益  
 $A_c$  闭环增益



# 开环增益和闭环增益

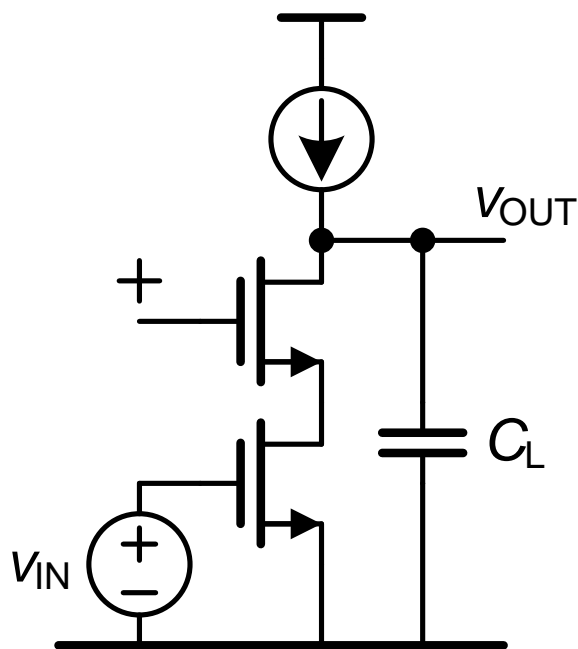


$$\left. \begin{aligned} V_{\varepsilon} &= V_{in} - HV_{out} \\ V_{out} &= GV_{\varepsilon} \end{aligned} \right\} \Rightarrow A_c = \frac{V_{out}}{V_{in}} = \frac{G}{1 + GH} \approx \frac{1}{H}$$

如果环路增益  $GH = T \gg 1$

Ref.: P. Gray, P.Hurst, S.Lewis, R. Meyer: Design of analog integrated circuits,  
4th ed., Wiley 2001

