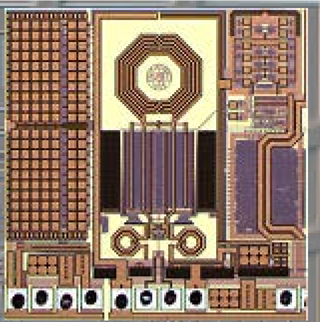

模拟集成电路设计原理

(Principle of Analog Integrated Circuit Design, INF0130025.02)

2014年09月18日

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集成电路器件和模型

- pn结模型
- BJT型晶体管模型
- MOS型晶体管模型
- MOS型与BJT型晶体管的比较

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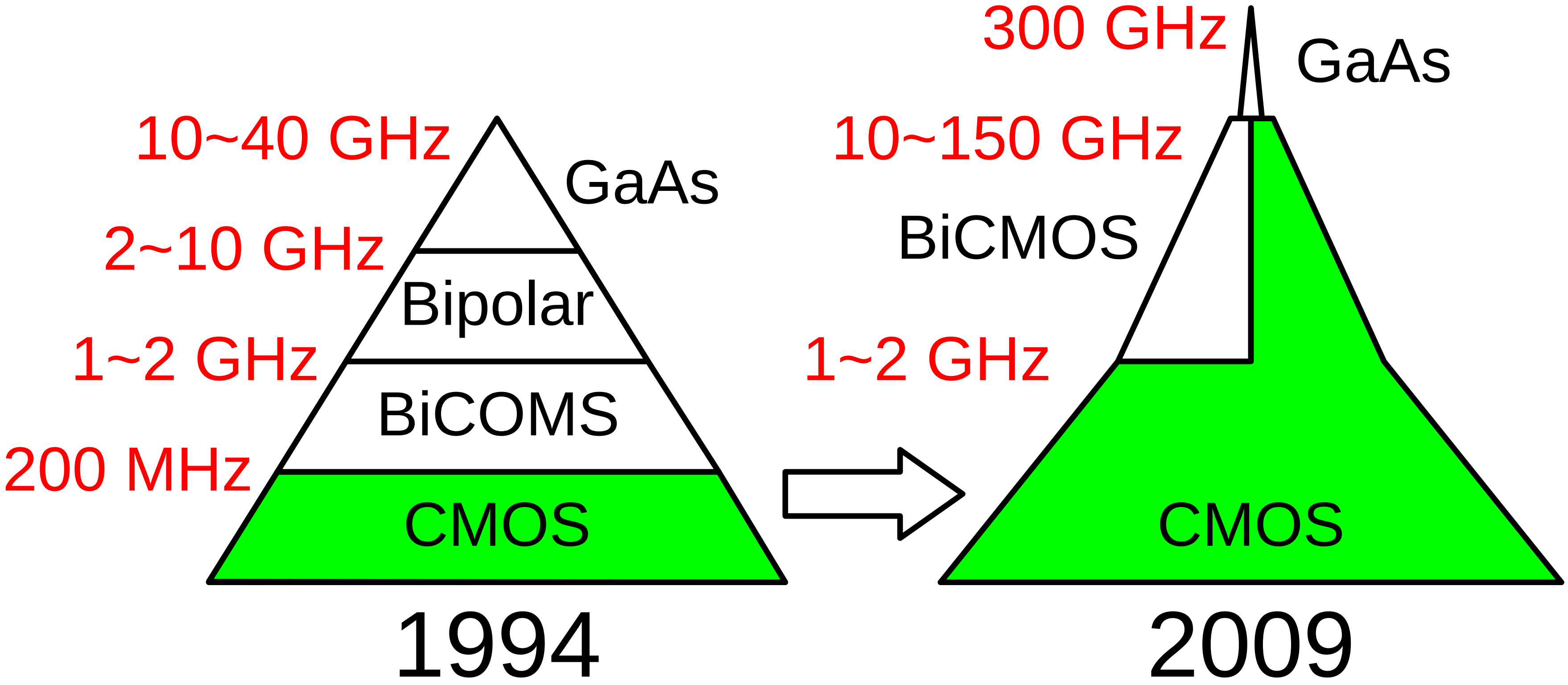
从双极型到MOS晶体管



Ref.: Toshiba & ISSCC2009

SIA线路图

Year	L_{min} µm	Bits/chip Gb/chip	Trans/chip millions/chip	Clock MHz	Wiring
1995	0.35	0.064	4	300	4-5
1998	0.25	0.256	7	450	5
2001	0.18	1	13	600	5-6
2004	0.13	4	25	800	6
2007	0.09	16	50	1000	6-7
2010	0.065	64	90	1100	7-8
2013	0.045				
2016	0.032				

Semiconductor Industry Association

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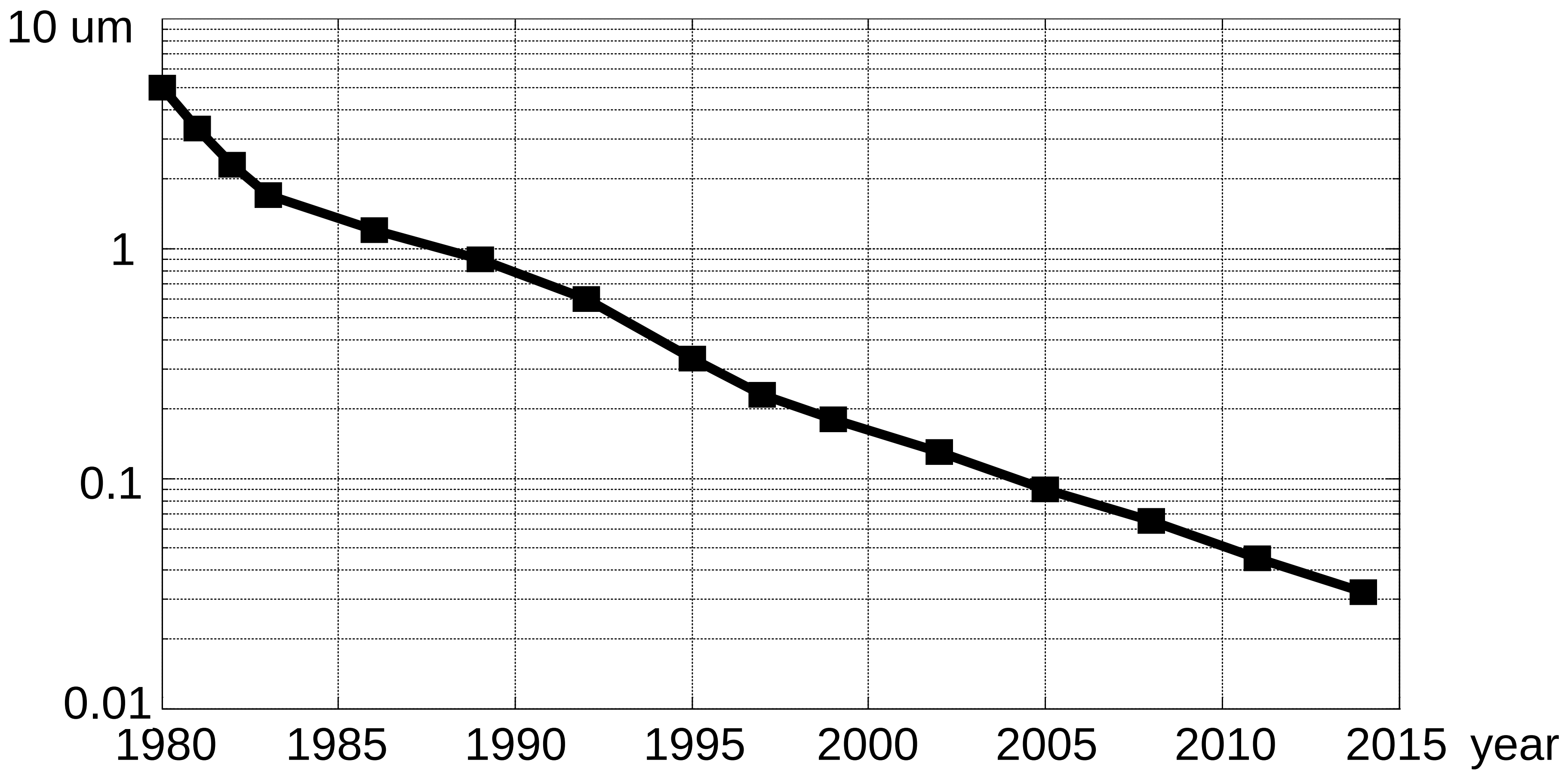
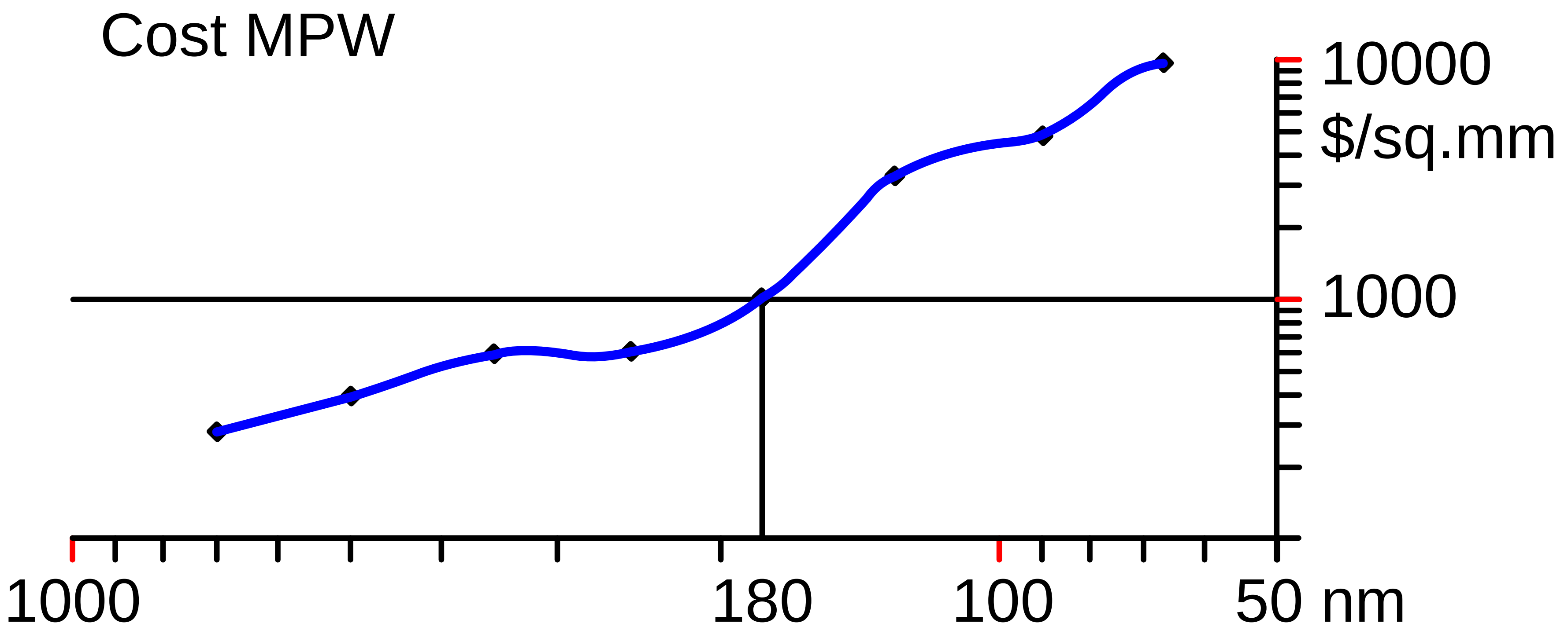
摩尔定律(The law of Moore)



使用不同特征尺寸的MPW价格



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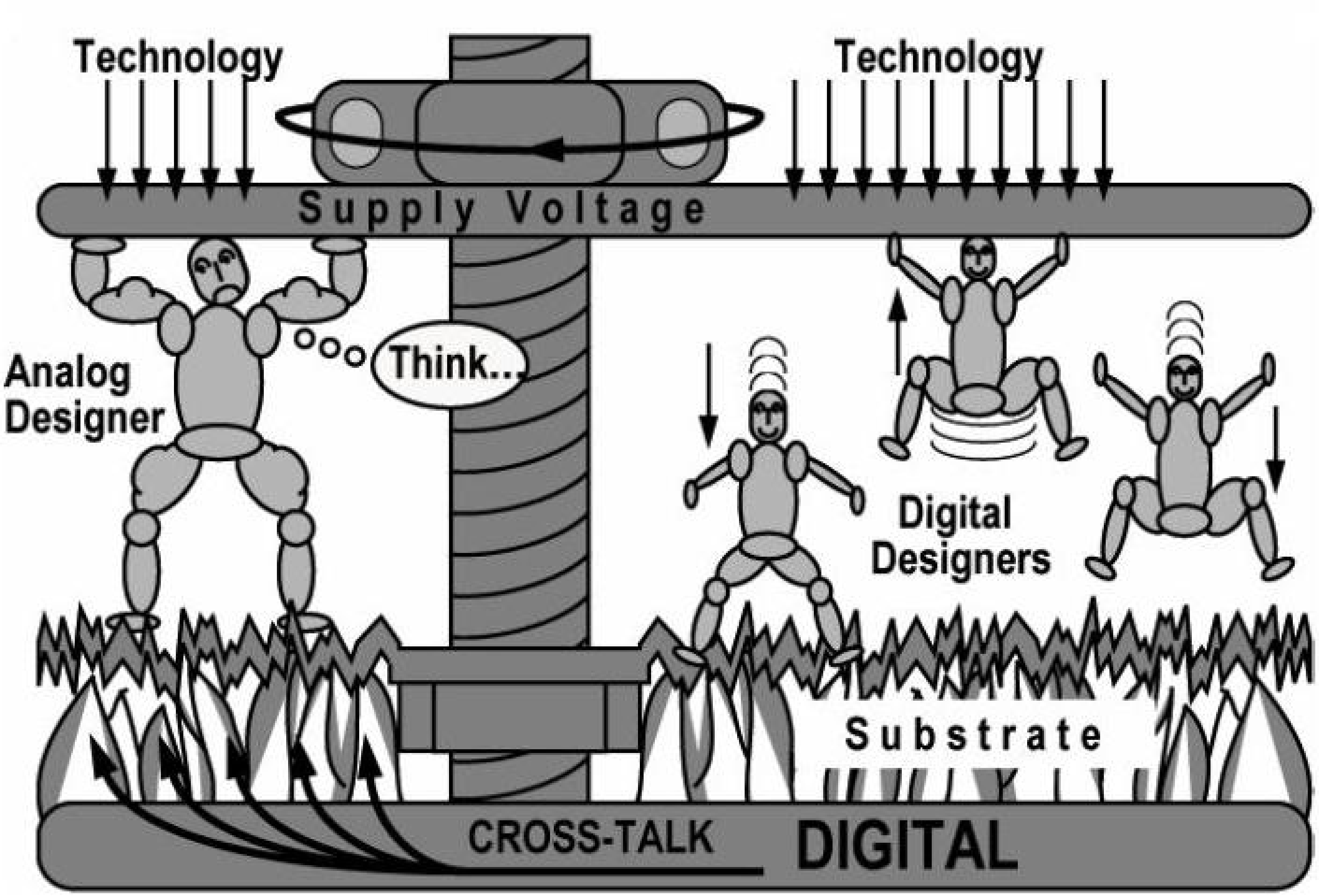
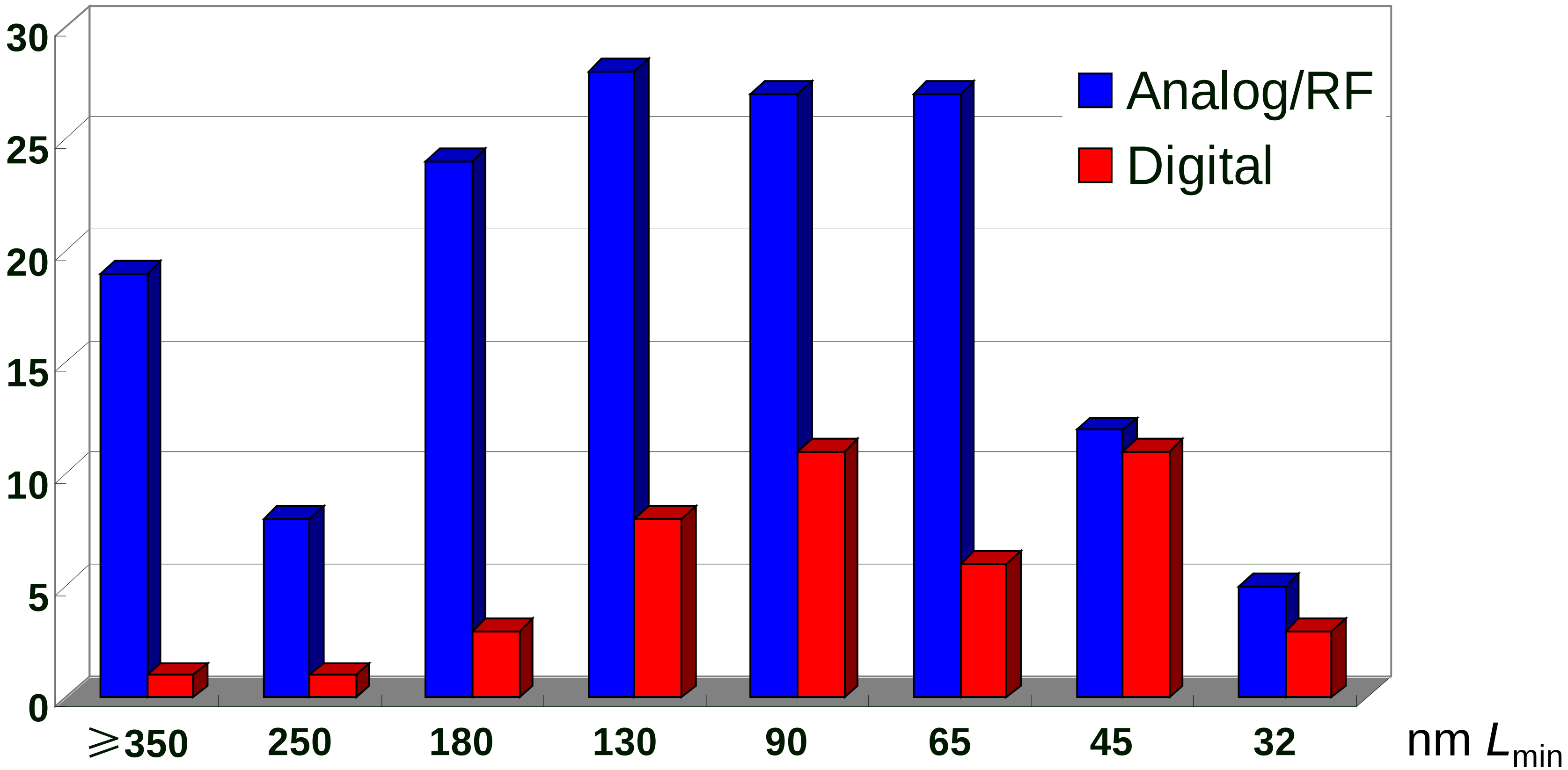
在深亚微米工艺下模拟设计的挑战



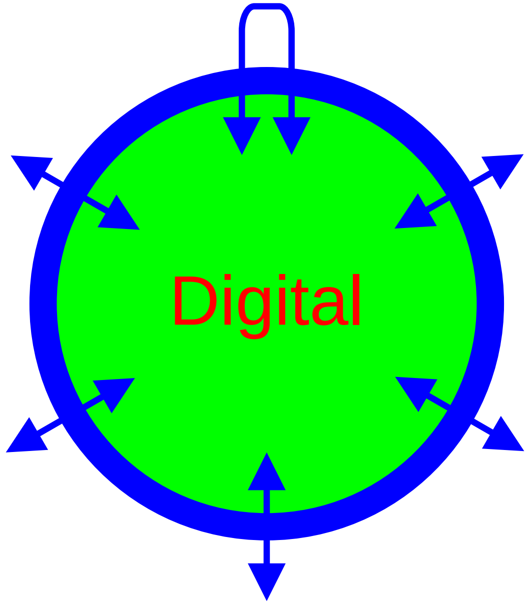
Ref.: SNUG2004, San Jose

ISSCC 2009论文分布情况



混合信号“蛋壳”

Analog
Real
World



Ref.: SNUG2004, San Jose

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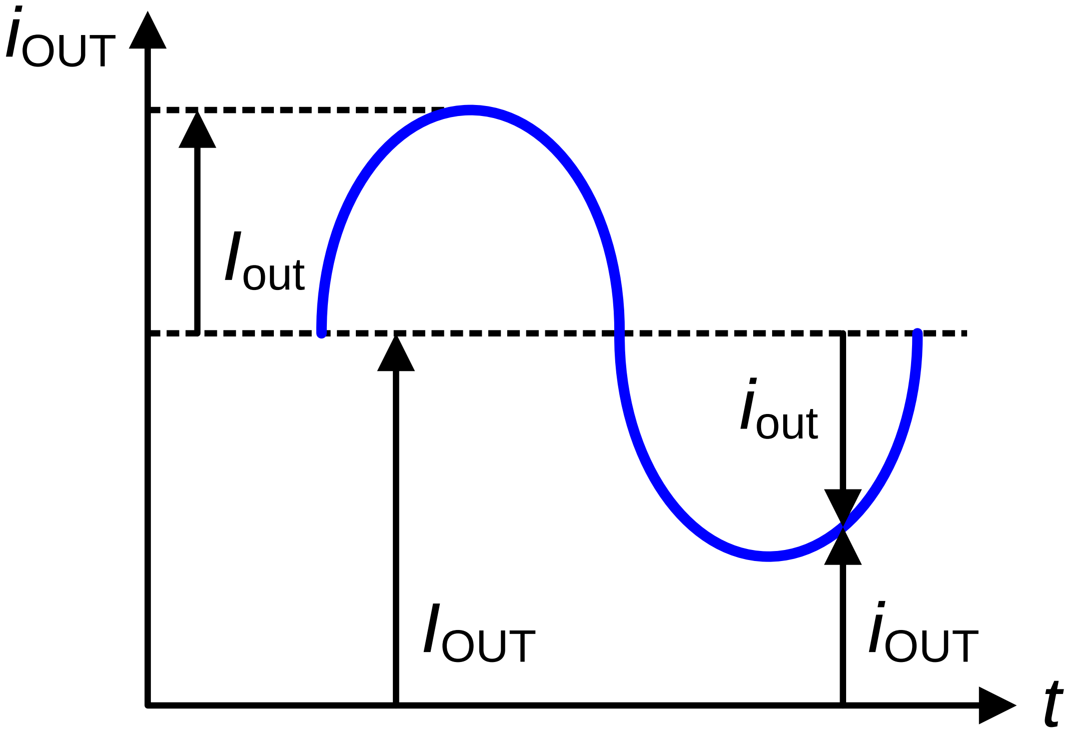
符号说明

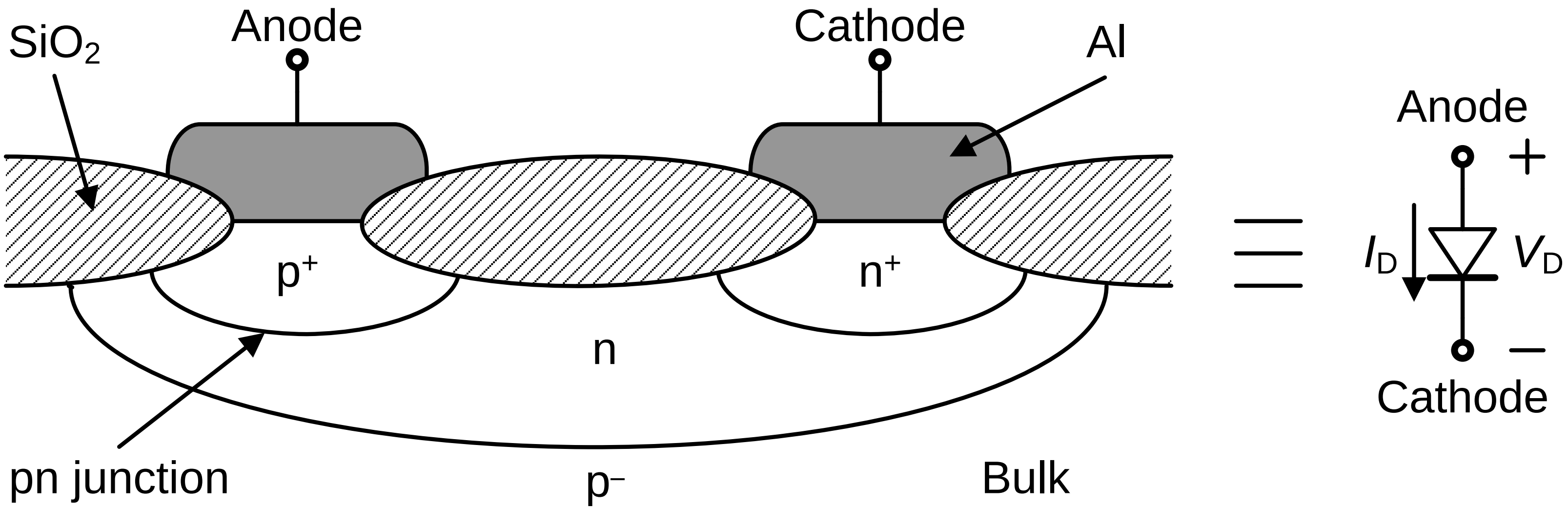
i_{OUT} total instantaneous value
I_{OUT} DC or average value
I_{out} amplitude of AC value
i_{out} instantaneous value of AC component



目 录

- pn结模型
  - □耗尽区宽度
  - □耗尽区电容
  - □击穿电压
  - □*I*-*V*关系曲线
  - □扩散电容
  - □小信号模型

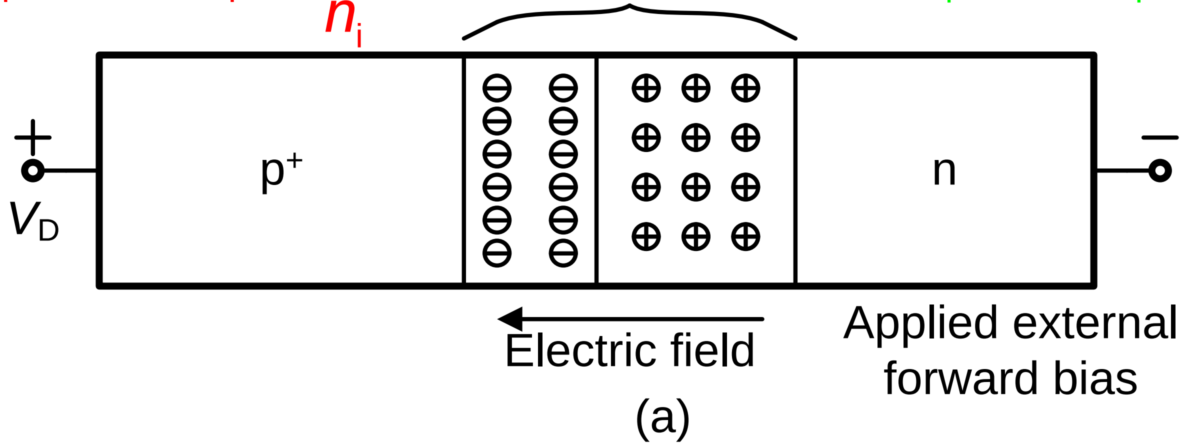
pn结的剖面图和符号



A cross section of a pn junction.