# 运放成为运放的原因？



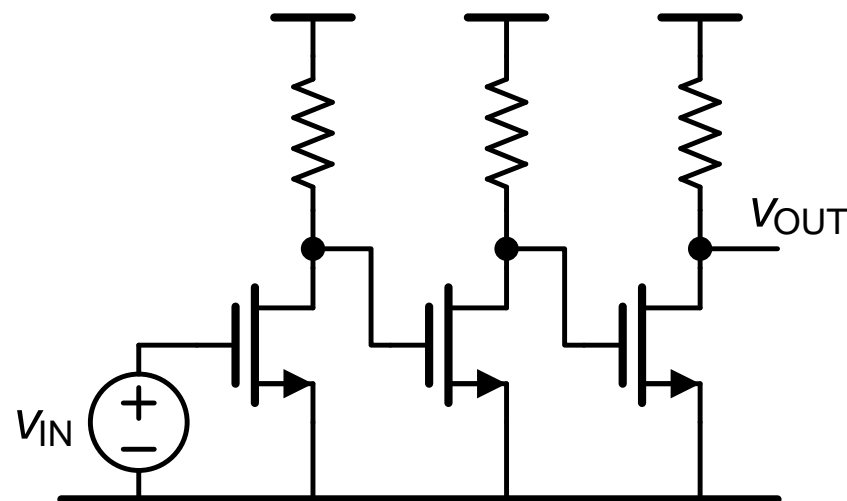
## 运算放大器

单级点放大器

高阻抗 = 高增益

增益与带宽交换

任何增益下都稳定



## 宽带放大器

多级点放大器

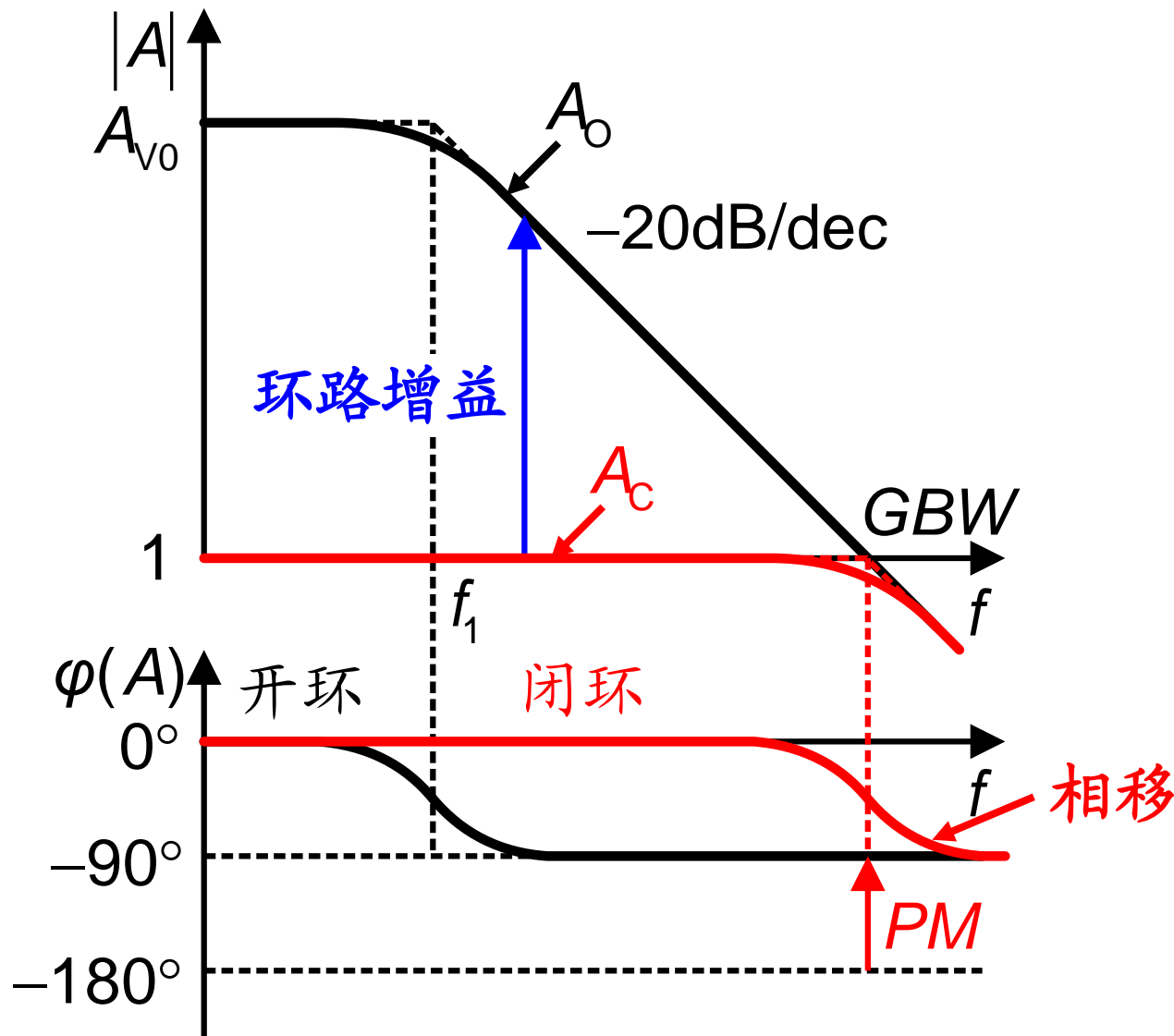
低阻抗节点

高带宽

只在某些增益下稳定

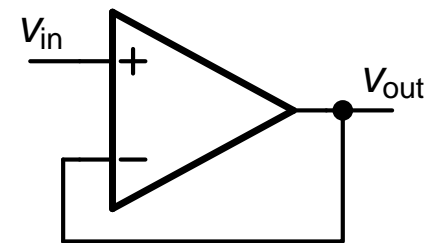


# 单极点系统



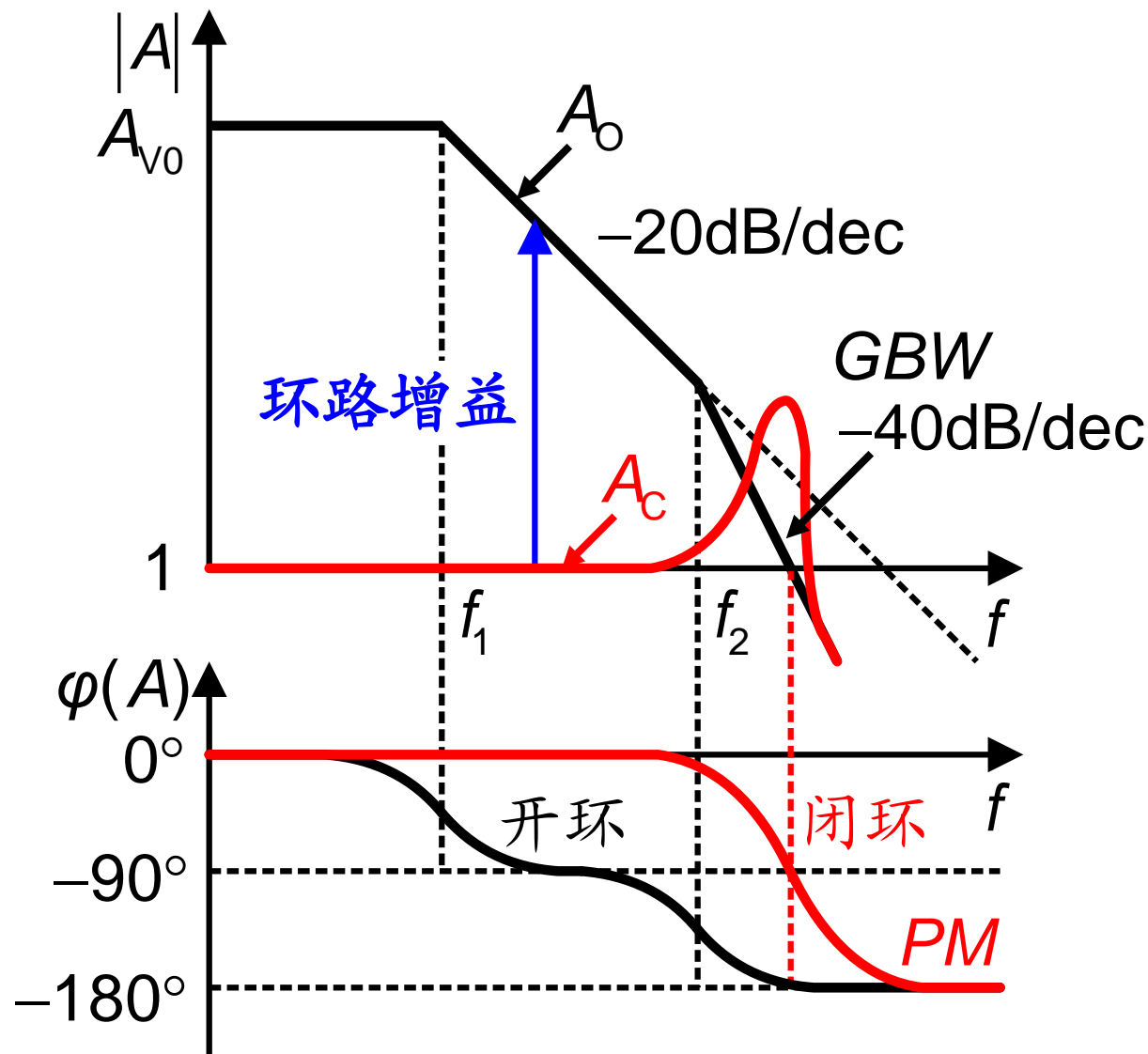
$A_o$  开环增益

$A_c$  闭环增益



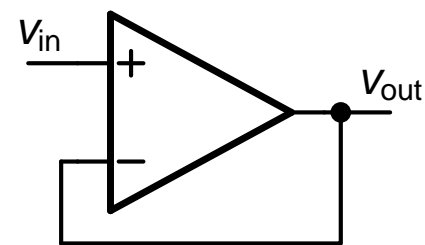
$PM$  相位裕度

# 双极点系统



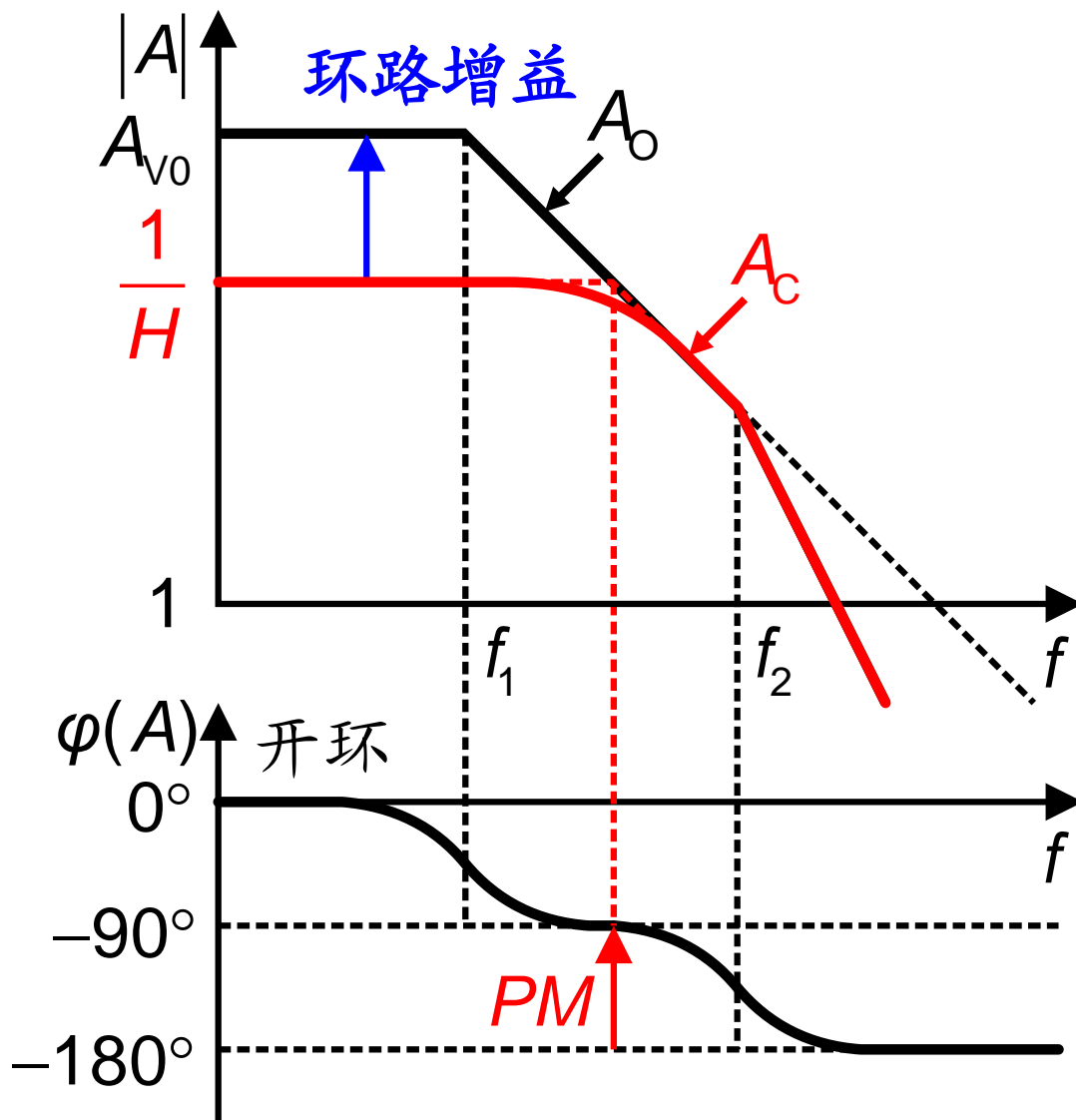
$A_o$  开环增益

$A_c$  闭环增益



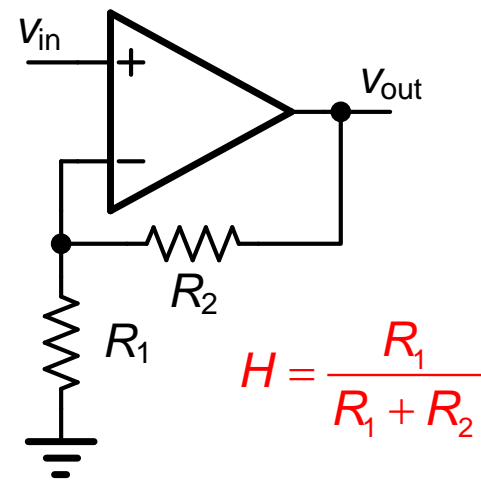
$PM$  相位裕度

# 环路增益与相位裕度的关系 1



$A_0$  开环增益

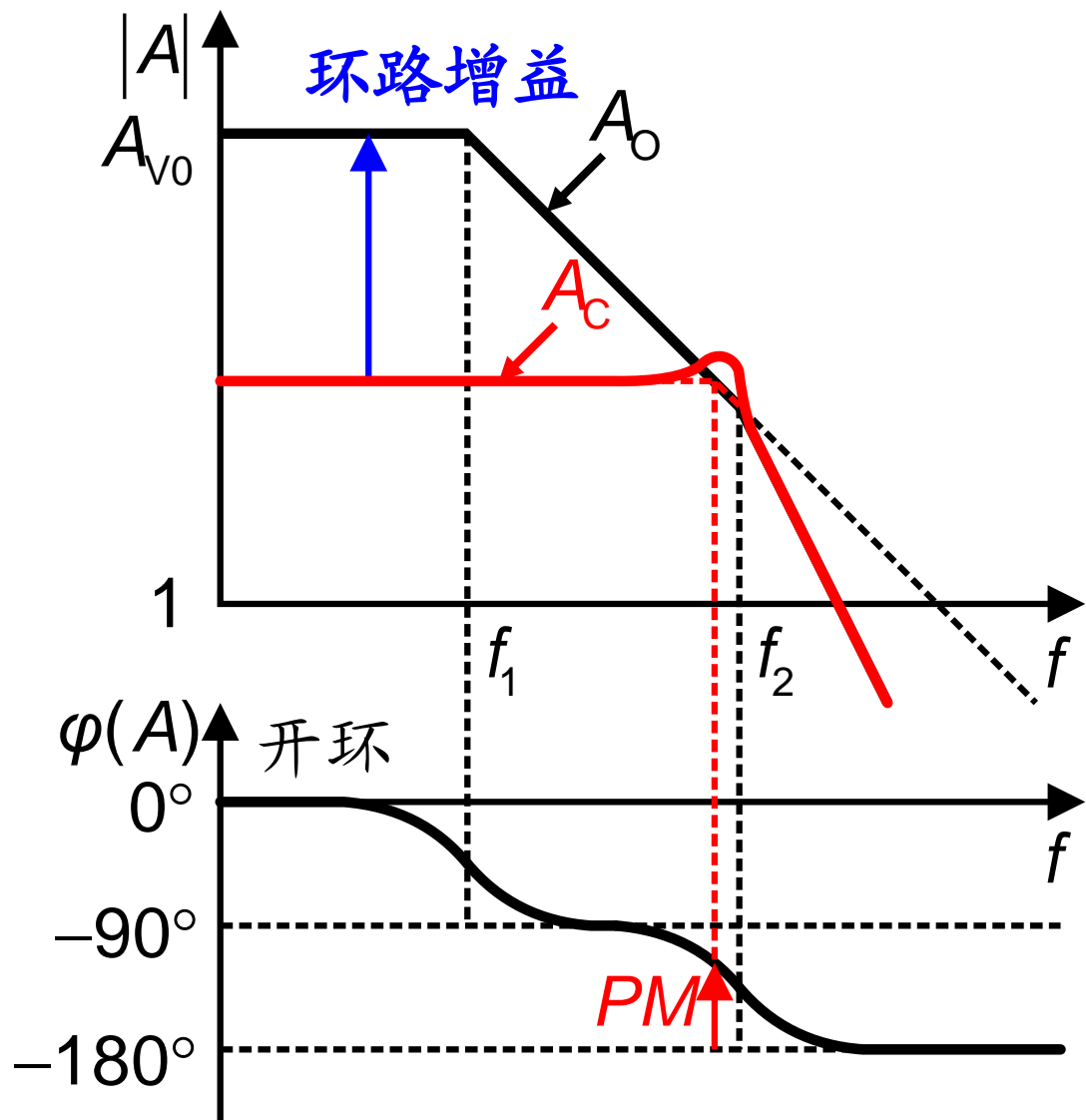
$A_C$  闭环增益



$$H = \frac{R_1}{R_1 + R_2}$$

PM相位裕度

# 环路增益与相位裕度的关系 2

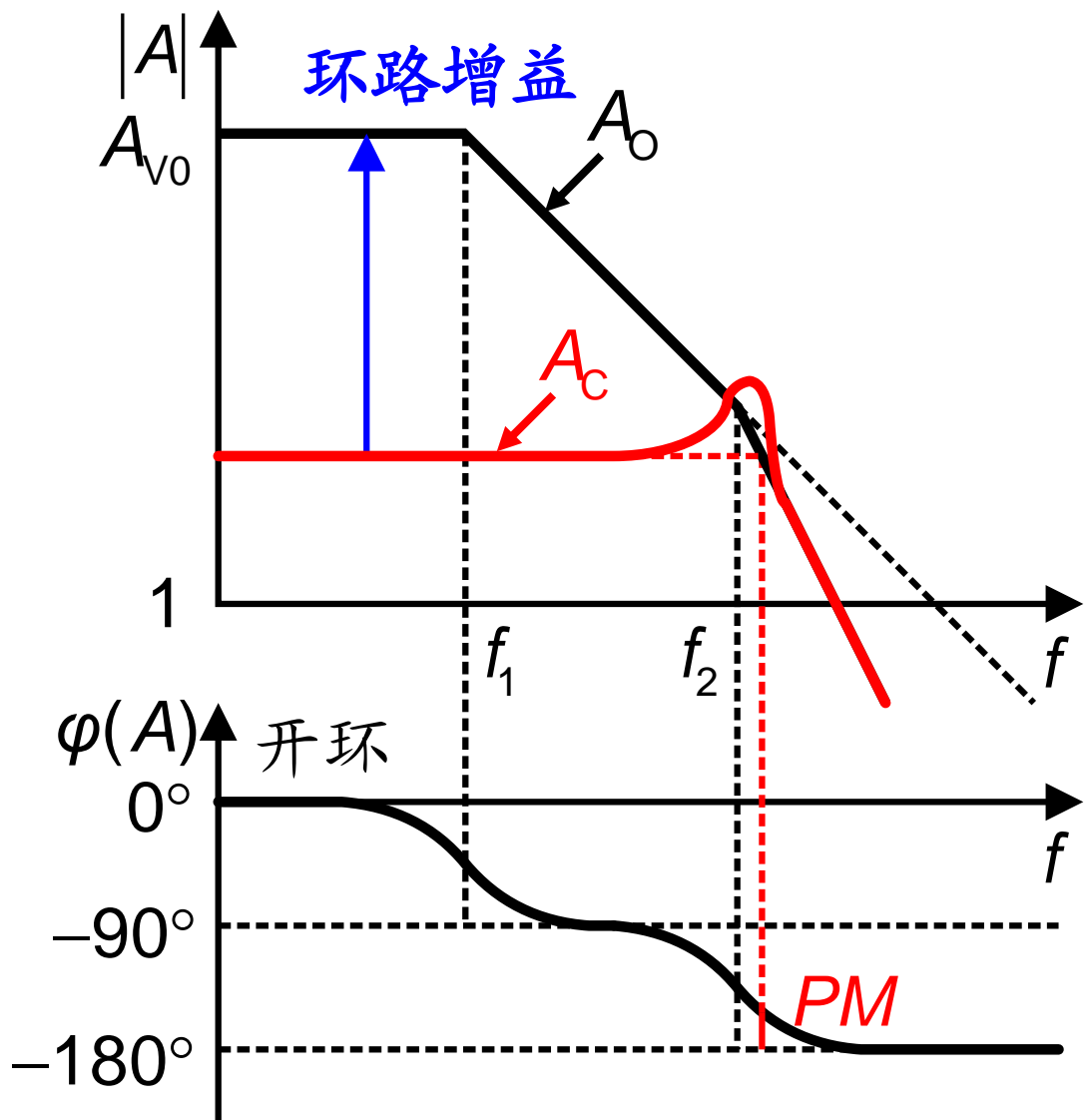


$A_o$  开环增益

$A_c$  闭环增益

$PM$  相位裕度

# 环路增益与相位裕度的关系 3

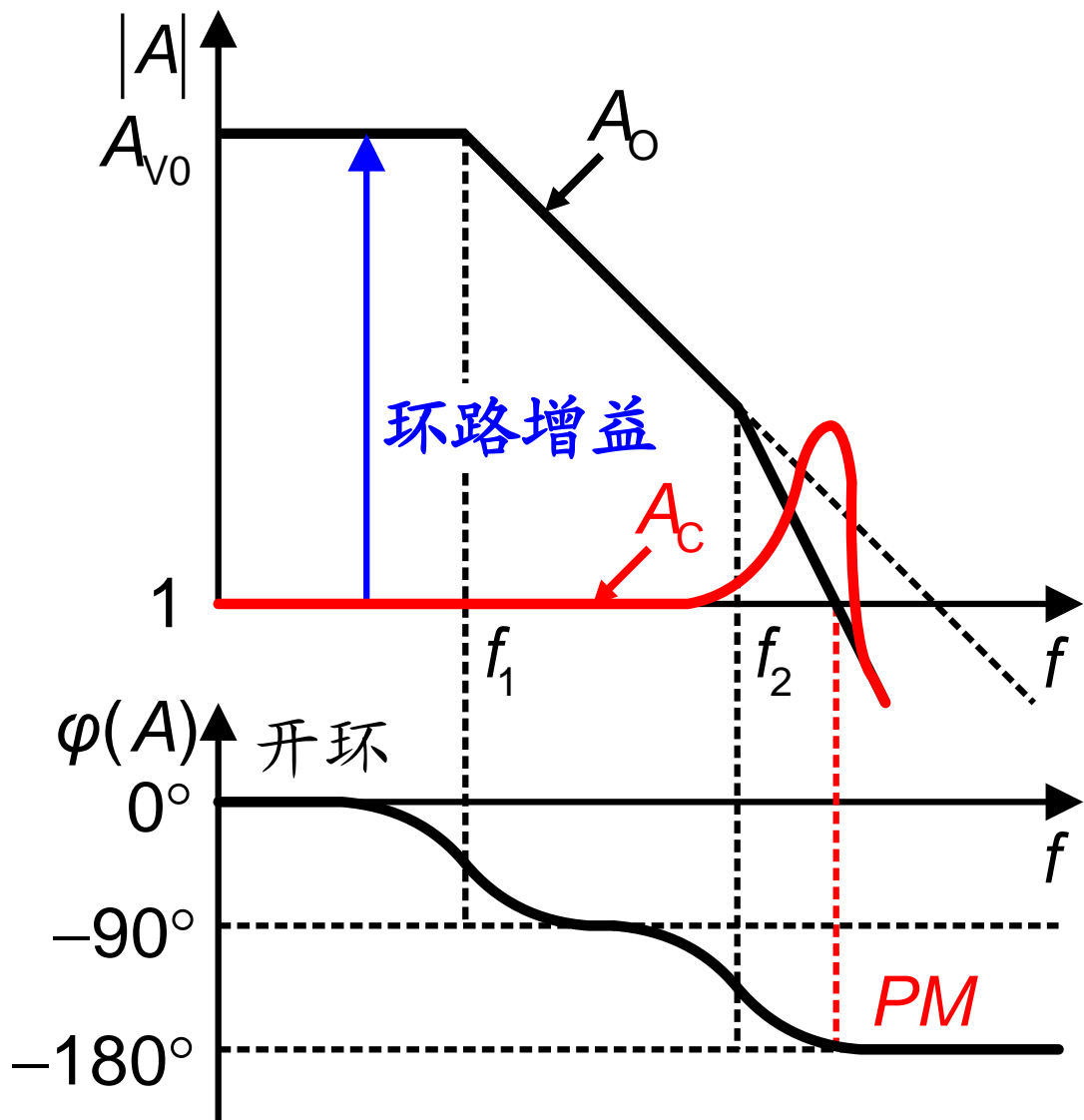


$A_o$  开环增益

$A_c$  闭环增益

PM相位裕度

# 环路增益与相位裕度的关系 4



$A_o$  开环增益

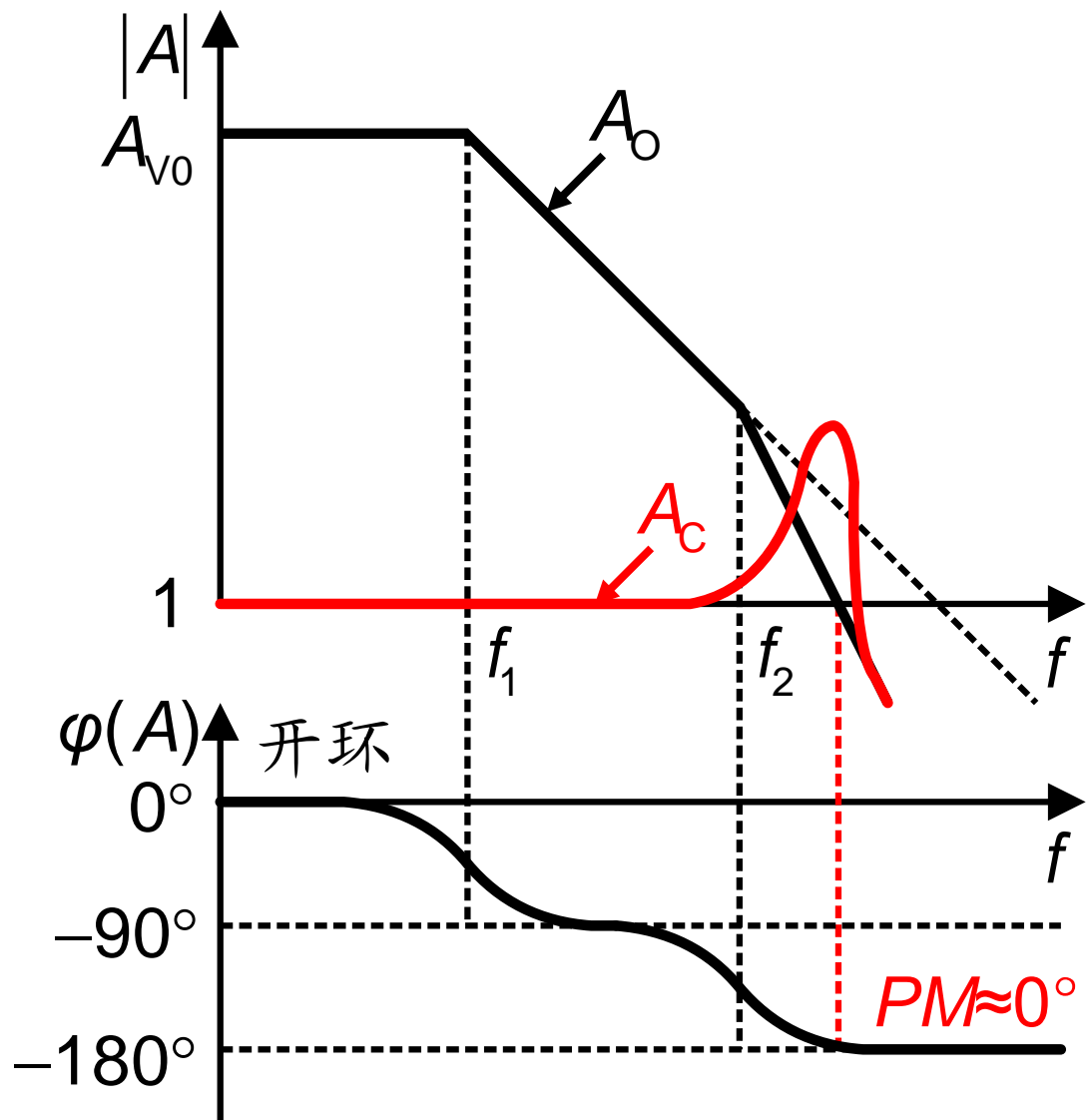
$A_c$  闭环增益

最坏情况

$$A_c = 1$$

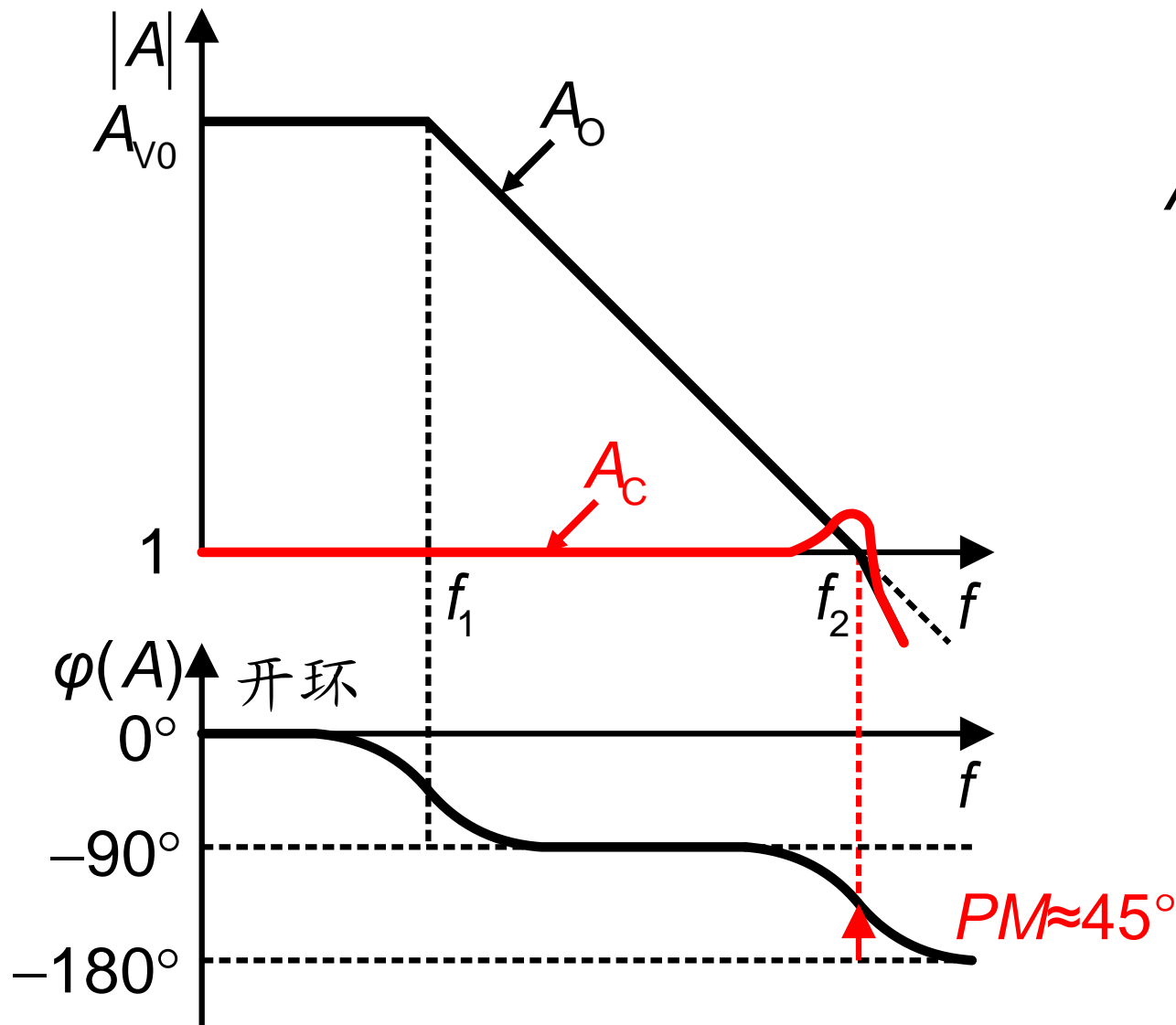
$PM$  相位裕度

# 当 $f_2$ 频率较低时



$A_c$  闭环增益

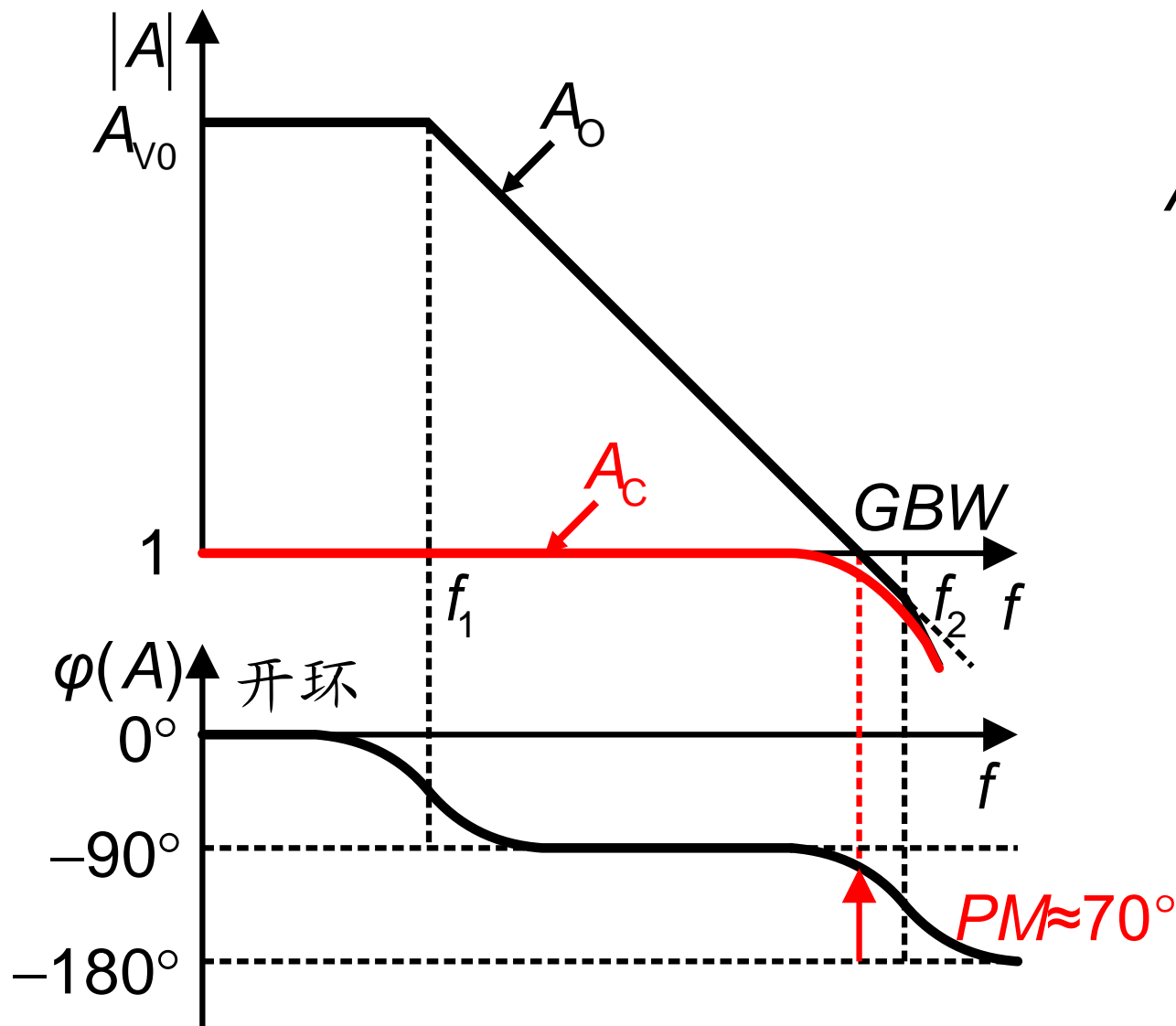