正偏pn结示意图：突变结

$$\Phi_F(p) = -V_T \ln\frac{N_A}{n_i} \qquad \Phi_F(n) = V_T \ln\frac{N_D}{n_i}$$



(a)

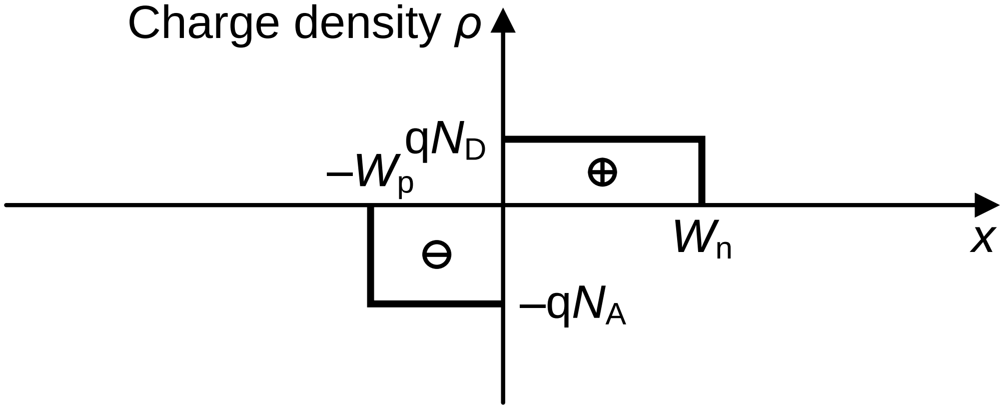
0 V偏压条件下，pn结中内建电势为：$\Phi_0 = V_T \ln\frac{N_A N_D}{n_i^2}$

其中，n_i为本征载流子浓度：$n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}}$

在300 K时，$V_T = \frac{kT}{q} \approx 26\ \mathrm{mV}$

$$\boxed{n_i \simeq 1.5 \times 10^{10}\mathrm{cm}^{-3}}$$

空间电荷密度



由电中性原则知，耗尽区内：$W_p N_A = W_n N_D$

由泊松方程，当$-W_p<x<0$，
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho}{\varepsilon_{si}} = \frac{qN_A}{\varepsilon_{si}}$$

其中 $\varepsilon_{si} = K_{si}\varepsilon_0$，积分得：
$$E(x) = -\frac{dV(x)}{dx} = -\left(\frac{qN_A}{\varepsilon_{si}}x + C_1\right)$$

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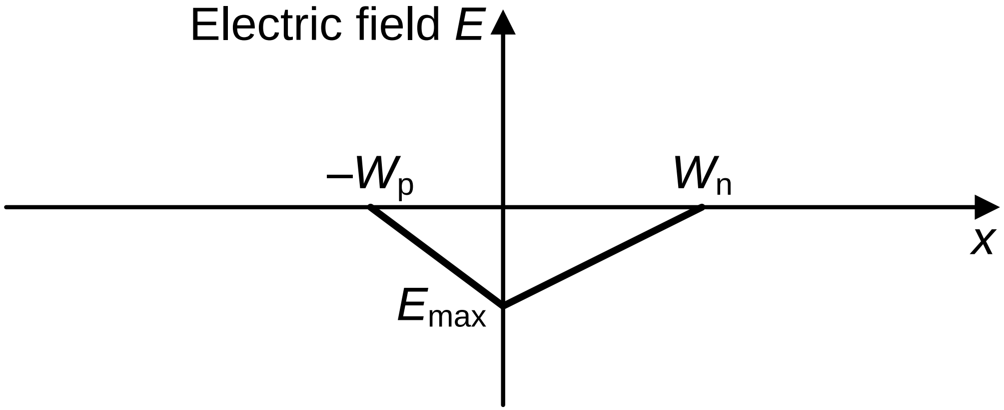
电场强度



第一个边界条件$E(-W_p)=0$，

$$E(x)=-\frac{qN_A}{\varepsilon_{si}}(x+W_p)=-\frac{dV(x)}{dx}$$

$$V(x)=\frac{qN_A}{\varepsilon_{si}}\left(\frac{x^2}{2}+W_px\right)+C_2$$

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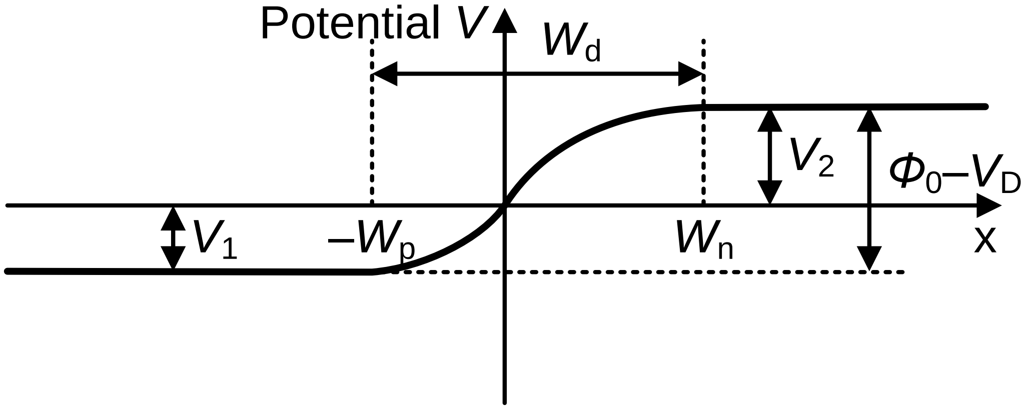
静电势



第二个边界条件$V(0)=0$，　$V(x) = \frac{qN_A}{\varepsilon_{si}}\left(\frac{x^2}{2} + W_p x\right)$

令$V(-W_p)=-V_1$，　$V_1 = \frac{qN_A}{\varepsilon_{si}}\frac{W_p^2}{2}$

同理，从$x=0$到$x=W_n$的电压为V_2，　$V_2 = \frac{qN_D}{\varepsilon_{si}}\frac{W_n^2}{2}$

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耗尽区宽度

pn结上的总静电势为：

$$\Phi_0-V_D=V_1+V_2=\frac{q}{2\varepsilon_{si}}\left(N_AW_p^2+N_DW_n^2\right)=\frac{qW_p^2N_A}{2\varepsilon_{si}}\left(1+\frac{N_A}{N_D}\right)$$

耗尽区总宽度：

$$W_d=W_p+W_n=\boxed{\left[\frac{2\varepsilon_{si}(N_A+N_D)}{qN_AN_D}\right]^{1/2}(\Phi_0-V_D)^{1/2}}$$

其中，p型区和n型区的宽度分别为：

$$W_p=\left[\frac{2\varepsilon_{si}\left(\Phi_0-V_D\right)}{qN_A\left(1+\frac{N_A}{N_D}\right)}\right]^{1/2}\qquad W_n=\left[\frac{2\varepsilon_{si}\left(\Phi_0-V_D\right)}{qN_D\left(1+\frac{N_D}{N_A}\right)}\right]^{1/2}$$

例题1:

一个硅材料pn结的掺杂溶度N_A=10^{16}原子/cm^{3}和 N_D=10^{17}原子/cm^3，在10 V反偏电压下，计算结的内建势和耗尽区宽度。

解：在300K时，内建势：

$$\Phi_0 = V_T \ln\frac{N_A N_D}{n_i^2} = 0.026\ln\frac{10^{16}\times10^{17}}{2.25\times10^{20}} = 757\ \text{mV}$$

p型区的耗尽区宽度

$$W_p = \left[\frac{2\varepsilon_{si}\left(\Phi_0 - V_D\right)}{qN_A\left(1+\dfrac{N_A}{N_D}\right)}\right]^{1/2} = \left[\frac{2\times1.04\times10^{-12}\times10.757}{1.6\times10^{-19}\times10^{16}\times1.1}\right]^{1/2} = 1.1\times10^{-4}\ \text{cm}$$

$$= 1.1\ \mu\text{m}$$

n型区的耗尽区宽度 $W_n = W_p(N_A/N_D) = 0.11\ \mu\text{m}$

耗尽区内的固定电荷

$$Q_\mathrm{j} = Q^+ = Q^- = \mathrm{q}N_\mathrm{D}W_\mathrm{n} = \left[\frac{2\mathrm{q}N_\mathrm{D}\varepsilon_\mathrm{si}(\Phi_0 - V_\mathrm{D})}{\left(1+\dfrac{N_\mathrm{D}}{N_\mathrm{A}}\right)}\right]^{1/2}$$

当$N_\mathrm{A} >> N_\mathrm{D}$时 $Q_\mathrm{j} \approx [2\mathrm{q}\varepsilon_\mathrm{si}(\Phi_0 - V_\mathrm{D})N_\mathrm{D}]^{1/2}$

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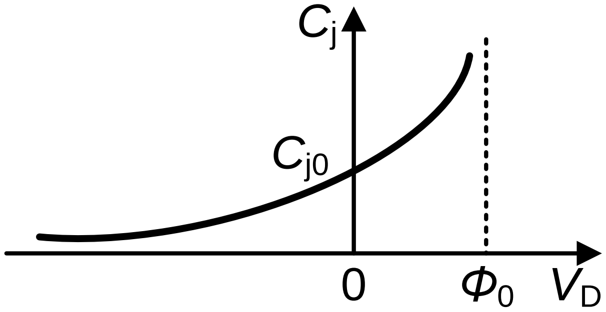
耗尽电容

$$C_j=\left|\frac{dQ_j}{dV_D}\right|=\left[\frac{q\varepsilon_{si}}{2(\Phi_0-V_D)}\frac{N_AN_D}{N_A+N_D}\right]^{1/2}=\frac{\varepsilon_{si}}{W_d}=\boxed{\frac{C_{j0}}{\sqrt{1-\dfrac{V_D}{\Phi_0}}}}$$

其中， $C_{j0}=\sqrt{\dfrac{q\varepsilon_{si}}{2\Phi_0}\dfrac{N_AN_D}{N_A+N_D}}$

当$N_A>>N_D$时，$C_{j0}\approx\sqrt{\dfrac{q\varepsilon_{si}N_D}{2\Phi_0}}$

$$Q_j\approx 2C_{j0}\Phi_0\sqrt{1-\frac{V_D}{\Phi_0}}$$



Depletion capacitance as a function of externally applied junction voltage

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例题2:

一个硅材料pn结的掺杂溶度$N_A=10^{16}$原子/cm^{3}和 $N_D=10^{17}$原子/cm^3，计算面积10 μm × 10 μm的pn结在0 V偏压下的耗尽电容？在3 V反偏电压下，pn结耗尽电容是多少？

解：0 V偏压下，面耗尽电容为，

$$C_{j0}=\sqrt{\frac{q\varepsilon_{si}}{2\Phi_0}\frac{N_AN_D}{N_A+N_D}}=\sqrt{\frac{1.6\times10^{-19}\times1.04\times10^{-12}\times10^{17}}{2\times0.757\times11}}$$

$$=3.16\times10^{-8}\ \text{F/cm}^2=0.316\ \text{fF/μm}^2$$

面积10 μm × 10 μm的pn结的耗尽电容为31.6 fF

3 V反偏电压下，耗尽电容为，

$$C_{T,j}=\frac{C_{T,j0}}{\sqrt{1-\frac{V_D}{\Phi_0}}}=\frac{3.16\ \text{fF}}{\sqrt{1-\frac{-3}{0.757}}}=1.42\ \text{fF}$$

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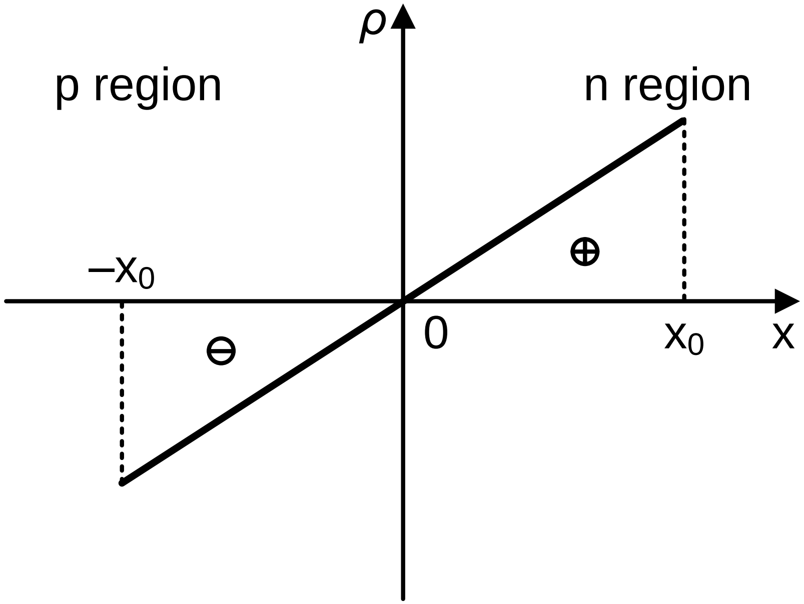
练习1：缓变pn结

缓变pn结的掺杂浓度梯度为*a*，请推导缓变pn结的空间电荷和耗尽电容。



Space charge density in a linearly graded pn junclian

击穿电压

耗尽区内的最大电场：

$$E_{\max} = -\frac{qN_A}{\varepsilon_{si}} W_p$$

当忽略Φ_0，

$$\left|E_{\max}\right| \approx \left[\frac{2qN_A N_D V_D}{\varepsilon_{si}(N_A + N_D)}\right]^{1/2}$$

击穿时的临界电场为E_{crit}，则击穿电压为：

$$BV = \frac{\varepsilon_{si}(N_A + N_D)}{2qN_A N_D} E_{crit}^2$$

当$N_A >> N_D$时，

$$BV \approx \frac{\varepsilon_{si} E_{crit}^2}{2qN_D}$$

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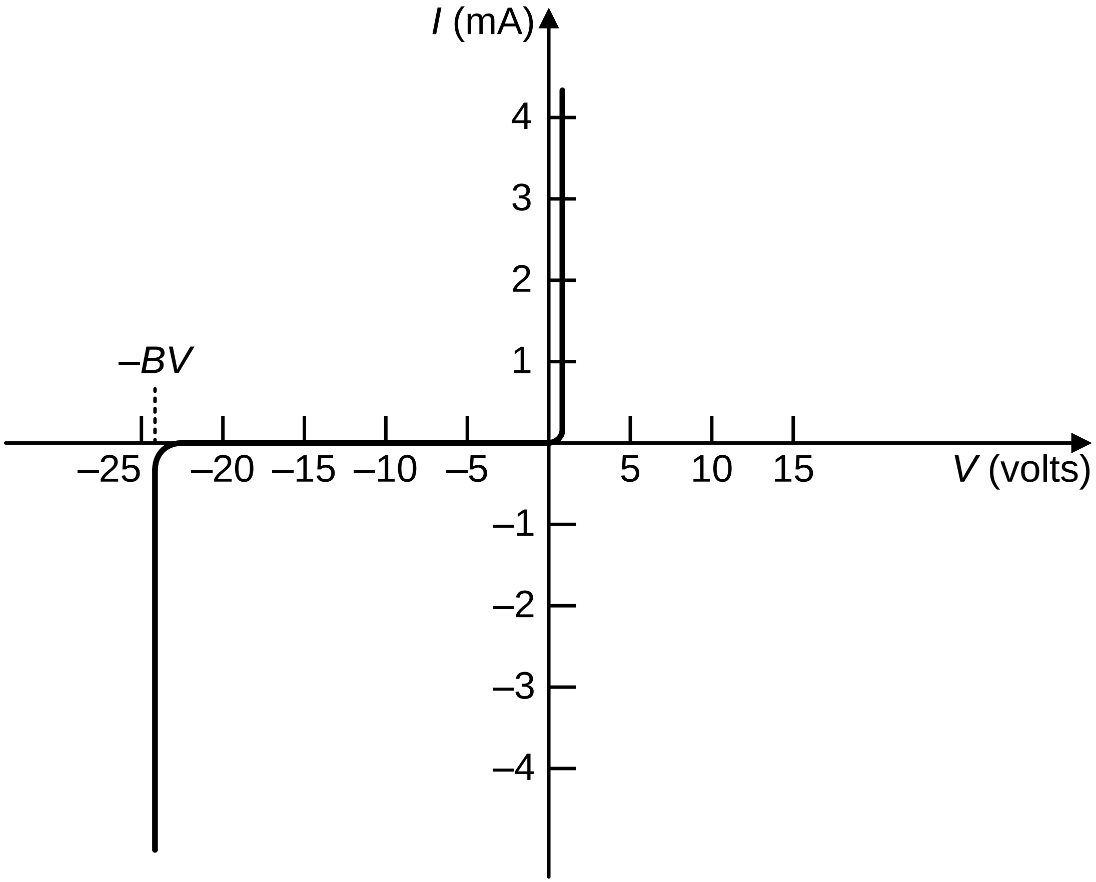
击穿电压反偏电流



击穿电压附近的反偏电流为：

$$I_{RA} = MI_R$$

其中I_R为pn结正常反偏电流，M为倍增因子，n=3~6

$$M = \frac{1}{1-\left(\frac{-V_D}{BV}\right)^n}$$

Typical *I-V* characteristic of a junction diode showing avalanche breakdown

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例题2:

一个硅材料pn结的掺杂溶度$N_A=10^{16}$原子/cm^{3}和 $N_D=10^{17}$原子/cm^3，假设临界电场$E_{crit}=3\times10^5$ V/cm，计算击穿电压。

解：击穿电压：

$$BV=\frac{\varepsilon_{si}(N_A+N_D)}{2qN_AN_D}E_{crit}^2=\frac{1.04\times10^{-12}\times11}{2\times1.6\times10^{-19}\times10^{17}}\times9\times10^{10}$$

$$=32\ \text{V}$$

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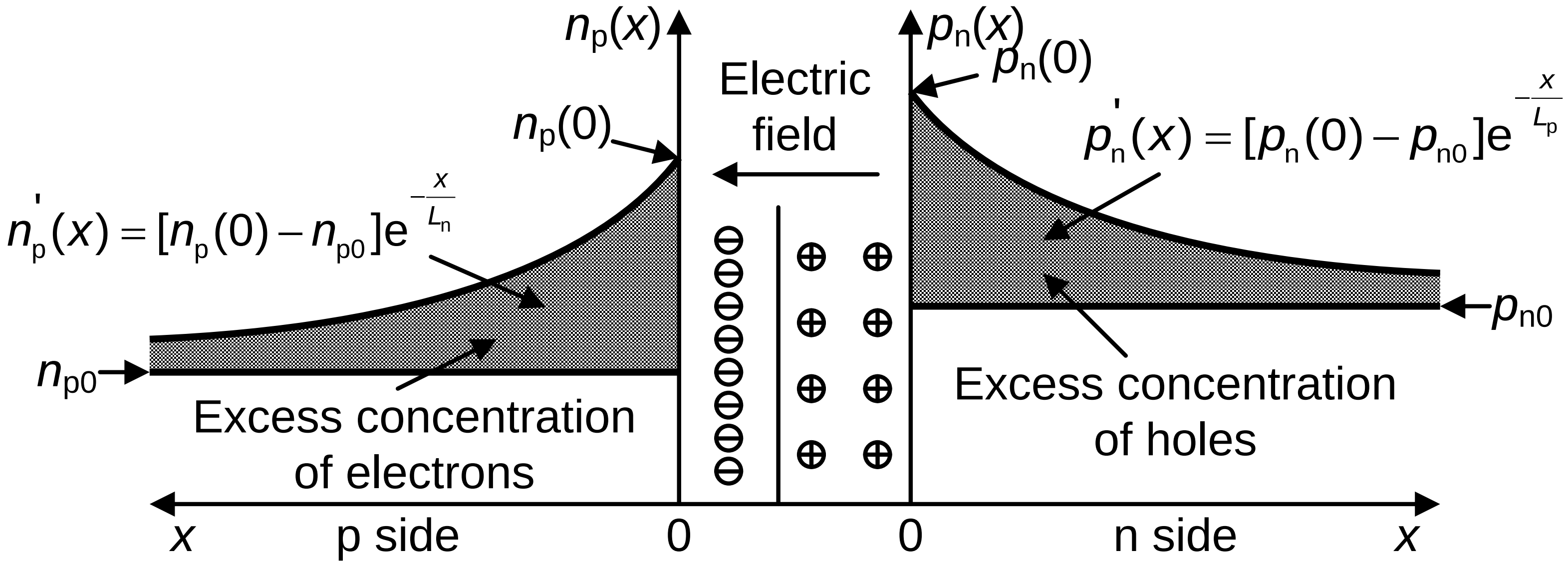
pn结中少数载流子分布图



Impurity concentration profile for diffused pn junction

p型和n型半导体中的少子浓度分别为:

$$n_p(0) = n_{p0}e^{\frac{V_D}{V_T}} \qquad p_n(0) = p_{n0}\,e^{\frac{V_D}{V_T}}$$

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非平衡少子的扩散

非平衡少子与扩散的距离之间是指数关系：

n型区的空穴：

$$p_n^{'}(x) = p_n^{'}(0)\mathrm{e}^{-\frac{x}{L_p}} = [p_n(0) - p_{n0}]\mathrm{e}^{-\frac{x}{L_p}} = p_{n0}(\mathrm{e}^{\frac{V_D}{V_T}} - 1)\mathrm{e}^{-\frac{x}{L_p}}$$

p型区的电子：

$$n_p^{'}(x) = n_p^{'}(0)\mathrm{e}^{-\frac{x}{L_n}} = [n_p(0) - n_{p0}]\mathrm{e}^{-\frac{x}{L_n}} = n_{p0}(\mathrm{e}^{\frac{V_D}{V_T}} - 1)\mathrm{e}^{-\frac{x}{L_n}}$$

其中L_p和L_n分别是n型和p型中空穴和电子的扩散长度。

$$p_n^{'}(x) = p_n(x) - p_{n0} \qquad n_p^{'}(x) = n_p(x) - n_{p0}$$

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非平衡少子的扩散电流密度

n型区的空穴：

$$J_{\mathrm{p}}(0)=-\mathrm{q}D_{\mathrm{p}}\frac{\mathrm{d}p_{\mathrm{n}}^{'}(x)}{\mathrm{d}x}\bigg|_{x=0}=\frac{\mathrm{q}D_{\mathrm{p}}p_{\mathrm{n0}}}{L_{\mathrm{p}}}\left(\mathrm{e}^{\frac{V_{\mathrm{D}}}{V_{\mathrm{T}}}}-1\right)$$

p型区的电子：

$$J_{\mathrm{n}}(0)=-\mathrm{q}D_{\mathrm{n}}\frac{\mathrm{d}n_{\mathrm{p}}^{'}(x)}{\mathrm{d}x}\bigg|_{x=0}=\frac{\mathrm{q}D_{\mathrm{n}}n_{\mathrm{p0}}}{L_{\mathrm{n}}}\left(\mathrm{e}^{\frac{V_{\mathrm{D}}}{V_{\mathrm{T}}}}-1\right)$$

其中，D_{n}和D_{p}分别是电子和空穴扩散系数

pn结的总电流密度

$$J(0)=J_{\mathrm{p}}(0)+J_{\mathrm{n}}(0)=\mathrm{q}\left(\frac{D_{\mathrm{p}}p_{\mathrm{n0}}}{L_{\mathrm{p}}}+\frac{D_{\mathrm{n}}n_{\mathrm{p0}}}{L_{\mathrm{n}}}\right)\left(\mathrm{e}^{\frac{V_{\mathrm{D}}}{V_{\mathrm{T}}}}-1\right)$$

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I-V关系曲线

横切面积为A的pn结电流:

$$I_D = AJ_n(0) = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)(e^{\frac{V_D}{V_T}} - 1)$$

$$= qA\left(\frac{D_p n_i^2}{L_p N_D} + \frac{D_n n_i^2}{L_n N_A}\right)(e^{\frac{V_D}{V_T}} - 1) = \boxed{I_s(e^{\frac{V_D}{V_T}} - 1)}$$

其中I_S为pn结反向饱和电流，为常数。

非平衡少数载流子电荷

n型区的空穴：

$$Q_{\rm p}=\mathrm{q}A\int_0^{\infty}p_{\rm n}^{'}(x)\mathrm{d}x=\mathrm{q}A\int_0^{\infty}[p_{\rm n}(0)-p_{\rm n0}]\mathrm{e}^{-\frac{x}{L_{\rm p}}}\mathrm{d}x$$

$$=\mathrm{q}AL_{\rm p}[p_{\rm n}(0)-p_{\rm n0}]=\frac{\mathrm{q}AL_{\rm p}n_{\rm i}^2}{N_{\rm D}}(\mathrm{e}^{\frac{V_{\rm D}}{V_{\rm T}}}-1)$$

p型区的电子：

$$Q_{\rm n}=\mathrm{q}A\int_0^{\infty}n_{\rm p}^{'}(x)\mathrm{d}x=\mathrm{q}A\int_0^{\infty}[n_{\rm p}(0)-n_{\rm p0}]\mathrm{e}^{-\frac{x}{L_{\rm n}}}\mathrm{d}x$$

$$=\mathrm{q}AL_{\rm n}[n_{\rm p}(0)-n_{\rm p0}]=\frac{\mathrm{q}AL_{\rm n}n_{\rm i}^2}{N_{\rm A}}(\mathrm{e}^{\frac{V_{\rm D}}{V_{\rm T}}}-1)$$

扩散电容

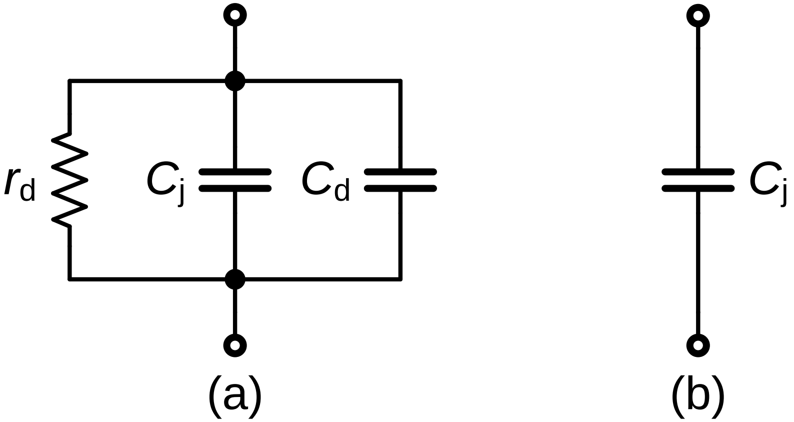
$$C_{\mathrm{d}}=\frac{\mathrm{d}Q_{\mathrm{d}}}{\mathrm{d}V_{\mathrm{D}}}=\frac{\mathrm{d}(Q_{\mathrm{n}}+Q_{\mathrm{p}})}{\mathrm{d}V_{\mathrm{D}}}=\frac{\mathrm{q}An_{\mathrm{i}}^{2}}{V_{\mathrm{T}}}\mathrm{e}^{\frac{V_{\mathrm{D}}}{V_{\mathrm{T}}}}\left(\frac{L_{\mathrm{n}}}{N_{\mathrm{A}}}+\frac{L_{\mathrm{p}}}{N_{\mathrm{D}}}\right)$$

$$\approx\frac{I_{\mathrm{D}}}{V_{\mathrm{T}}}\frac{L_{\mathrm{n}}^{2}L_{\mathrm{p}}+L_{\mathrm{p}}^{2}L_{\mathrm{n}}\frac{N_{\mathrm{A}}}{N_{\mathrm{D}}}}{L_{\mathrm{p}}D_{\mathrm{n}}+L_{\mathrm{n}}D_{\mathrm{p}}\frac{N_{\mathrm{A}}}{N_{\mathrm{D}}}}$$

重掺杂一侧少子的电荷存储可以忽略，假设n区重掺杂，$N_{\mathrm{D}}>>N_{\mathrm{A}}$，扩散电荷$Q_{\mathrm{d}}\approx Q_{\mathrm{n}}$：其中$\tau_{\mathrm{T}}$为二极管的渡越时间

$$C_{\mathrm{d}}\approx\frac{L_{\mathrm{n}}^{2}}{D_{\mathrm{n}}}\frac{I_{\mathrm{D}}}{V_{\mathrm{T}}}=\tau_{\mathrm{T}}\frac{I_{\mathrm{D}}}{V_{\mathrm{T}}}$$

pn结的小信号模型



The small-singal model for a junction.
(a) forward bias. (b) reverse bias.