# 通过增加 $f_2$ , 提高 $PM$ 。( $f_2=GBW$ )



$A_c$  闭环增益



# 通过增加 $f_2$ , 提高 $PM$ 。( $f_2 \approx 3GBW$ )

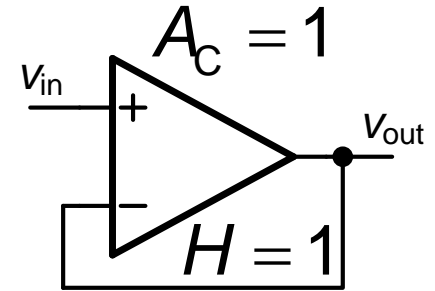


$A_c$  闭环增益

$f_2 \approx 3GBW$

# 当 $f_2 \approx 3GBW$ 时，计算 $PM$

开环增益  $A_o = \frac{A_{V0}}{(1 + j\frac{f}{f_1})(1 + j\frac{f}{f_2})}$



闭环增益  $A_c = \frac{A_o}{1 + A_o} \approx \frac{A_{V0}}{1 + A_{V0}} \frac{1}{1 + j\frac{f}{GBW} + j^2 \frac{f^2}{GBWf_2}}$

$$\approx \frac{1}{1 + j2\zeta \frac{f}{f_r} + j^2 \frac{f^2}{f_r^2}}$$

$f_r = \sqrt{GBWf_2}$  自激振荡频率

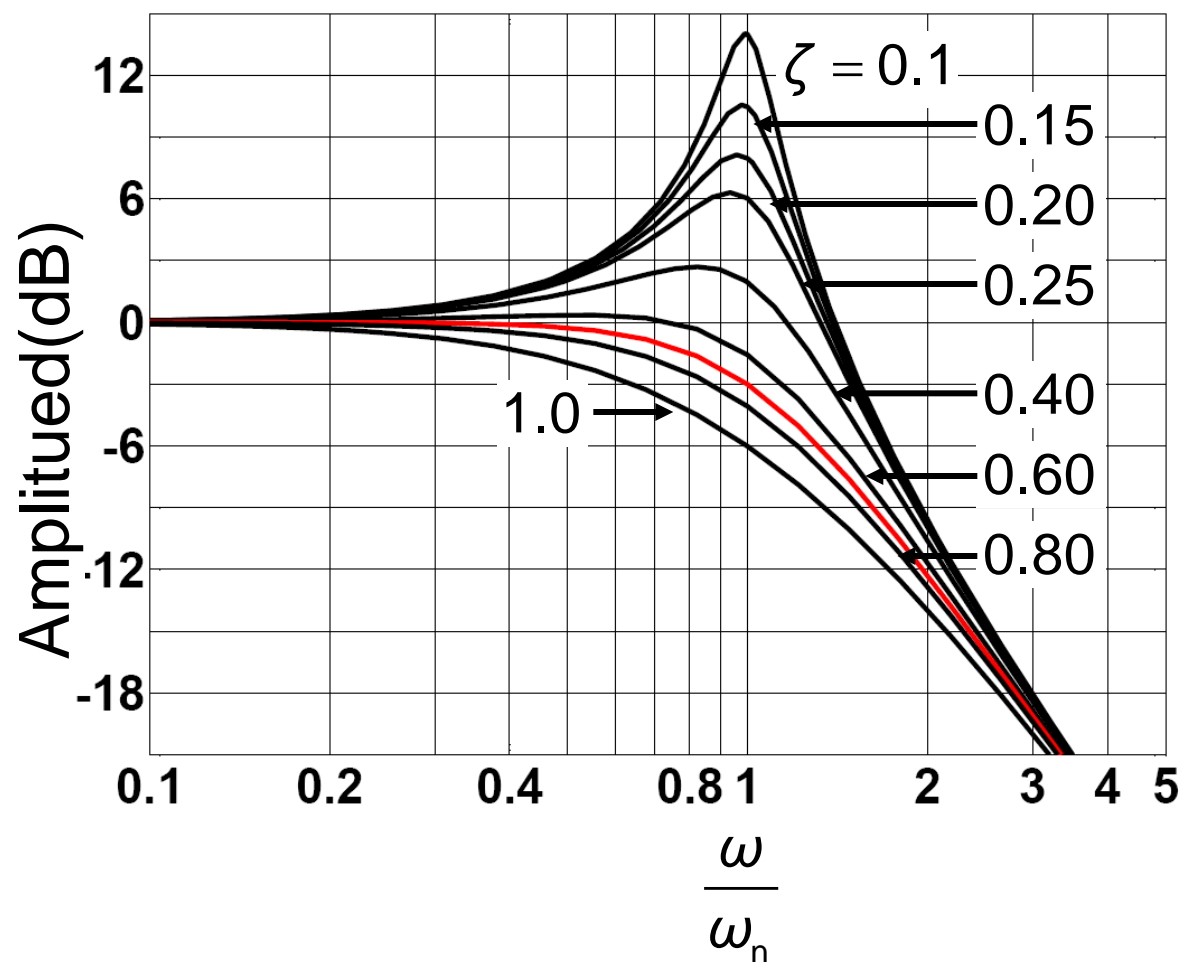
$\zeta = \frac{1}{2} \sqrt{\frac{f_2}{GBW}}$  阻尼系数 (=  $1/2Q$ )

# $PM$ , $\zeta$ , $P_f$ 和 $P_t$

$$f_r = \sqrt{GBW f_2} \quad PM(^{\circ}) = 90^{\circ} - \arctan \frac{GBW}{f_2} = \arctan \frac{f_2}{GBW}$$

$\frac{f_2}{GBW}$	$PM(^{\circ})$	$\zeta = \frac{1}{2} \sqrt{\frac{f_2}{GBW}}$	$P_f(\text{dB})$	$P_t(\text{dB})$
0.5	27	0.35	3.6	2.3
1	45	0.5	1.25	1.3
1.5	56	0.61	0.28	0.73
2	63	$\sqrt{2}/2$	0	0.37
3	72	0.87	0	0.04
<b>4</b>	<b>76</b>	<b>1</b>		
<b>5</b>	<b>79</b>			