正偏输出阻抗r_d

$$\boxed{\frac{1}{r_d}=\frac{dI_D}{dV_D}=I_s\frac{e^{V_D/V_T}}{V_T}\approx\frac{I_D}{V_T}}$$

$$\boxed{C_d=\frac{\tau_T}{r_d}}$$

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例题3:

二极管的渡越时间为100 pS，正向偏置电流为1 mA，问其小信号输出阻抗和扩散电容分别是多少？

解:

小信号是输出阻抗:

$$r_{\mathrm{d}} = \frac{V_{\mathrm{T}}}{I_{\mathrm{D}}} = \frac{26\ \mathrm{mV}}{1\ \mathrm{mA}} = 26\ \Omega$$

扩散电容:

$$C_{\mathrm{d}} = \frac{\tau_{\mathrm{T}}}{r_{\mathrm{d}}} = \frac{100\ \mathrm{pS}}{26\ \Omega} = 3.8\ \mathrm{pF}$$

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- BJT型晶体管模型
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    - □正向工作区
    - □厄利效应
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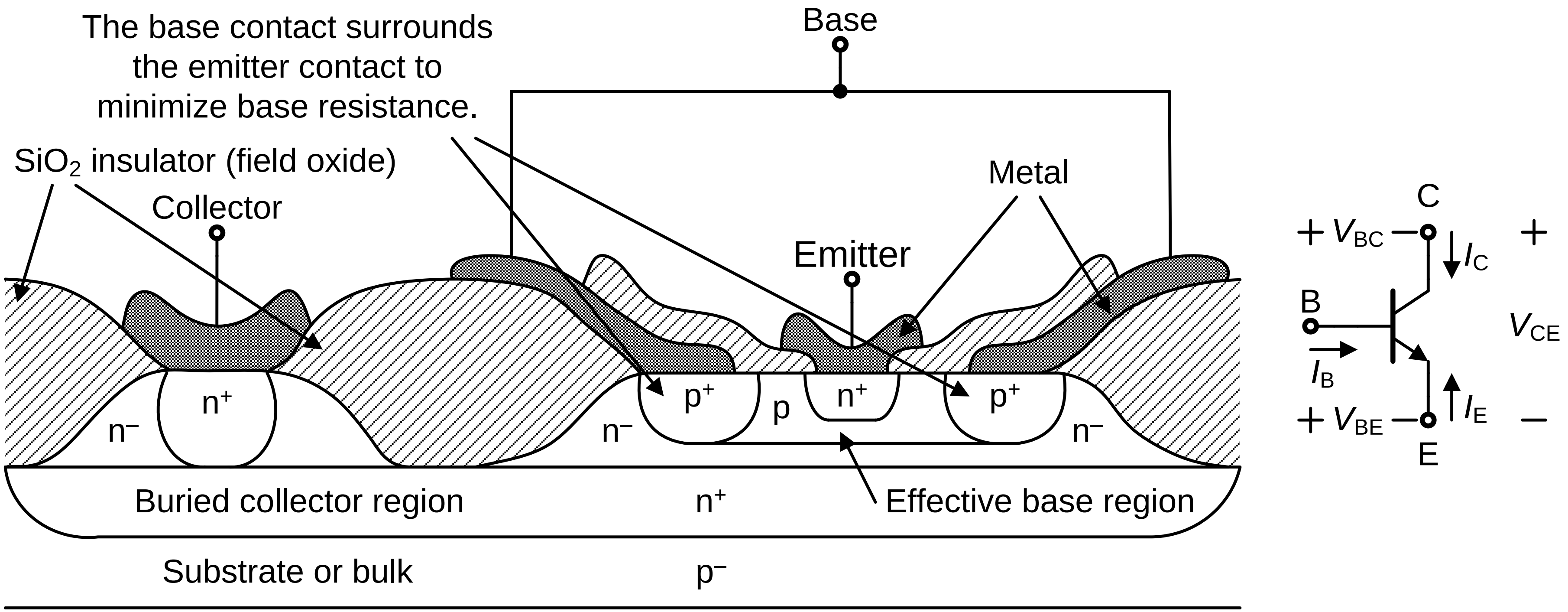
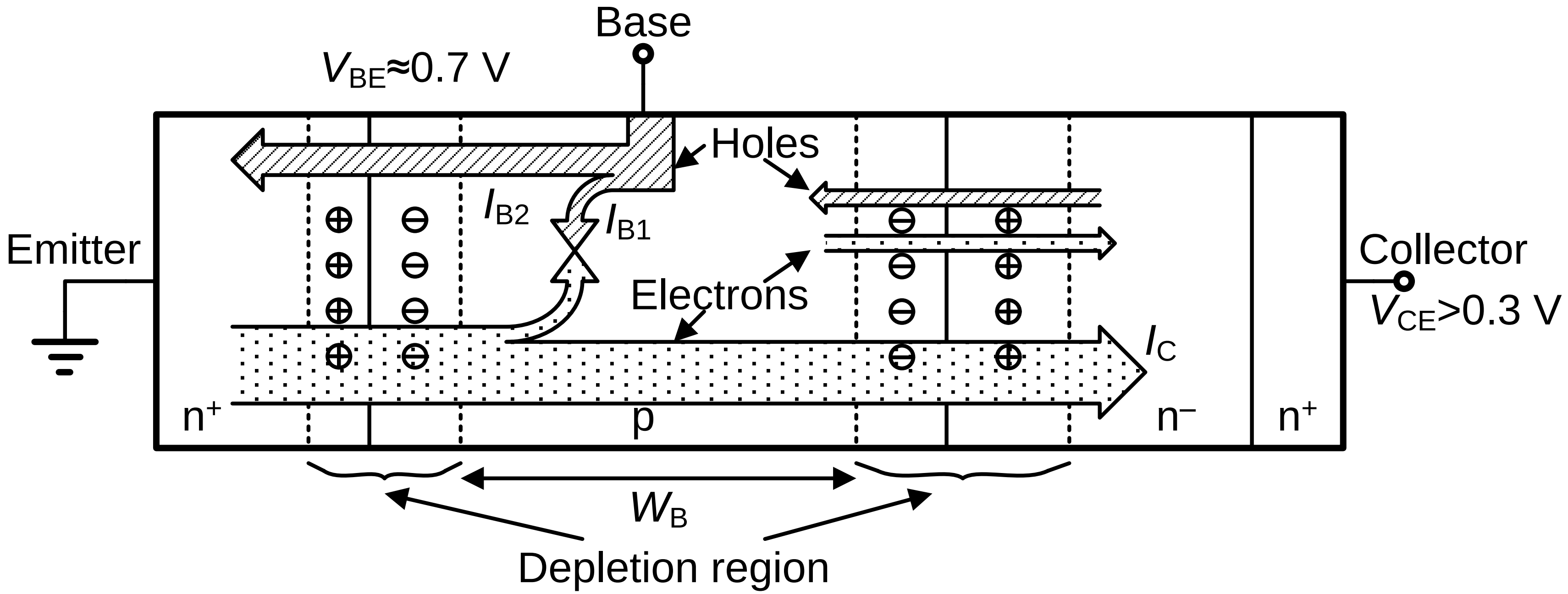
    - □击穿电压
    - □电流增益β_F与工作条件的关系
  - □小信号模型
    - □低频小信号模型
    - □高频小信号模型
    - □模拟评价指标
  - □器件模型概要

BJT型晶体管的剖面图和符号



A cross section of an npn bipolar-junction transistor.

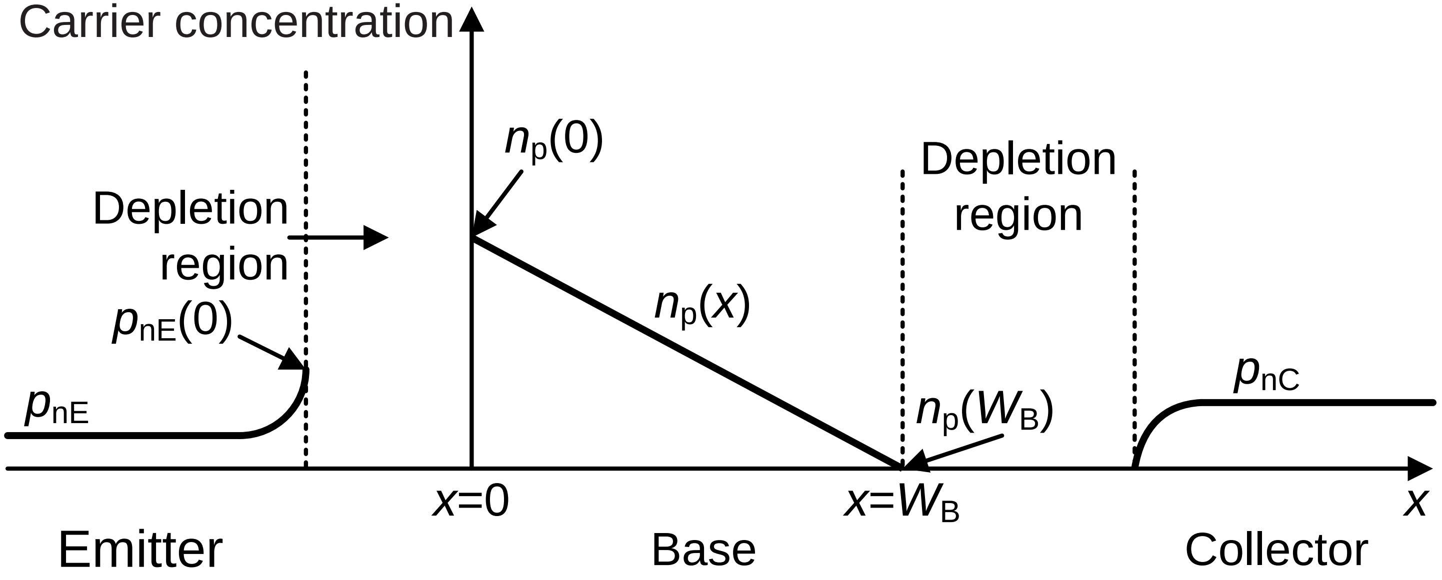
npn型BJT晶体管的电流



Various components of the currents of an npn transistor.

发射结正偏，集电结反偏

少数载流子溶度分布图



Carrier concentrations along the cross section

基区中耗尽区边缘的少数载流子浓度:

$$n_p(0) = n_{p0} e^{\frac{V_{BE}}{V_T}} \qquad n_p(W_B) = n_{p0} e^{\frac{V_{BC}}{V_T}} \approx 0$$

集电极电流

基区中电子引起的扩散电流密度:

$$J_n = qD_n \frac{dn_p(x)}{dx} = -qD_n \frac{n_p(0)}{W_B}$$

集电极电流:

$$I_C = qAD_n \frac{n_p(0)}{W_B} = \frac{qAD_n n_{p0}}{W_B} e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}}$$

其中A为发射区面积，I_S为基极-发射极结反向饱和电流

$$I_S = \frac{qAD_n n_{p0}}{W_B} = \frac{qAD_n n_i^2}{W_B N_B} = \frac{qA\overline{D_n} n_i^2}{Q_B} \qquad n_{p0} = \frac{n_i^2}{N_B}$$

$Q_B(=W_B N_B)$是基区中单位面积的掺杂原子数，

$\overline{D_n}$ 是电子平均扩散系数。

基极电流之一：复合电流

基区中少数载流子电荷为：

$$Q_\mathrm{e}=\frac{1}{2}n_\mathrm{p}(0)W_\mathrm{B}qA$$

基极复合电流为：

$$I_\mathrm{B1}=\frac{Q_\mathrm{e}}{\tau_\mathrm{b}}=\frac{1}{2}\frac{n_\mathrm{p}(0)W_\mathrm{B}qA}{\tau_\mathrm{b}}=\frac{1}{2}\frac{n_\mathrm{p0}W_\mathrm{B}qA}{\tau_\mathrm{b}}\mathrm{e}^{\frac{V_\mathrm{BE}}{V_\mathrm{T}}}$$

其中τ_b是基区中少数载流子的存活时间。

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基极电流之二：发射区的空穴电流

由基区注入发射区的空穴所引起扩散电流的：

$$I_{\mathrm{B2}}=\frac{\mathrm{q}AD_{\mathrm{p}}}{L_{\mathrm{p}}}p_{\mathrm{nE}}(0)=\frac{\mathrm{q}AD_{\mathrm{p}}}{L_{\mathrm{p}}}p_{\mathrm{nE0}}\mathrm{e}^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}}=\frac{\mathrm{q}AD_{\mathrm{p}}}{L_{\mathrm{p}}}\frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{E}}}\mathrm{e}^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}}$$

其中L_{p}是空穴在发射区的扩散长度，D_{p}是空穴的扩散系数。$p_{\mathrm{nE}}(0)$是发射区耗尽层边缘的空穴浓度。

基极总电流:

$$I_{\mathrm{B}}=I_{\mathrm{B1}}+I_{\mathrm{B2}}=\left(\frac{1}{2}\frac{n_{\mathrm{p0}}W_{\mathrm{B}}\mathrm{q}A}{\tau_{\mathrm{b}}}+\frac{\mathrm{q}AD_{\mathrm{p}}}{L_{\mathrm{p}}}\frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{E}}}\right)\mathrm{e}^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}}$$

共射极电流增益

正向放大区的电流增益：典型值为50~500

$$\beta_\mathrm{F}=\frac{I_\mathrm{C}}{I_\mathrm{B}}=\frac{\dfrac{qAD_\mathrm{n}n_\mathrm{p0}}{W_\mathrm{B}}}{\dfrac{1}{2}\dfrac{n_\mathrm{p0}W_\mathrm{B}qA}{\tau_\mathrm{b}}+\dfrac{qAD_\mathrm{p}}{L_\mathrm{p}}\dfrac{n_\mathrm{i}^2}{N_\mathrm{E}}}=\frac{1}{\dfrac{W_\mathrm{B}^2}{2\tau_\mathrm{b}D_\mathrm{n}}+\dfrac{D_\mathrm{p}}{D_\mathrm{n}}\dfrac{W_\mathrm{B}}{L_\mathrm{p}}\dfrac{N_\mathrm{B}}{N_\mathrm{E}}}$$

发射极与基极和集电极电流关系：

$$I_\mathrm{E}=-(I_\mathrm{C}+I_\mathrm{B})=-\left(I_\mathrm{C}+\frac{I_\mathrm{C}}{\beta_\mathrm{F}}\right)=-\frac{I_\mathrm{C}}{\alpha_\mathrm{F}}$$

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共基极电流增益

$$\alpha_F = -\frac{I_C}{I_E} = \frac{\beta_F}{1+\beta_F} = \frac{1}{1+\dfrac{1}{\beta_F}} = \frac{1}{1+\dfrac{W_B^2}{2\tau_b D_n}+\dfrac{D_p}{D_n}\dfrac{W_B}{L_p}\dfrac{N_B}{N_E}} \approx \alpha_T \gamma$$

基区输运系数： $$\alpha_T = \frac{1}{1+\dfrac{W_B^2}{2\tau_b D_n}}$$

发射极注入效率： $$\gamma = \frac{1}{1+\dfrac{D_p}{D_n}\dfrac{W_B}{L_p}\dfrac{N_B}{N_E}}$$

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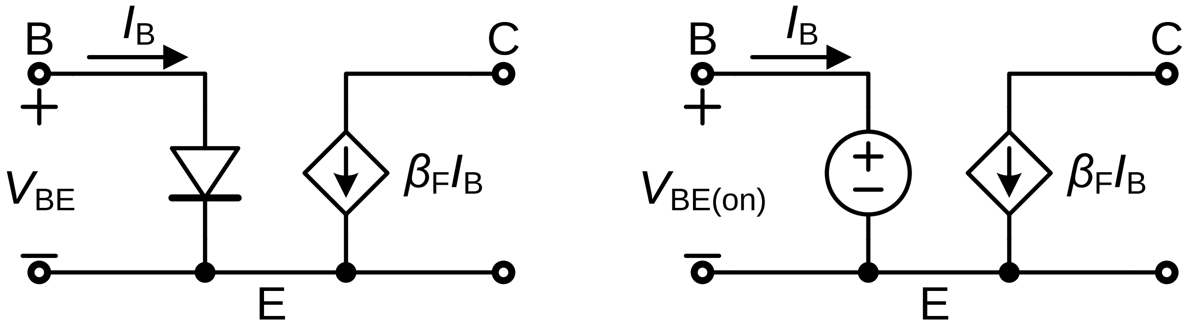
大信号模型

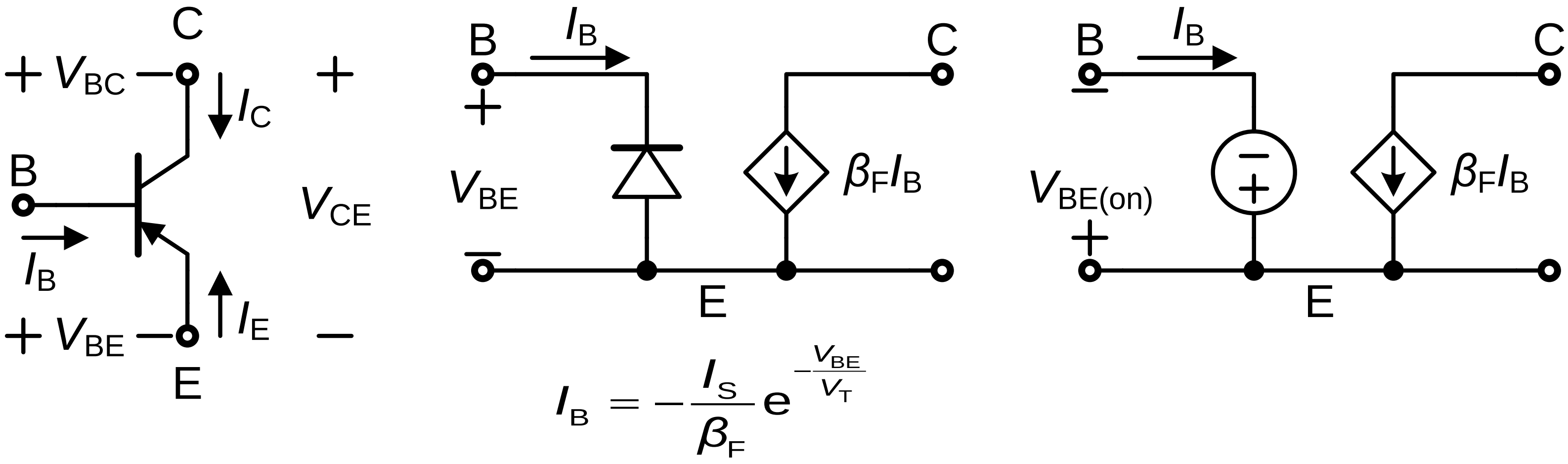


$$I_B = \frac{I_S}{\beta_F} e^{\frac{V_{BE}}{V_T}}$$

(a) (b)

Large-signal models of npn transistors for use in bias calculations.
(a) Circuit incorporating an input diode.
(b) Simplified circuit with an input voltage source.

pnp型BJT晶体管的大信号模型



$$I_B = -\frac{I_S}{\beta_F} e^{-\frac{V_{BE}}{V_T}}$$

(a) (b) (c)

(a) pnp Bipolar transistor sign convention. (b),(c) Large-signal models of pnp transistors corresponding to npn transistors

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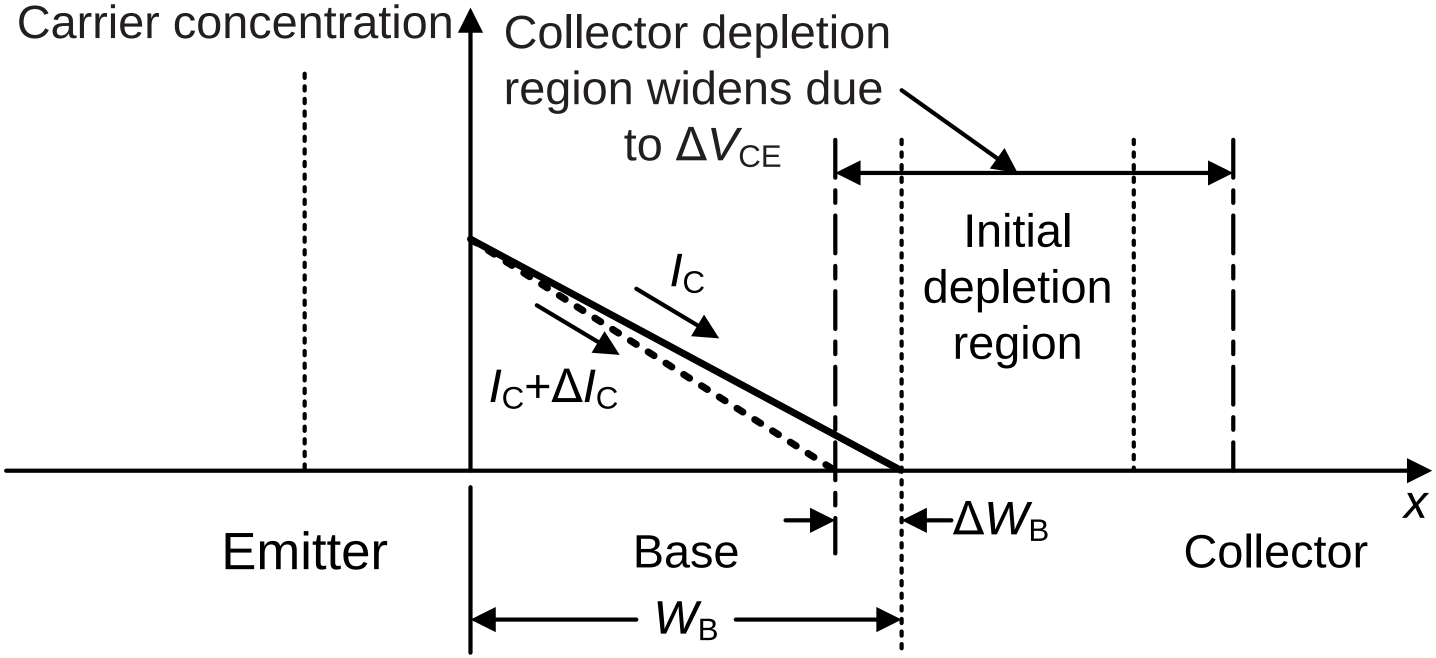
厄利效应：(1)



Effect of increases in V_{CE} on the collector depletion region and base width of a bipolar transistor.

集电极电流:

$$I_C = \frac{qA\overline{D}_n n_i^2}{Q_B} e^{\frac{V_{BE}}{V_T}}$$

厄利效应：(2)

$$\frac{\partial I_\mathrm{C}}{\partial V_\mathrm{CE}}=-\frac{qA\overline{D_\mathrm{n}}n_\mathrm{i}^2}{Q_\mathrm{B}^2}\mathrm{e}^{\frac{V_\mathrm{BE}}{V_\mathrm{T}}}\frac{\mathrm{d}Q_\mathrm{B}}{\mathrm{d}V_\mathrm{CE}}=-\frac{I_\mathrm{C}}{Q_\mathrm{B}}\frac{\mathrm{d}Q_\mathrm{B}}{\mathrm{d}V_\mathrm{CE}}$$

对于一个均匀掺杂基区晶体管$Q_\mathrm{B}=W_\mathrm{B}N_\mathrm{B}$，且$V_\mathrm{BE}$固定时

$$\frac{\partial I_\mathrm{C}}{\partial V_\mathrm{CE}}=-\frac{I_\mathrm{C}}{W_\mathrm{B}}\frac{\mathrm{d}W_\mathrm{B}}{\mathrm{d}V_\mathrm{CE}}$$

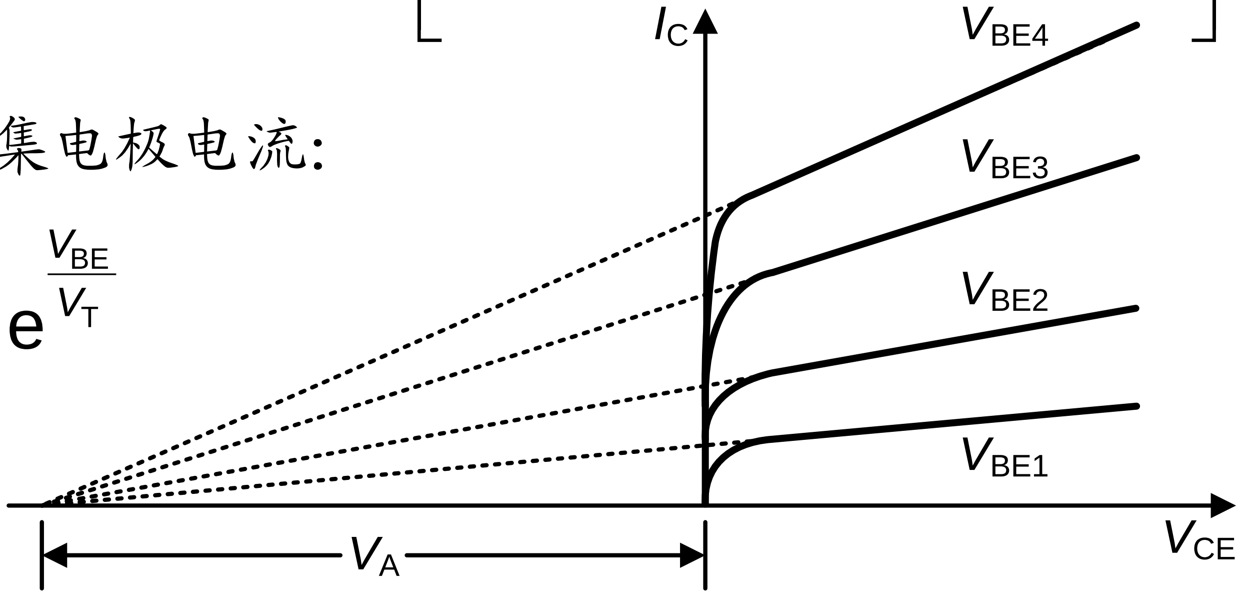
$$\left|\frac{\mathrm{d}W_\mathrm{B}}{\mathrm{d}V_\mathrm{CE}}\right|=\left|\frac{\mathrm{d}W_\mathrm{B}}{\mathrm{d}V_\mathrm{CB}}\right|=\left[\frac{\varepsilon_\mathrm{si}}{2qN_\mathrm{B}\left(1+\frac{N_\mathrm{B}}{N_\mathrm{C}}\right)(\Phi_0+V_\mathrm{CB})}\right]^{1/2}$$

厄利电压

$$V_A = \frac{I_C}{\dfrac{\partial I_C}{\partial V_{CE}}} = -W_B \frac{\mathrm{d}V_{CE}}{\mathrm{d}W_B} = -W_B \left[\frac{2qN_B\left(1+\dfrac{N_B}{N_C}\right)(\Phi_0 + V_{CB})}{\varepsilon_{si}} \right]^{1/2}$$

考虑厄利效应的集电极电流:

$$I_C = I_S\left(1+\frac{V_{CE}}{V_A}\right)e^{\frac{V_{BE}}{V_T}}$$



Bipolar transistor output characteristics showing the Early voltage, V_A

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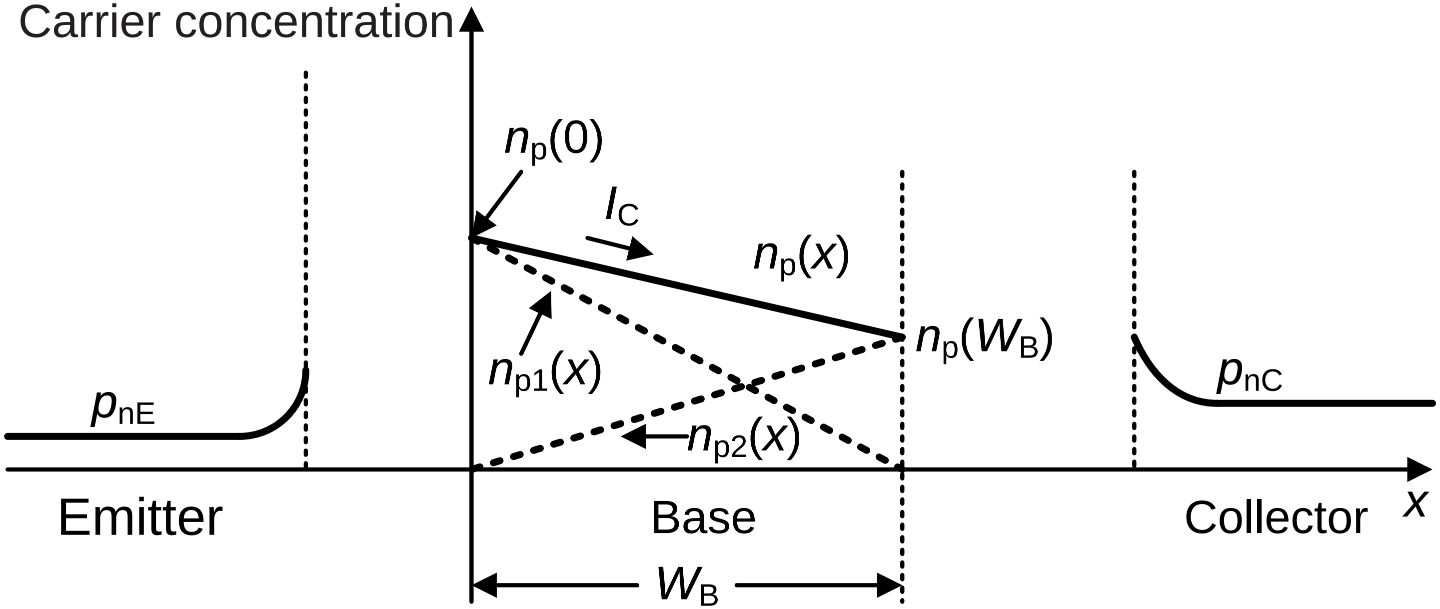
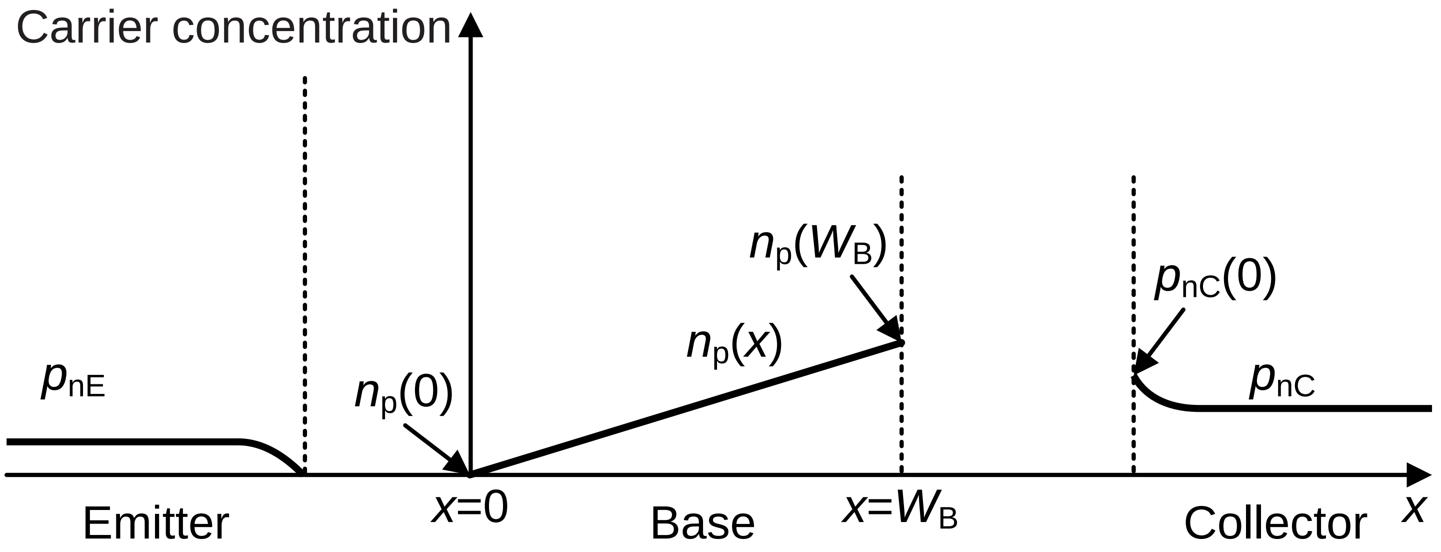
饱和区



Carrier concentrations in a saturated npn transistor. (Not to scale.)

发射结正偏，集电结正偏

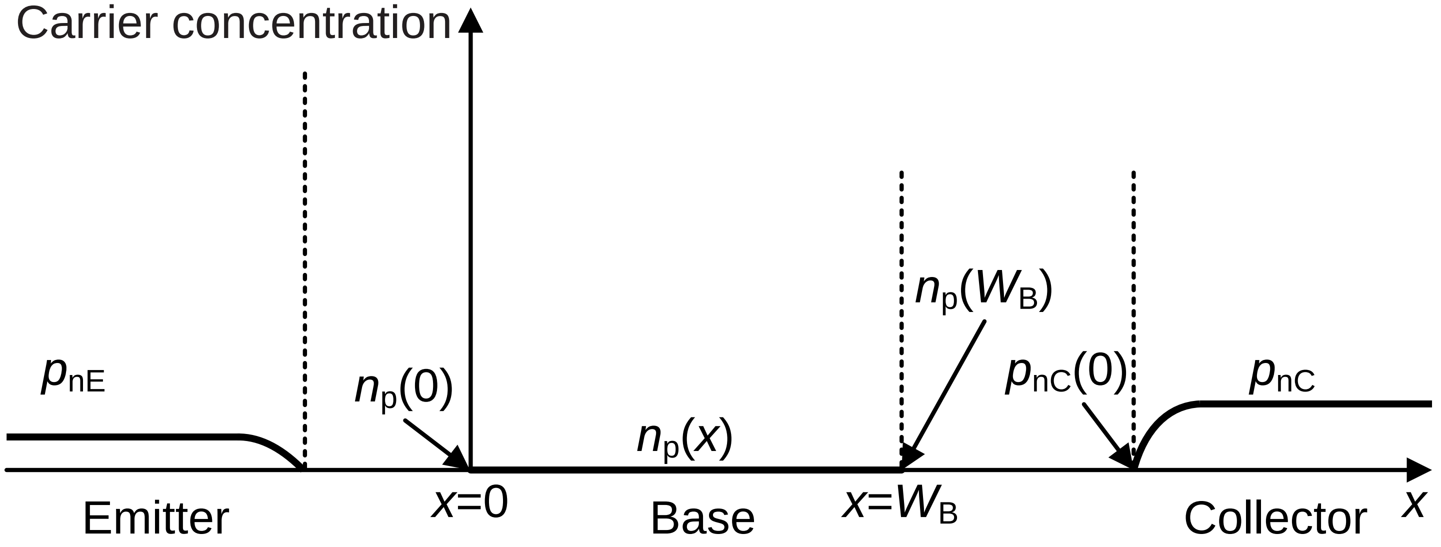
反向放大区



Carrier concentrations in a reversed npn transistor. (Not to scale.)

发射结反偏，集电结正偏

截止区



Carrier concentrations in a cutoff npn transistor. (Not to scale.)

发射结反偏，集电结反偏

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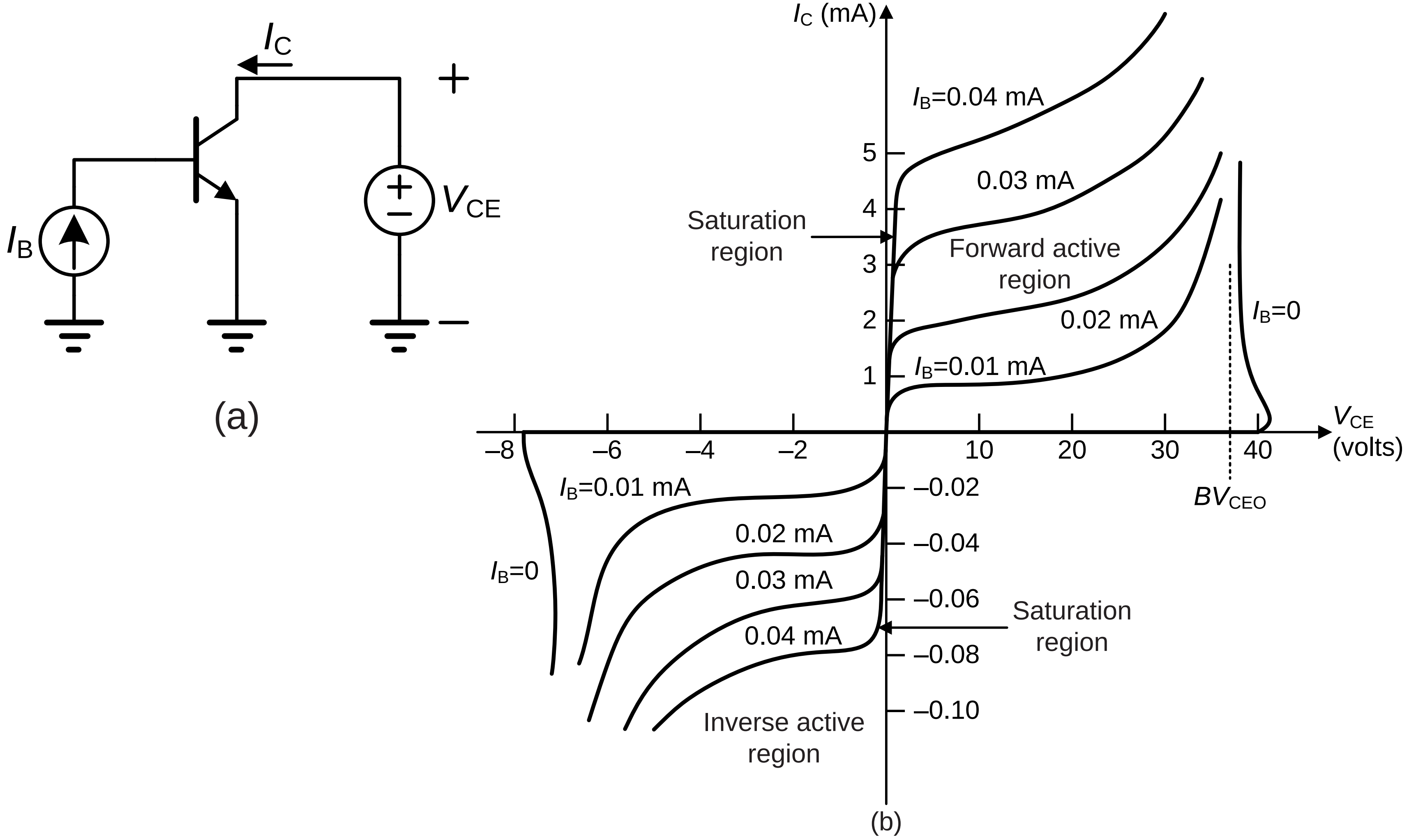
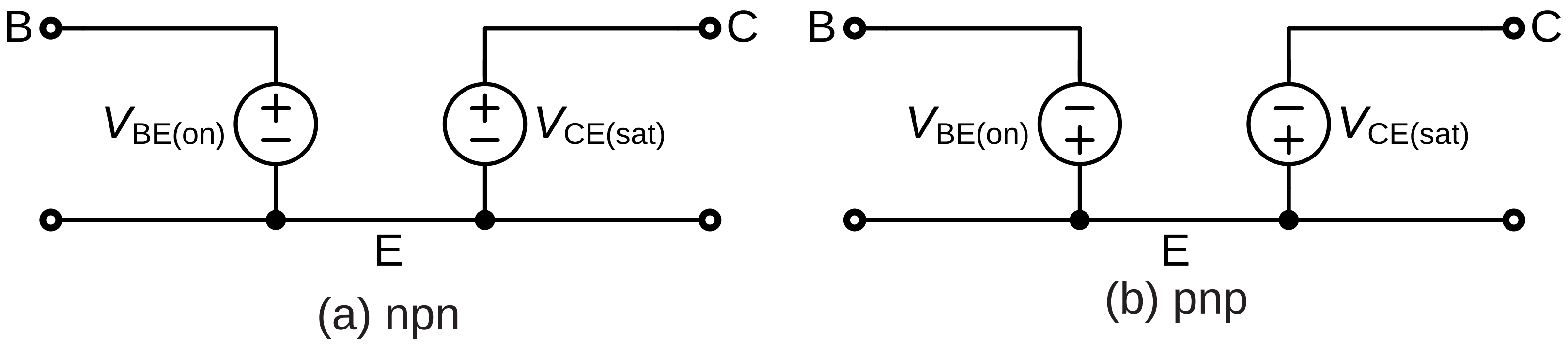
典型I_C-V_{CE}特性曲线图



(a) Test circuit. (b) Typical I_C-V_{CE} characteristics for an npn bipolar transistor. Note the different scales for positive and negative currents and voltages.

饱和区大信号模型



Large-signal models for bipolar transistors in the saturation region.

Ebers-Moll模型：(1)

基区中集电极耗尽区边缘的少数载流子浓度:

$$n_p(W_B) = n_{p0}\,e^{\frac{V_{BC}}{V_T}}$$

基极-集电极结正偏电流:

$$I_{CR} = -I_{CS}(e^{\frac{V_{BC}}{V_T}} - 1)$$

基极-发射极结正偏电流:

$$I_{EF} = -I_{ES}(e^{\frac{V_{BE}}{V_T}} - 1)$$

Ebers-Moll模型：正向/反向放大共基极电流增益α_F,α_R

$$\begin{cases} I_C = \alpha_F I_{ES}(e^{\frac{V_{BE}}{V_T}} - 1) - I_{CS}(e^{\frac{V_{BC}}{V_T}} - 1) \\ I_E = -I_{ES}(e^{\frac{V_{BE}}{V_T}} - 1) + \alpha_R I_{CS}(e^{\frac{V_{BC}}{V_T}} - 1) \end{cases}$$

Ebers-Moll模型：(2)

在正向放大区，V_{BE}为正，V_{BC}为负，

$$\begin{cases} I_{\mathrm{C}} = \alpha_{\mathrm{F}} I_{\mathrm{ES}}\left(e^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}} - 1\right) + I_{\mathrm{CS}} \\ I_{\mathrm{E}} = -I_{\mathrm{ES}}\left(e^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}} - 1\right) - \alpha_{\mathrm{R}} I_{\mathrm{CS}} \end{cases}$$

正向/反向放大区共射极电流增益：$\beta_{\mathrm{F}} = \dfrac{\alpha_{\mathrm{F}}}{1-\alpha_{\mathrm{F}}}$ $\beta_{\mathrm{R}} = \dfrac{\alpha_{\mathrm{R}}}{1-\alpha_{\mathrm{R}}}$

集电极电流：$I_{\mathrm{C}} = \alpha_{\mathrm{F}} I_{\mathrm{ES}}\left(e^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}} - 1\right) + I_{\mathrm{CS}} = \alpha_{\mathrm{F}}(-I_{\mathrm{E}} - \alpha_{\mathrm{R}} I_{\mathrm{CS}}) + I_{\mathrm{CS}}$

$$= -\alpha_{\mathrm{F}} I_{\mathrm{E}} + I_{\mathrm{CS}}(1-\alpha_{\mathrm{R}}\alpha_{\mathrm{F}}) = -\alpha_{\mathrm{F}} I_{\mathrm{E}} + I_{\mathrm{CO}}$$