# 闭环幅度频率响应

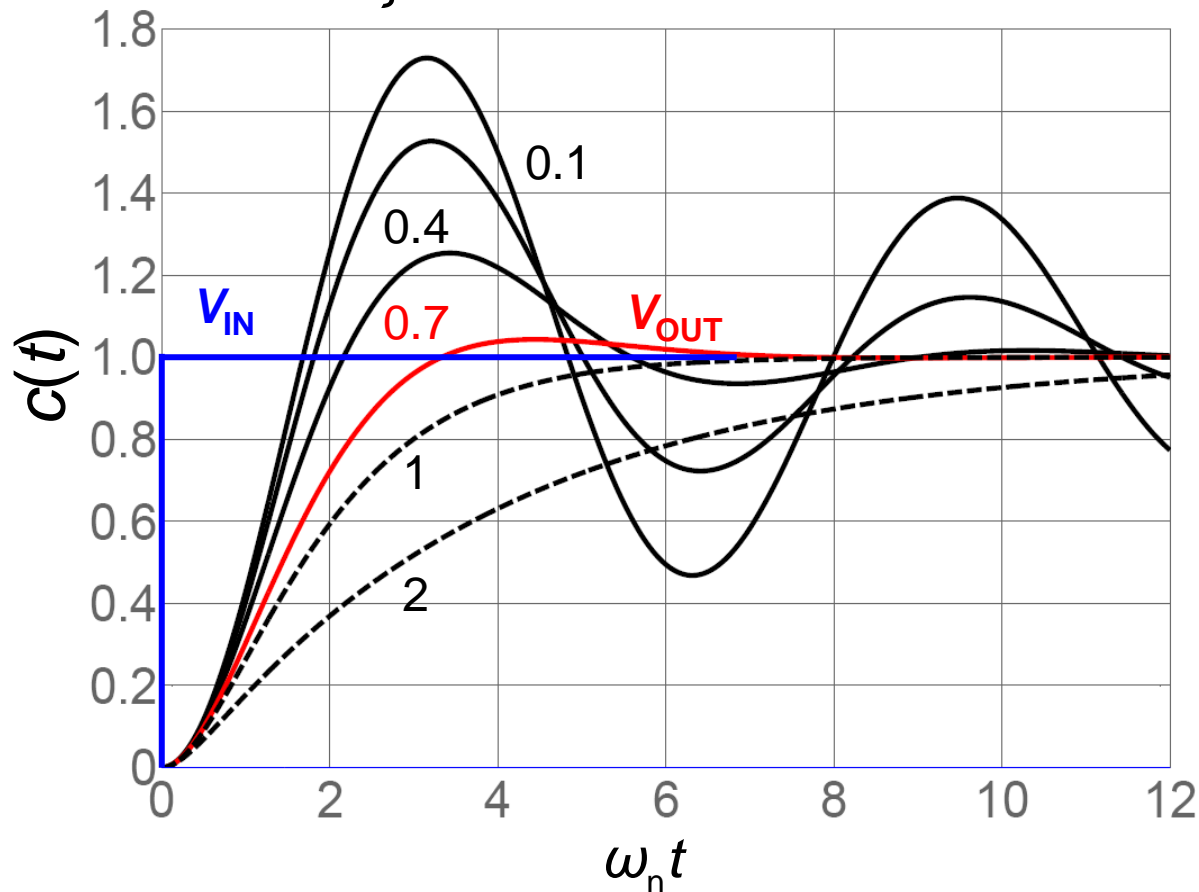


$$\zeta = Q = \sqrt{2}/2$$

$$P_f = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

# 闭环阶跃冲击响应

$\zeta = 0.1 \ 0.2 \ 0.4 \ 0.7 \ 1 \ 2$



$$\zeta = Q = \frac{\sqrt{2}}{2}$$

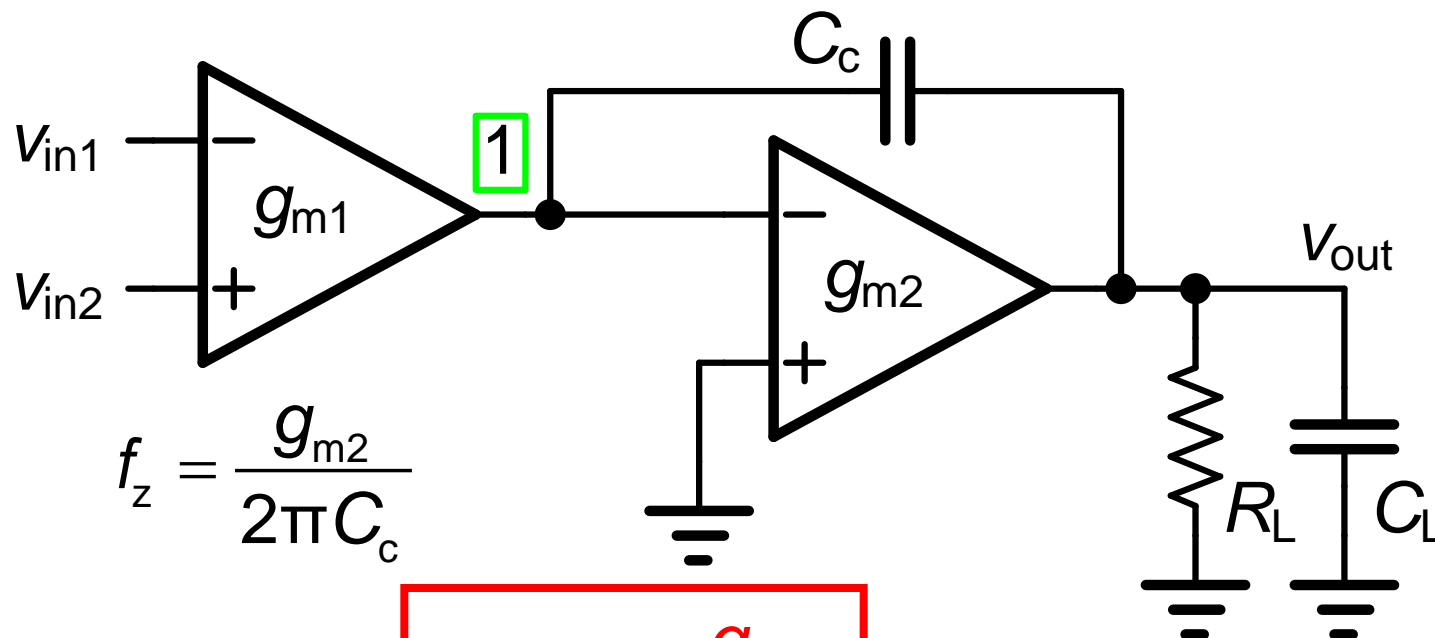
$$P_t = 1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

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- 三级运放的稳定性

# 通用两级放大器 1

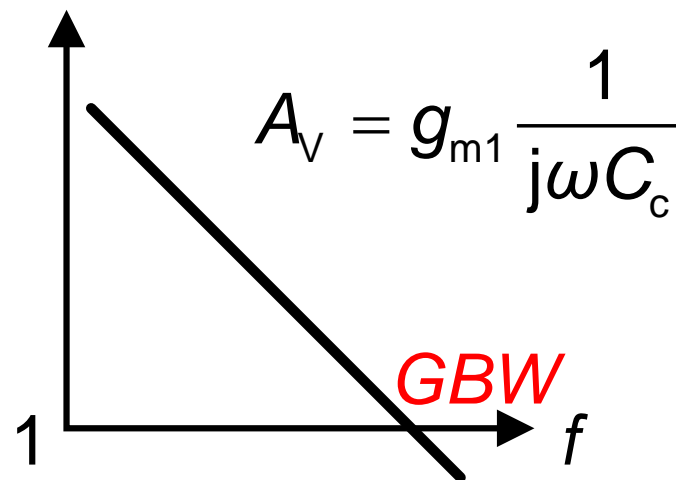


$$f_z = \frac{g_{m2}}{2\pi C_c}$$

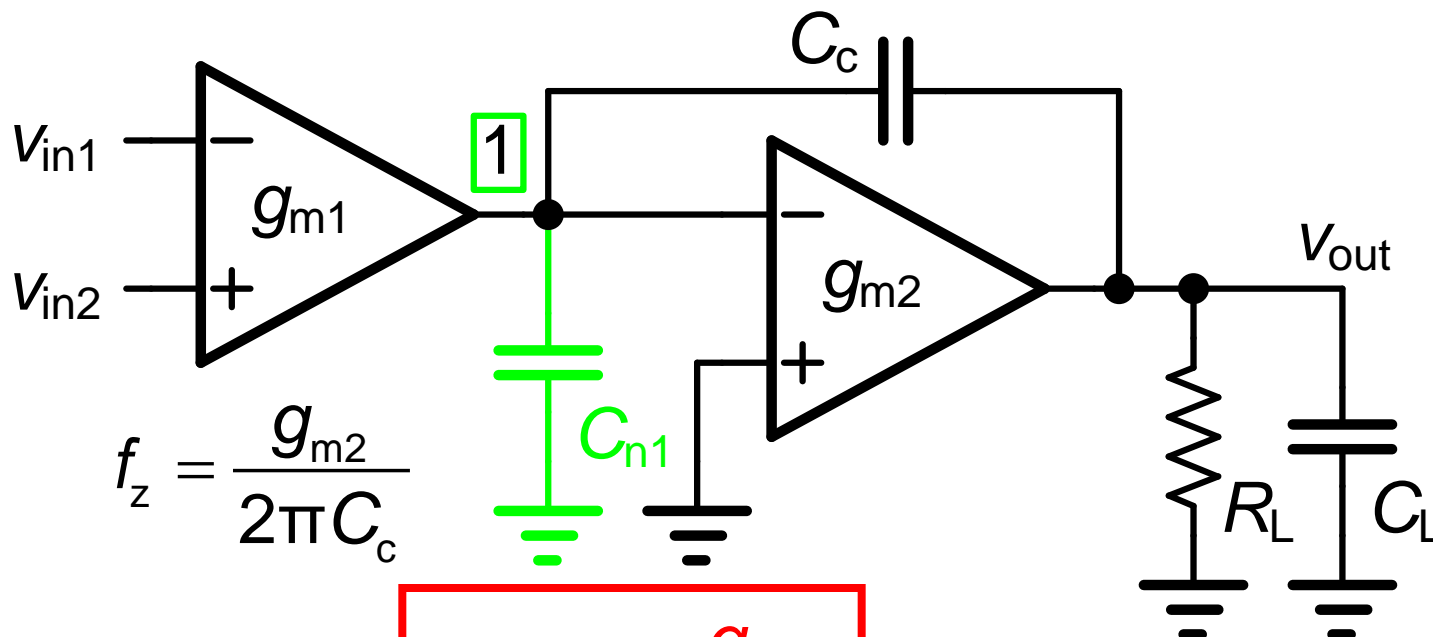
$$|A_V| = 1 \Rightarrow$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd} = \frac{g_{m2}}{2\pi C_L}$$



# 通用两级放大器 2

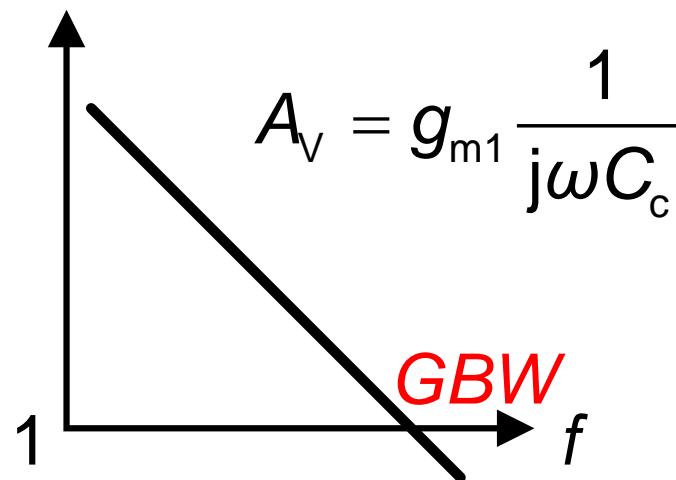


$$f_z = \frac{g_{m2}}{2\pi C_c}$$

$$|A_V| = 1 \Rightarrow$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd} = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \frac{C_{n1}}{C_c}}$$





# 初步设计两级运放

$$GBW = \frac{g_{m1}}{2\pi C_c} \quad f_{nd} = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \underbrace{\frac{C_{n1}}{C_c}}_{\approx 0.3}}$$

$$\left. \begin{array}{l} f_{nd} = 3GBW \\ \frac{C_{n1}}{C_c} \approx 0.3 \end{array} \right\} \Rightarrow \boxed{\frac{g_{m2}}{g_{m1}} \approx 4 \frac{C_L}{C_c}} \text{ 第二级需要很大的电流!}$$

当  $GBW = 100 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$  时

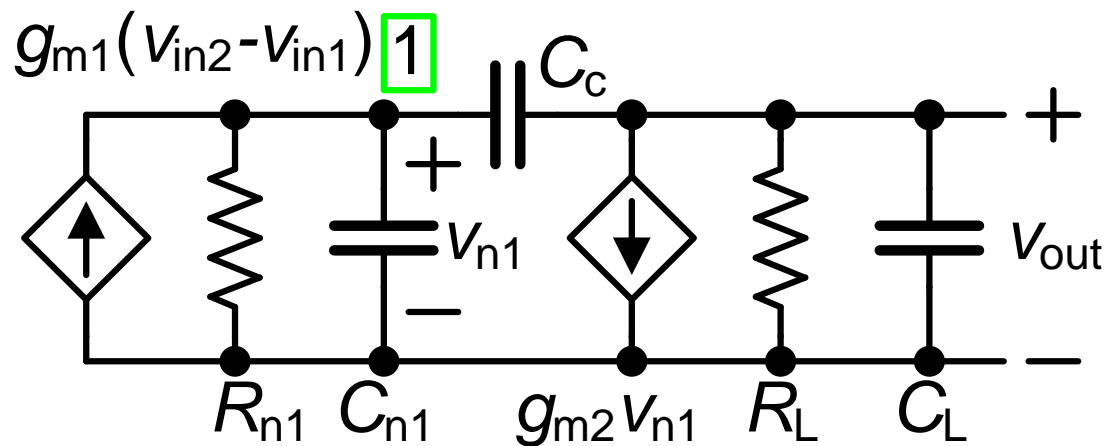
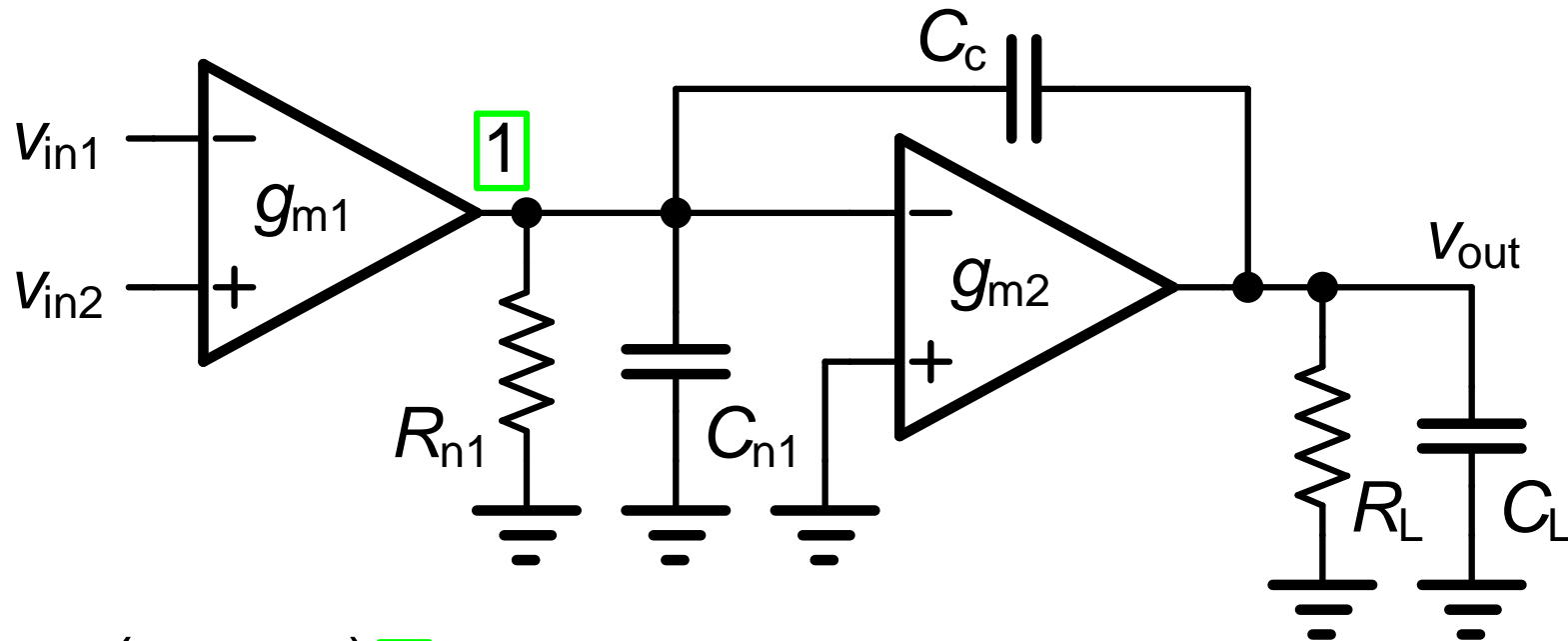
解：选择  $C_c = 1 \text{ pF}$

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- 三级运放的稳定性

# 通用两级运放：密勒OTA

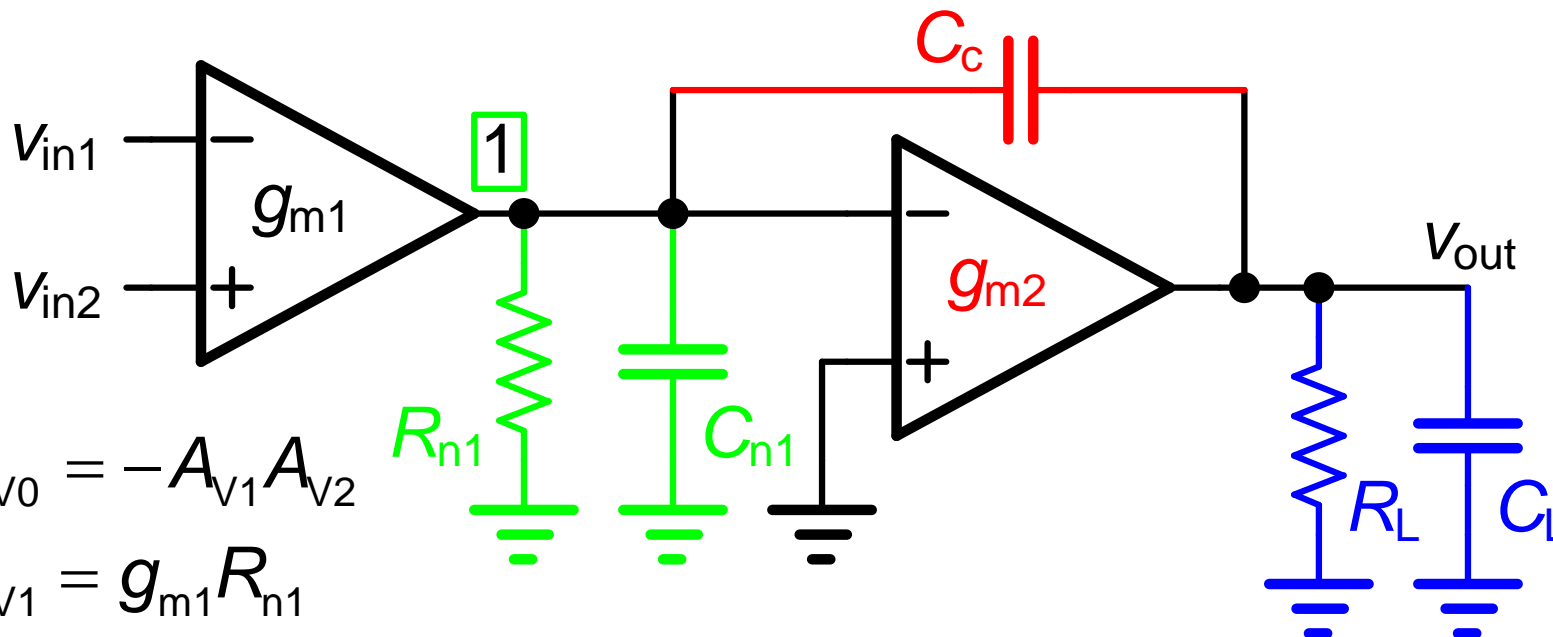


$$A_{V0} = -A_{V1}A_{V2}$$

$$A_{V1} = g_{m1}R_{n1}$$

$$A_{V2} = g_{m2}R_L$$

# 通用两级运放



$$A_{V0} = -A_{V1}A_{V2}$$

$$A_{V1} = g_{m1}R_{n1}$$

$$A_{V2} = g_{m2}R_L$$

$$1 - \frac{C_c}{g_{m2}}s$$

$$A_V = A_{V0} \frac{1 - \frac{C_c}{g_{m2}}s}{1 + (R_{n1}C_{n1} + R_{n1}C_c + A_{V2}R_{n1}C_c + R_L C_L + R_L C_c)s + R_{n1}R_L C C s^2}$$

$$CC = C_{n1}C_c + C_{n1}C_L + C_c C_L$$

# 零极点近似

$$A = A_0 \frac{1 - cs}{1 + as + bs^2}$$

零点  $z = \frac{1}{c}$

极点  $s_1 = -\frac{1}{a}$      $s_2 = -\frac{a}{b}$

$$a \uparrow \Rightarrow s_1 \downarrow \text{ and } s_2 \uparrow$$

如果  $s_2 \gg s_1$  :

$s$  很小

$$\downarrow$$

$$1 + as + bs^2 = 0$$

$$\downarrow$$

$$s_1 = -\frac{1}{a}$$

$s$  很大

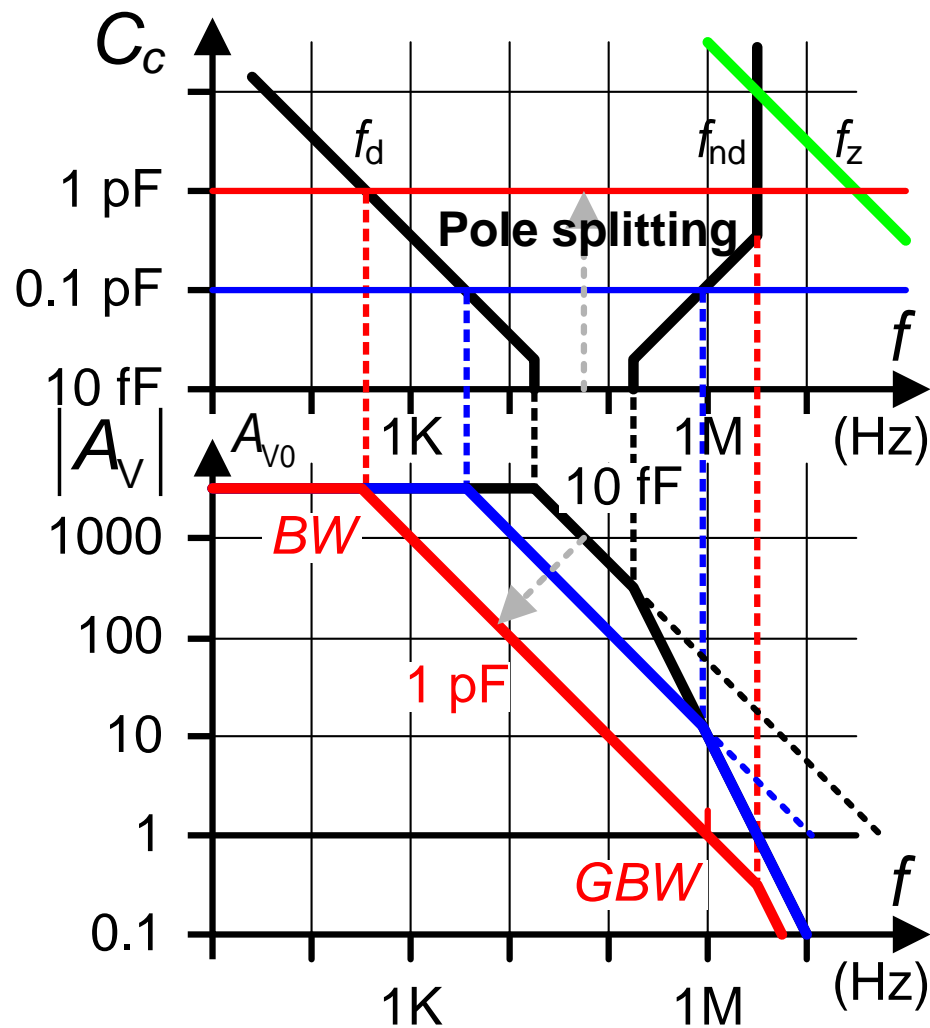
$$\downarrow$$

$$1 + as + bs^2 = 0$$

$$\downarrow$$

$$s_2 = -\frac{a}{b}$$

# 密勒OTA: 用 $C_c$ 进行极点分离



$C_c$ 取较大值,  
进行极点分离:

$$f_d = \frac{1}{2\pi A_{V2} R_{n1} C_c}$$

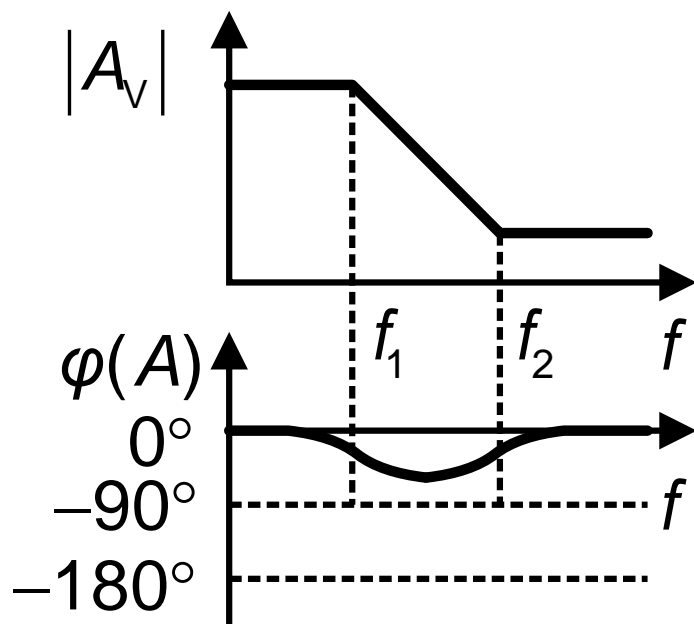
$$f_z = \frac{g_{m2}}{2\pi C_c} \text{ 为正零点!}$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

# 正零点的作用

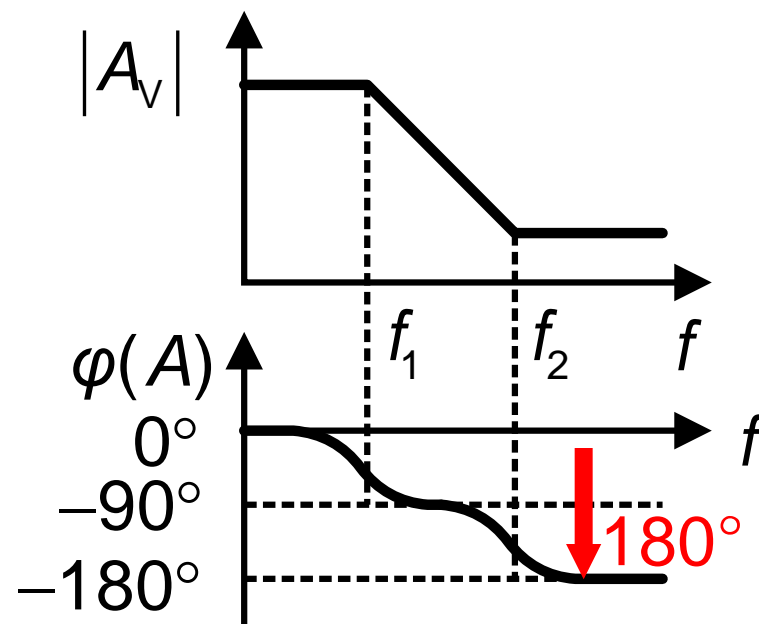
负零点

$$A_V = A_{V0} \frac{1 + jf/f_2}{1 + jf/f_1}$$



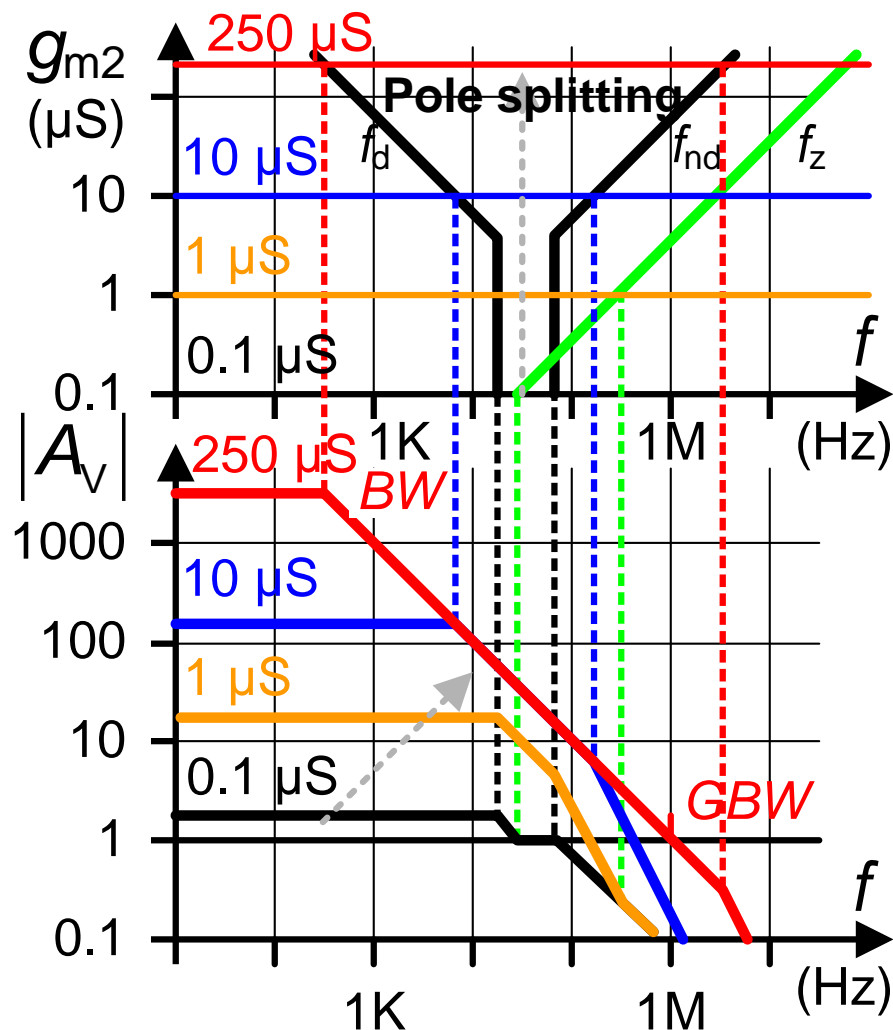
正零点

$$A_V = A_{V0} \frac{1 - jf/f_2}{1 + jf/f_1}$$



对于相位，正零点像一个负极点!!!

# 密勒OTA: 用 $g_{m2}$ 进行极点分离



$g_{m2}$  取较大值,  
进行极点分离:

$$f_d = \frac{1}{2\pi A_{V2} R_{n1} C_c}$$

$$f_z = \frac{g_{m2}}{2\pi C_c} \text{ 为正零点!}$$

$$GBW = \frac{g_{m1}}{2\pi C_c}$$



# 极点分离的方式

$$\frac{g_{m2}}{g_{m1}} \approx 4 \frac{C_L}{C_c} \quad \text{或表示为} \quad g_{m2} C_c \approx 4 g_{m1} C_L$$

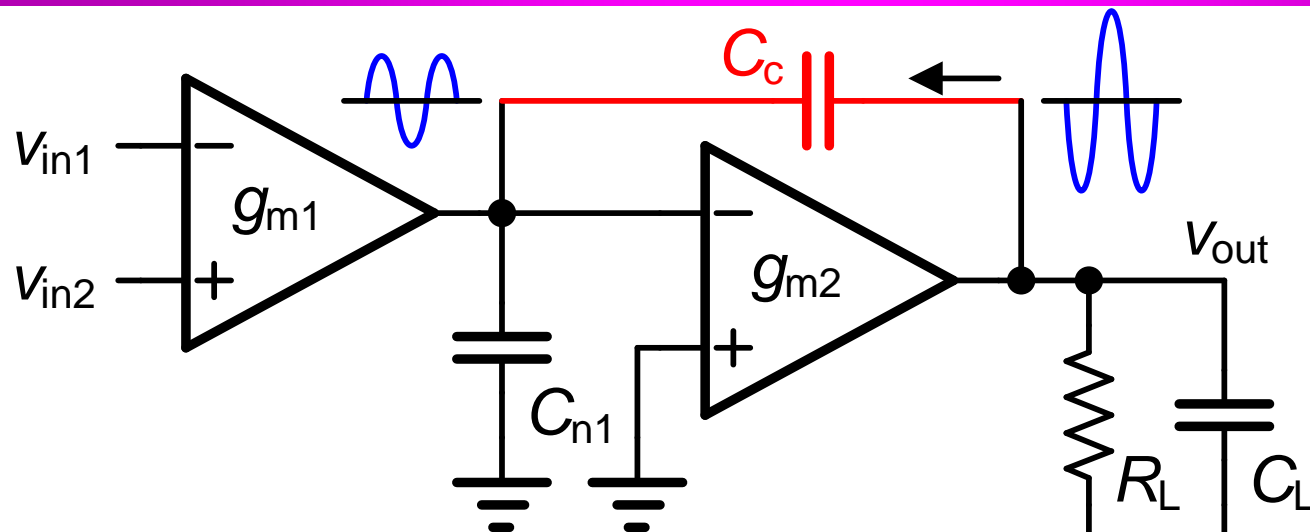
$g_{m2}$  和  $C_c$  二者均可

# 目录

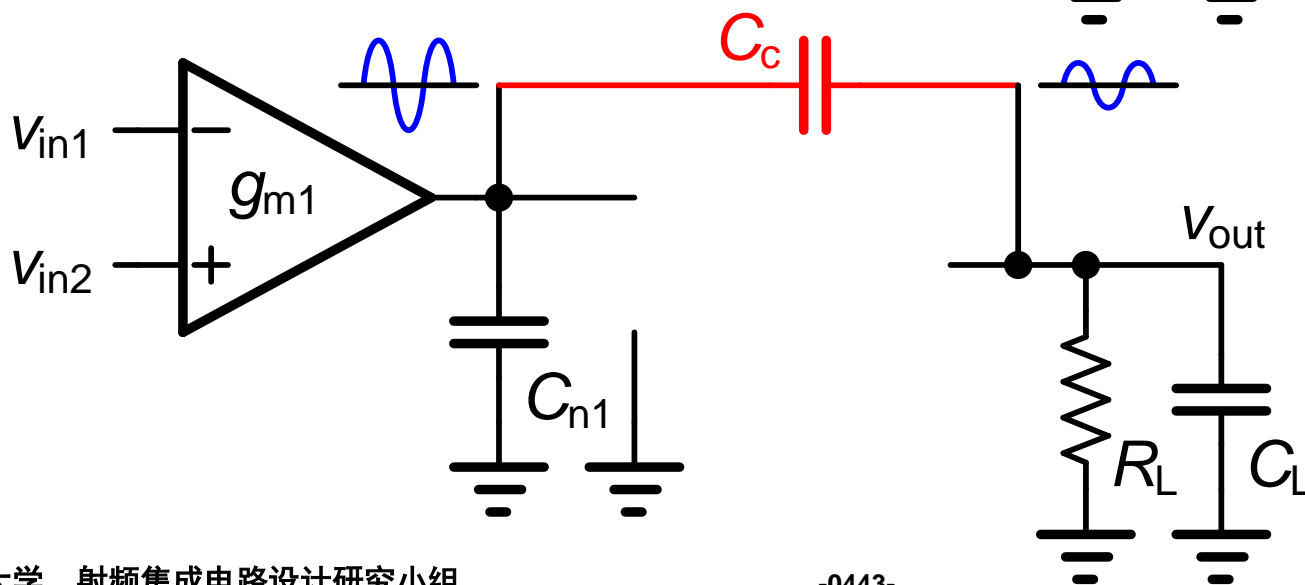
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- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

# 密勒效应的正反馈特性产生正零点



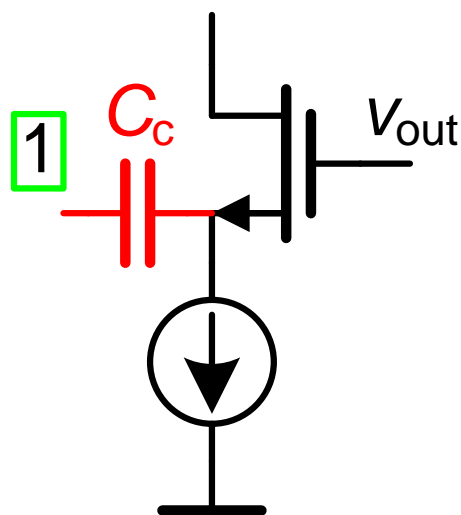
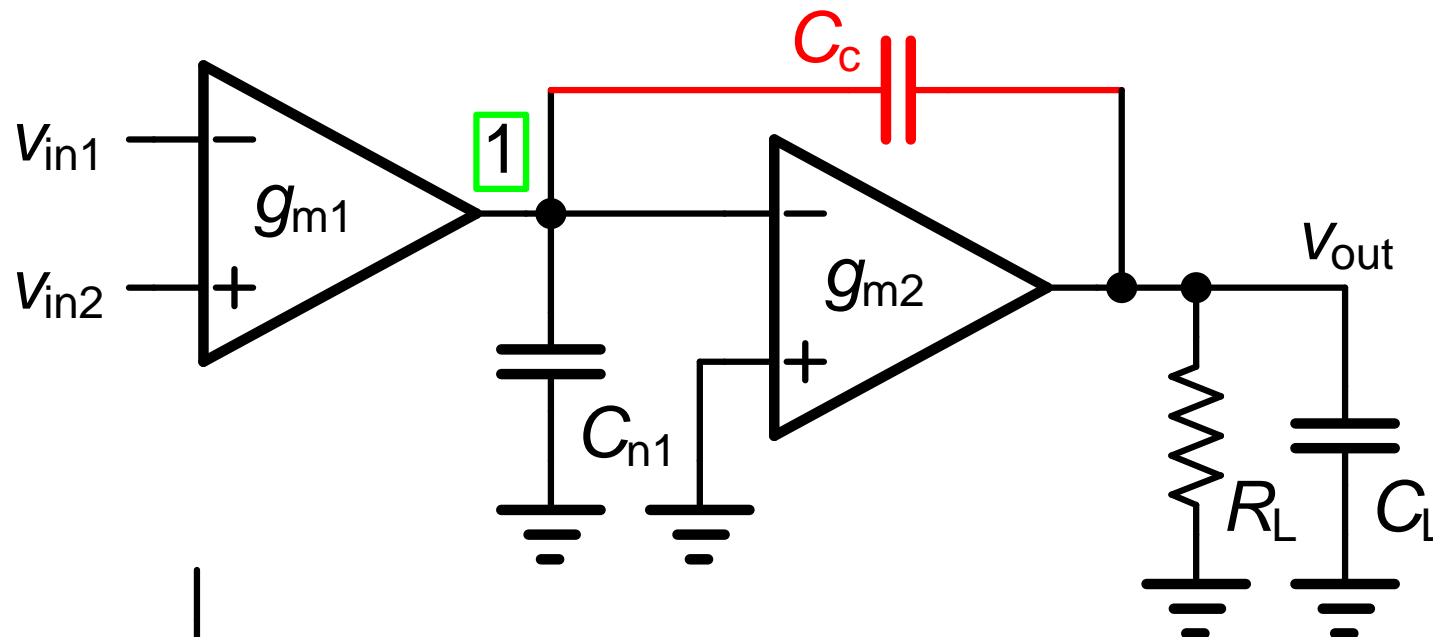
密勒效应为  
反馈



正反馈

↓  
截断!

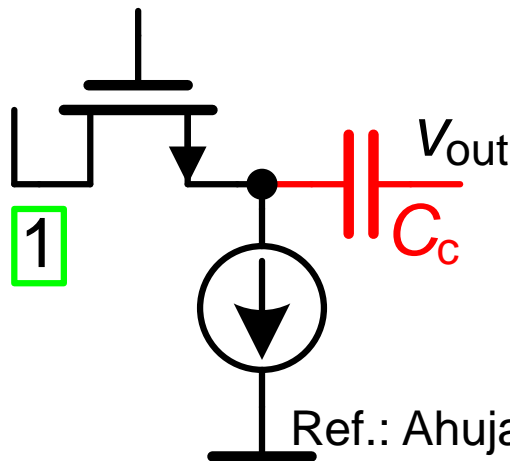
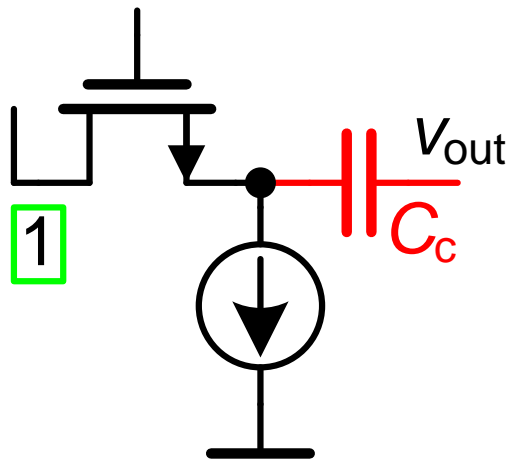
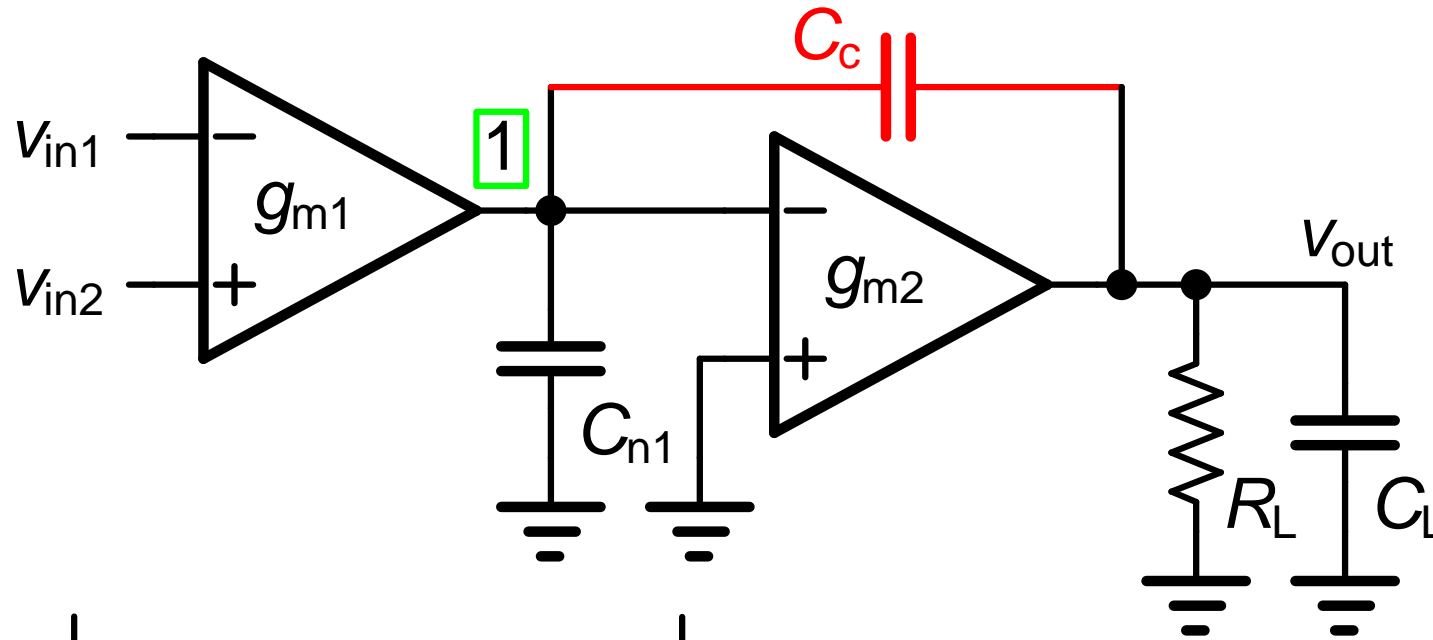
# 截断正馈通路 1



电压缓冲源极跟随器

Ref.: Tsividis, JSSC Dec.76, 748-753

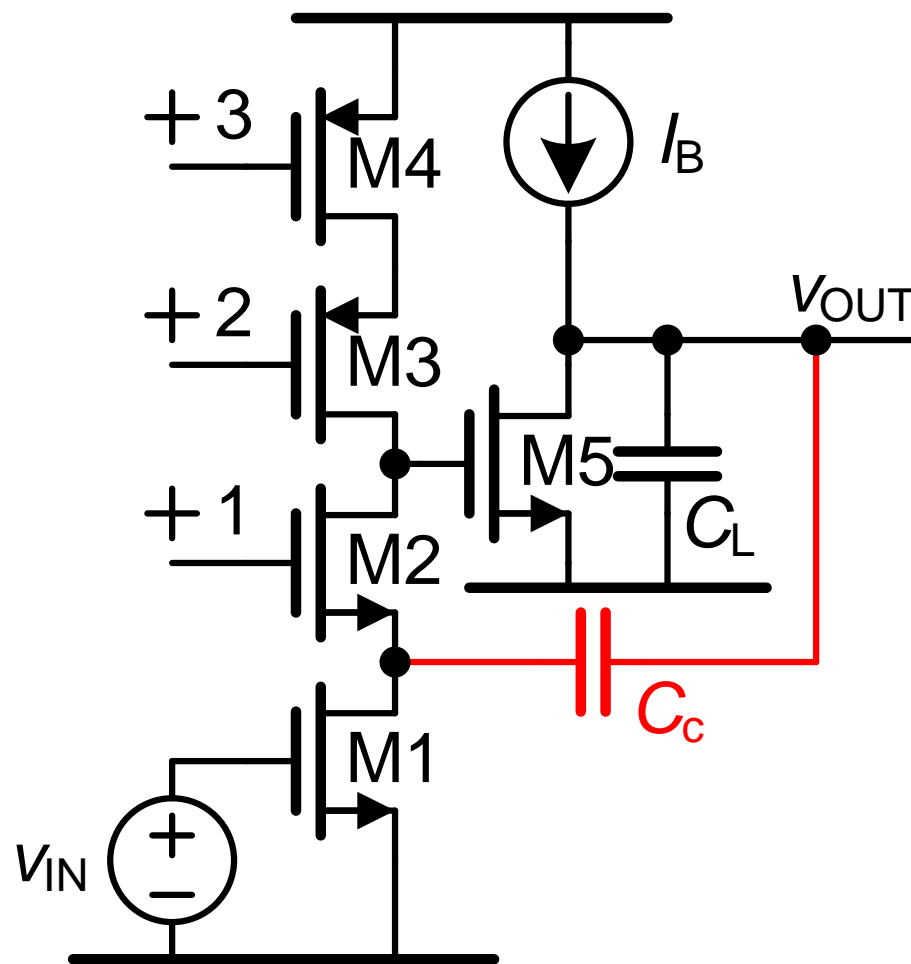
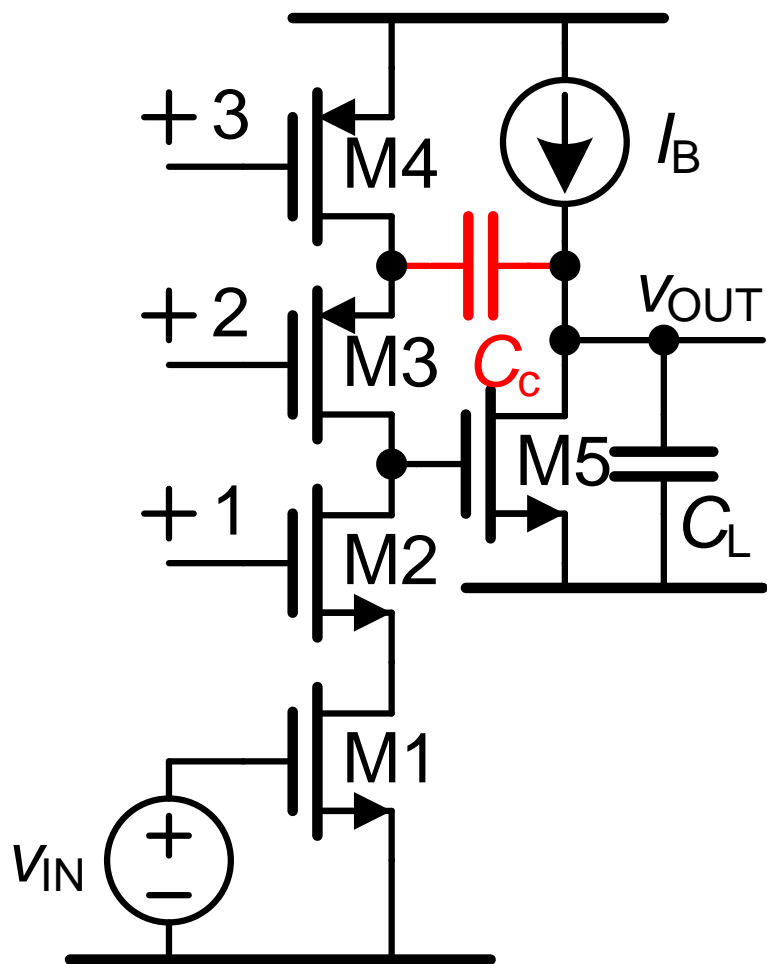
# 截断正馈通路 2



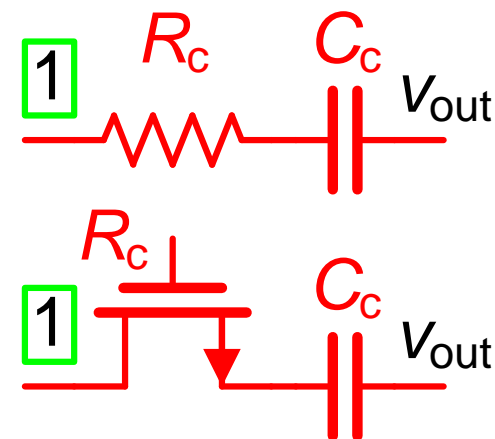
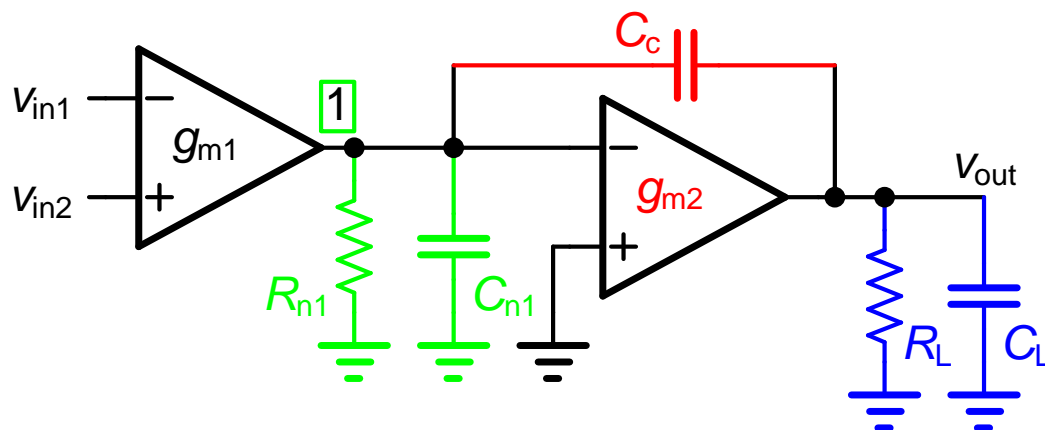
电流缓冲共源共栅

Ref.: Ahuja, JSSC Dec 83, 629-633

# 共源共栅密勒补偿



# 截断正馈通路 3



$$f_z = \frac{1}{2\pi C_c (1/g_{m2} - R_c)}$$

$R_c = 1/g_{m2}$  无穷远处零点

$R_c > 1/g_{m2}$  负零点

Ref.: Senderovics, JSSC Dec 78, 760-766

# 负零点补偿

$$R_c \gg 1/g_{m2} \Rightarrow f_z = -\frac{1}{2\pi C_c R_c}$$

$$f_z = 3GBW \Rightarrow R_c = \frac{1}{3g_{m1}}$$

选择  $\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}}$



# 练习：两级运放

已知： $GBW = 50 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$  和  $V_{GS1} - V_T = 0.2 \text{ V}$

求： $I_{DS1}$ 、 $I_{DS2}$ 、 $C_c$  和  $R_c$

$$\text{选择 } C_c = 1 \text{ pF} \Rightarrow g_{m1} = 2\pi C_c GBW = 315 \mu\text{S}$$

$$I_{DS1} = 31.5 \mu\text{A} \quad 1/g_{m1} \approx 3.2 \text{ k}\Omega$$

$$f_{nd} = 150 \text{ MHz} \Rightarrow g_{m2} = 2\pi C_L 4GBW = 8g_{m1} = 2520 \mu\text{S}$$

$$I_{DS2} = 252 \mu\text{A} \quad 1/g_{m2} \approx 400 \Omega$$

$$\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}} \Rightarrow 400 \Omega < R_c < 1 \text{ k}\Omega$$

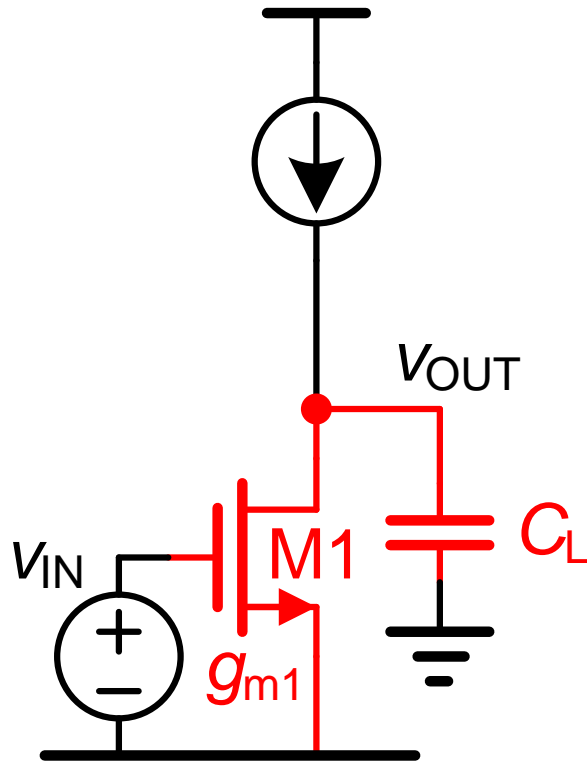
$$R_c \approx 400\sqrt{2.5} \approx 640 \Omega \pm 60\%$$

# 目录

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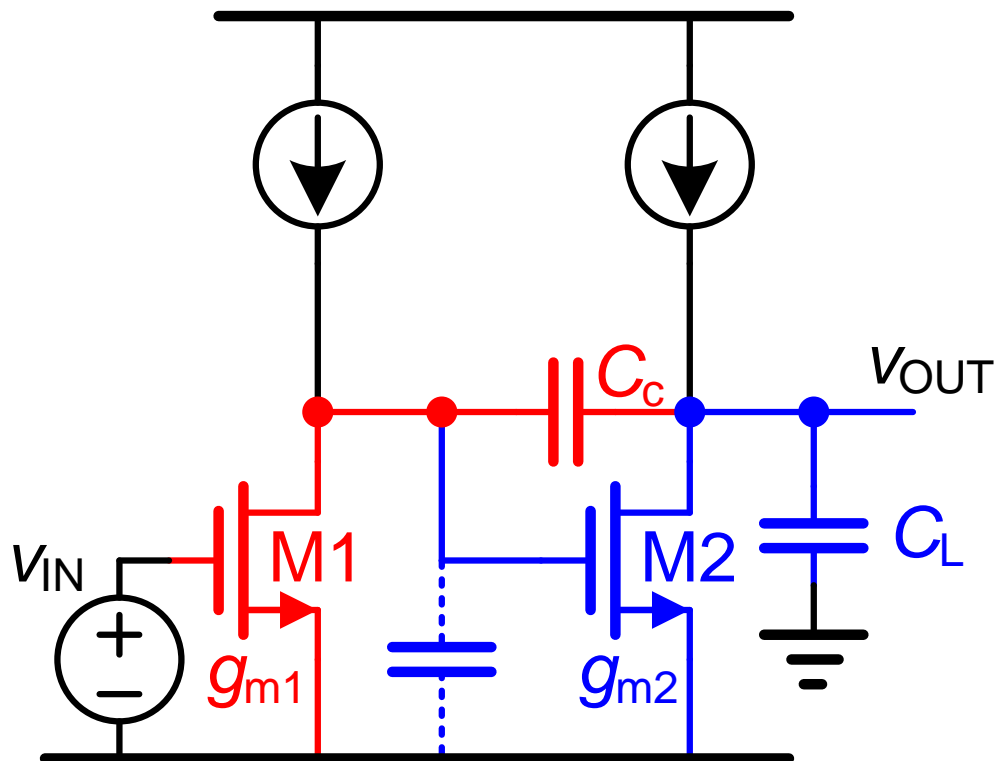
- 运算放大器的使用
- 两级运放的稳定性
- 极点分离
- 正零点的补偿
- 三级运放的稳定性

# 一级CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi C_L}$$

# 两级密勒CMOS OTA

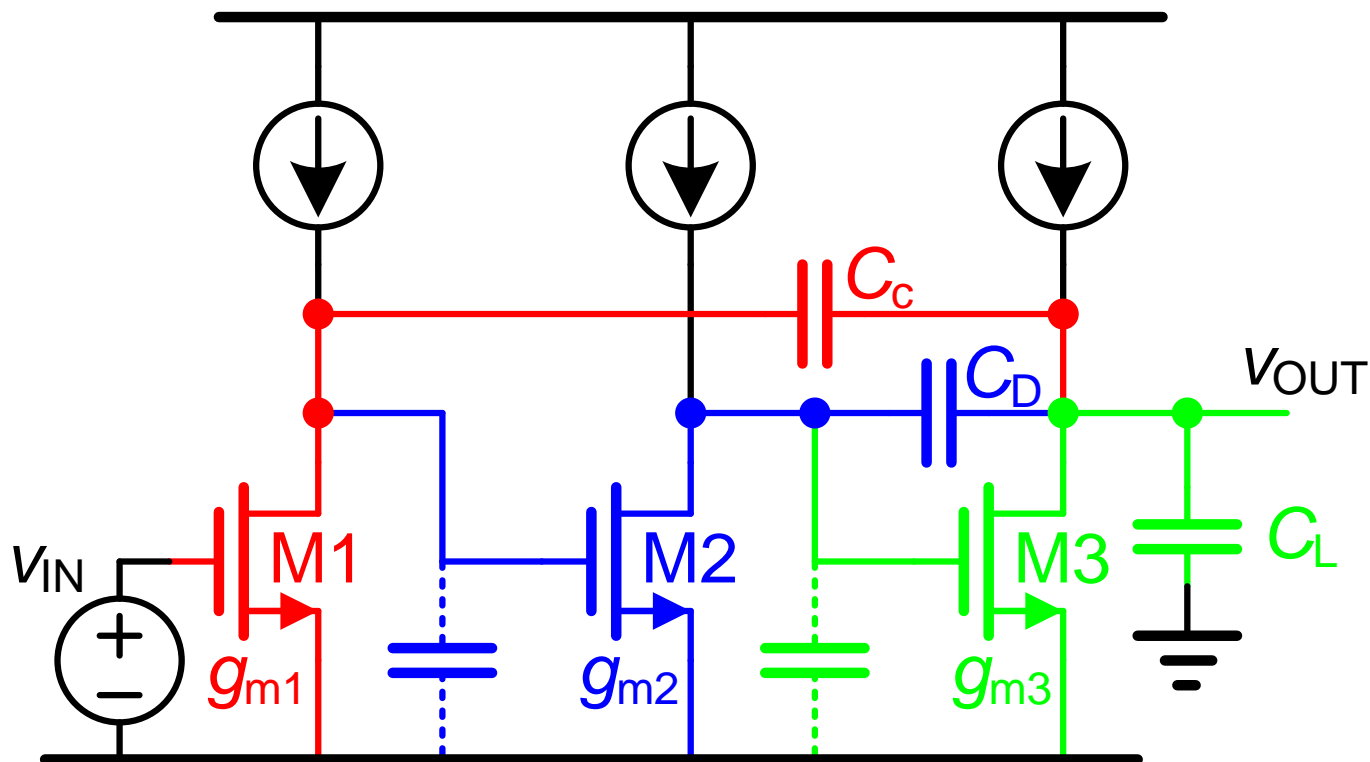


$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

# 三级嵌套密勒CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi C_c}$$

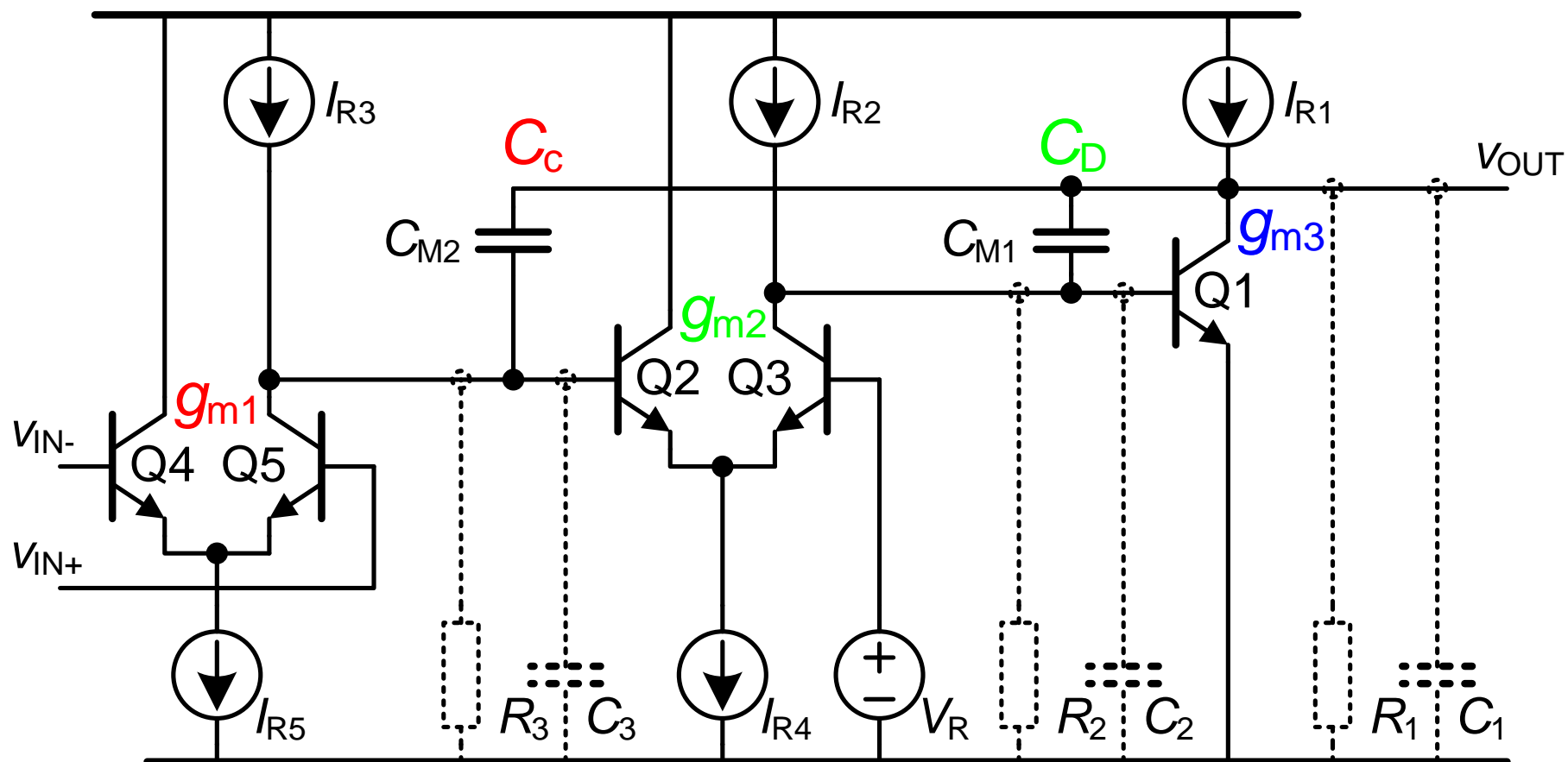
$$f_{nd1} = \frac{g_{m2}}{2\pi C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

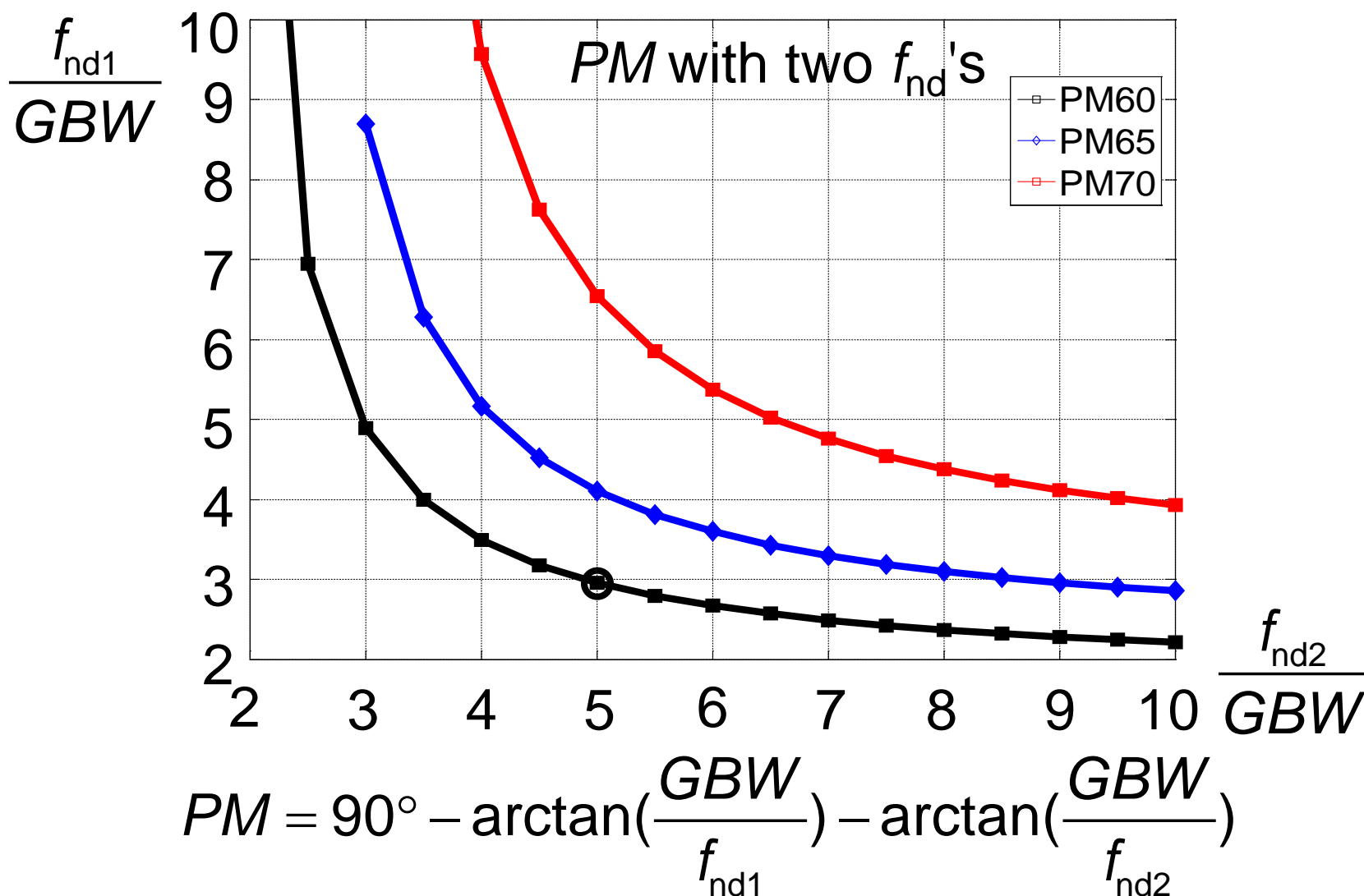
$$f_{nd2} = 5GBW$$

# 差分对构建嵌套密勒OTA

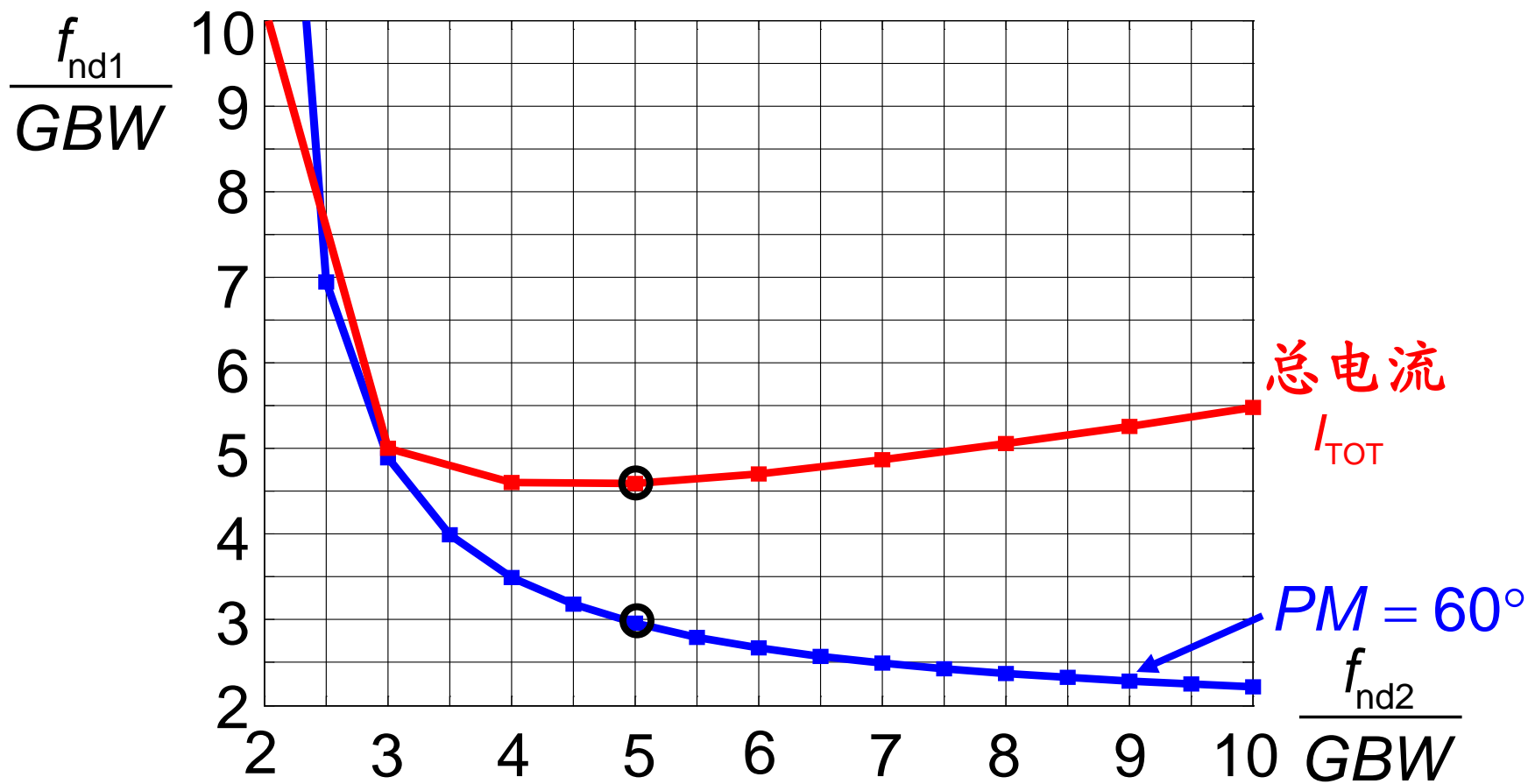


Ref.: Huijsing, JSSC Dec.85, pp.1144-1150

# PM与两个非主极点的关系



# 功耗与两个非主极点的关系



$$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3}$$



# 三级运放的初步设计

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi C_L}$$

$$f_{nd1} = 3GBW$$

$$f_{nd2} = 5GBW$$

选择  $C_D \approx C_c \Rightarrow$

$$\frac{g_{m2}}{g_{m1}} \approx 3 \qquad \frac{g_{m3}}{g_{m1}} \approx 5 \frac{C_L}{C_c}$$

输出级需要大电流!

# 练习：三级运放设计

已知： $GBW = 50 \text{ MHz}$ 、 $C_L = 2 \text{ pF}$  和  $V_{GS} - V_T = 0.2 \text{ V}$

求： $I_{DS1}$ 、 $I_{DS2}$ 、 $I_{DS3}$ 、 $C_c$  和  $C_D$

$$\text{选择 } C_c = C_D = 1 \text{ pF} \Rightarrow g_{m1} = 2\pi C_c GBW = 315 \text{ } \mu\text{S}$$

$$I_{DS1} = 31.5 \text{ } \mu\text{A}$$

$$f_{nd1} = 150 \text{ MHz} \Rightarrow g_{m2} = 2\pi C_D 3GBW = 3g_{m1} = 945 \text{ } \mu\text{S}$$

$$I_{DS2} = 94.5 \text{ } \mu\text{A}$$

$$f_{nd2} = 250 \text{ MHz} \Rightarrow g_{m3} = 2\pi C_L 5GBW = 10g_{m1} = 3150 \text{ } \mu\text{S}$$

$$I_{DS3} = 315 \text{ } \mu\text{A}$$

# 一/两/三级运放的比较

$$GBW = 50 \text{ MHz} \quad C_L = 2 \text{ pF}$$

单级:  $I_{DS1} = 31.5 \text{ } \mu\text{A}$

$$I_{TOT} = 2I_{DS1} = 63 \text{ } \mu\text{A}$$

两级: 选择  $C_c = 1 \text{ pF}$   $I_{DS1} = 31.5 \text{ } \mu\text{A}$   $I_{DS2} = 252 \text{ } \mu\text{A}$

$$I_{TOT} = 2I_{DS1} + I_{DS2} = 315 \text{ } \mu\text{A}$$

三级: 选择  $C_c = C_D = 1 \text{ pF}$

$$I_{DS1} = 31.5 \text{ } \mu\text{A} \quad I_{DS2} = 94.5 \text{ } \mu\text{A} \quad I_{DS3} = 315 \text{ } \mu\text{A}$$

$$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3} = 576 \text{ } \mu\text{A}$$