基极电流：

$$I_{\mathrm{CO}} = I_{\mathrm{CS}}(1-\alpha_{\mathrm{R}}\alpha_{\mathrm{F}})$$

$$I_{\mathrm{B}} = -(I_{\mathrm{C}} + I_{\mathrm{E}}) = \frac{1-\alpha_{\mathrm{F}}}{\alpha_{\mathrm{F}}} I_{\mathrm{C}} - \frac{I_{\mathrm{CO}}}{\alpha_{\mathrm{F}}} = \frac{I_{\mathrm{C}}}{\beta_{\mathrm{F}}} - \frac{I_{\mathrm{CO}}}{\alpha_{\mathrm{F}}}$$

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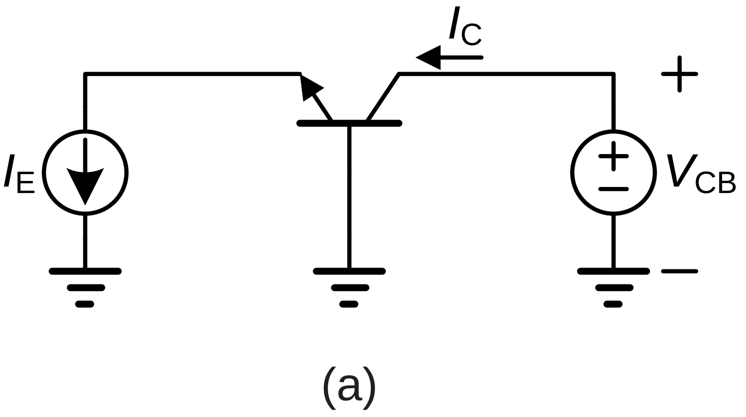
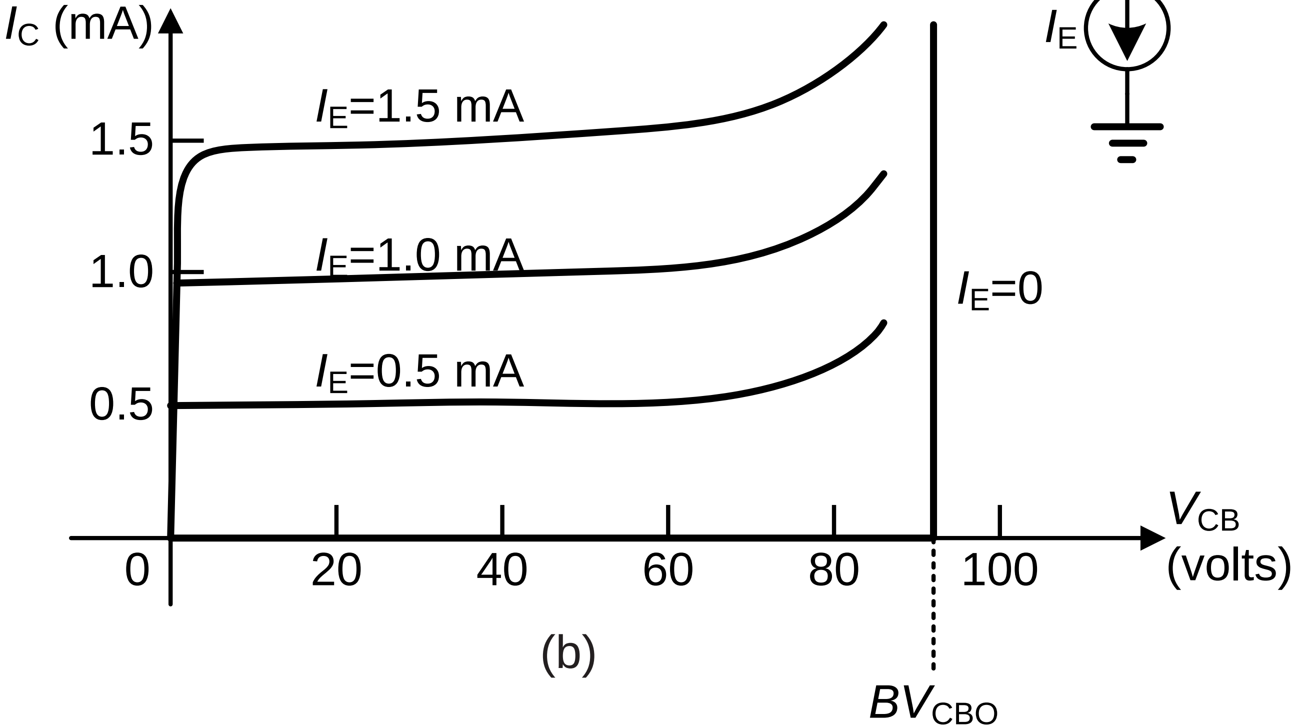
共基击穿电压





$$I_C = -\alpha_F I_E M$$

$$M = \frac{1}{1-\left(\frac{V_{CB}}{BV_{CBO}}\right)^n}$$

Common-base transistor connection. (a) Test circuit. (b) $I_C - V_{CB}$ characteristics.

共射与共基击穿电压的关系

$$I_{\mathrm{B}}=-(I_{\mathrm{C}}+I_{\mathrm{E}}) \qquad I_{\mathrm{C}}=\frac{M\alpha_{\mathrm{F}}}{1-M\alpha_{\mathrm{F}}}I_{\mathrm{B}}$$

令$M\alpha_{\mathrm{F}}=1$，且$V_{\mathrm{CB}}\approx V_{\mathrm{CE}}$，

$$\frac{\alpha_{\mathrm{F}}}{1-\left(\frac{BV_{\mathrm{CEO}}}{BV_{\mathrm{CBO}}}\right)^{n}}=1$$

$$\frac{BV_{\mathrm{CEO}}}{BV_{\mathrm{CBO}}}=\sqrt[n]{1-\alpha_{\mathrm{F}}}=\sqrt[n]{\frac{\alpha_{\mathrm{F}}}{\beta_{\mathrm{F}}}}\approx\frac{1}{\sqrt[n]{\beta_{\mathrm{F}}}}$$

$$BV_{\mathrm{CEO}}\approx\frac{BV_{\mathrm{CBO}}}{\sqrt[n]{\beta_{\mathrm{F}}}}$$

β_{F}和n典型为100和4

例题4:

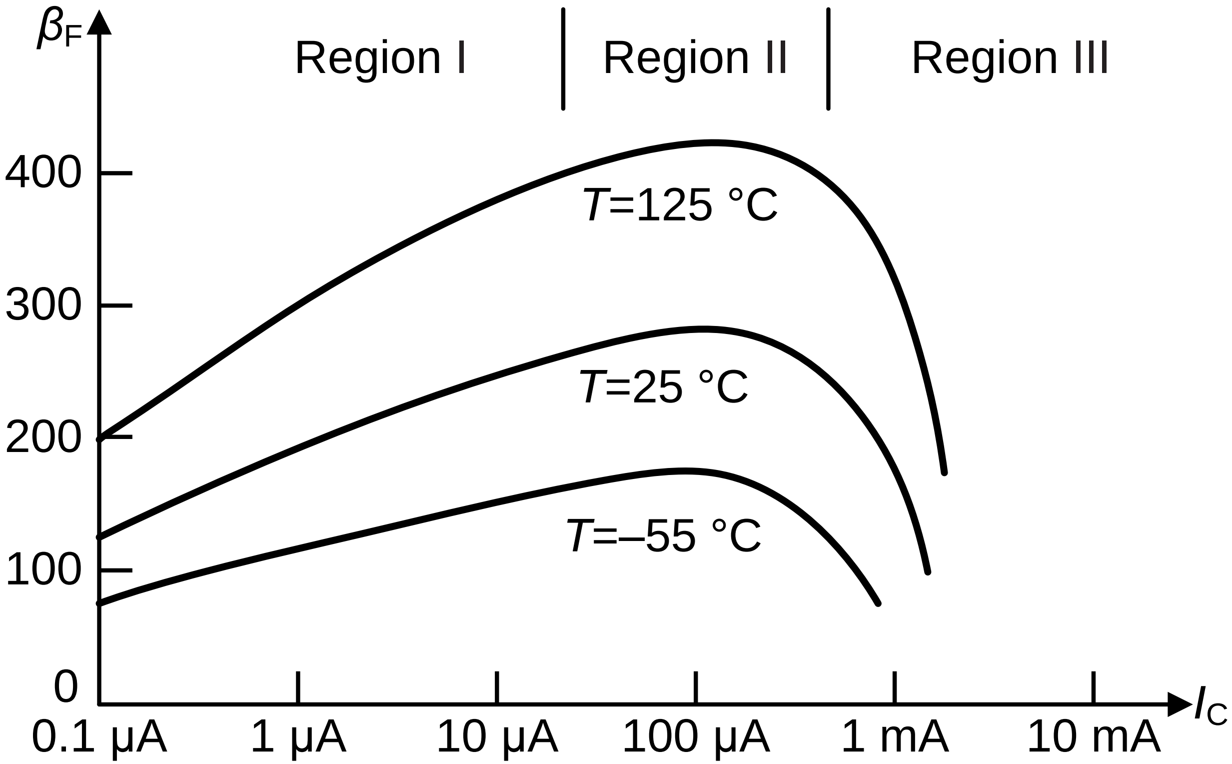
BJT晶体管的集电极掺杂浓度远小于基极浓度，当集电极浓度为2×10^{15}原子/cm^3，临界电场$E_{\mathrm{crit}}=3\times10^5$ V/cm，$\beta=100$和$n=4$，计算击穿电压BV_{CEO}。

解：因为$N_{\mathrm{B}}>>N_{\mathrm{C}}$，

$$BV_{\mathrm{CBO}}=\frac{\varepsilon_{\mathrm{si}}E_{\mathrm{crit}}^2}{2qN_{\mathrm{C}}}=\frac{1.04\times10^{-12}\times9\times10^{10}}{2\times1.6\times10^{-19}\times2\times10^{15}}=146\ \mathrm{V}$$

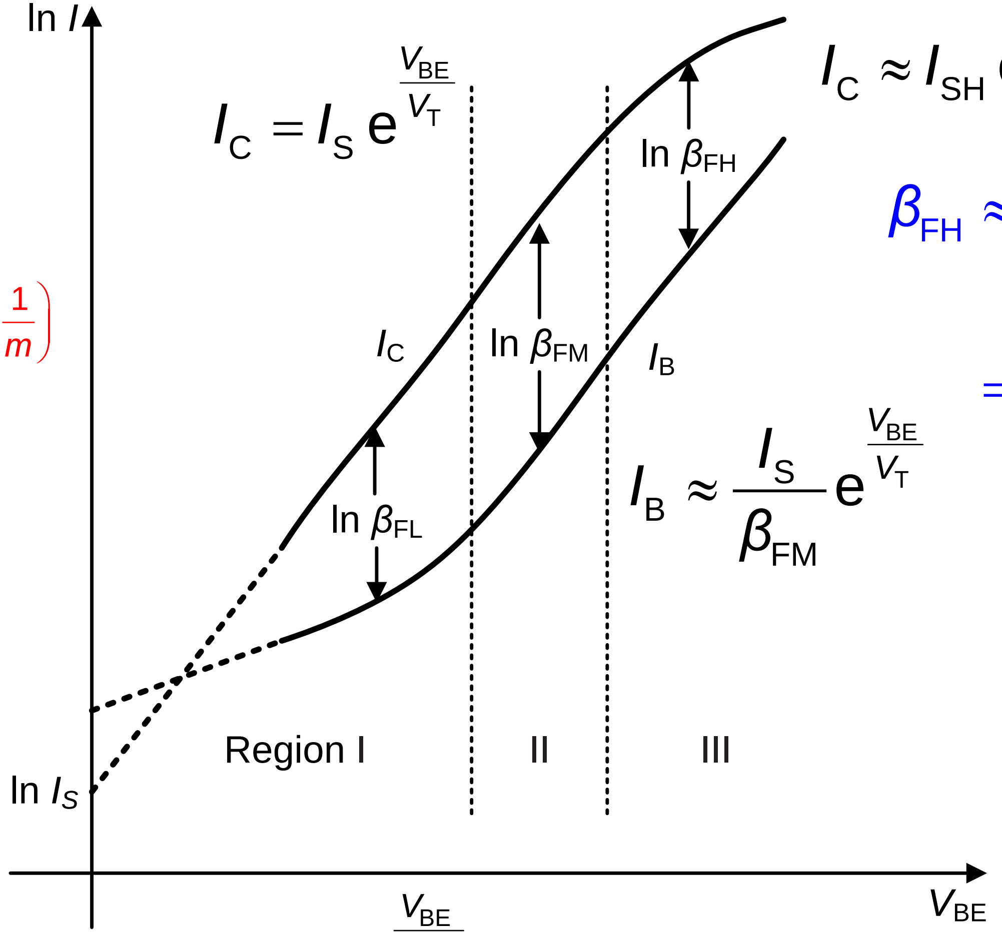
$$BV_{\mathrm{CEO}}\approx\frac{BV_{\mathrm{CBO}}}{\sqrt[n]{\beta_{\mathrm{F}}}}=\frac{146}{\sqrt[4]{100}}=46\ \mathrm{V}$$

电流增益β_F与工作条件的关系



Typical curves of β_F versus I_C for an npn integrated-circuit transistor with 6 μm^2 emitter area.

ln*I*-V_{BE}关系



$$I_C = I_S\, e^{\frac{V_{BE}}{V_T}}$$

$$I_C \approx I_{SH}\, e^{\frac{V_{BE}}{2V_T}}$$

$$I_B \approx \frac{I_S}{\beta_{FM}}\, e^{\frac{V_{BE}}{V_T}}$$

$$I_{BX} \approx I_{SX}\, e^{\frac{V_{BE}}{mV_T}}$$

$$\beta_{FL} \approx \frac{I_C}{I_{BX}} = \frac{I_S}{I_{SX}}\, e^{\frac{V_{BE}}{V_T}\left(1-\frac{1}{m}\right)}$$

$$= \frac{I_S}{I_{SX}}\left(\frac{I_C}{I_S}\right)^{[1-1/m]}$$

$$\beta_{FH} \approx \frac{I_{SH}}{I_S}\,\beta_{FM}\, e^{-\frac{V_{BE}}{2V_T}}$$

$$= \frac{I_{SH}^2}{I_S}\,\beta_{FM}\,\frac{1}{I_C}$$

$m=2$

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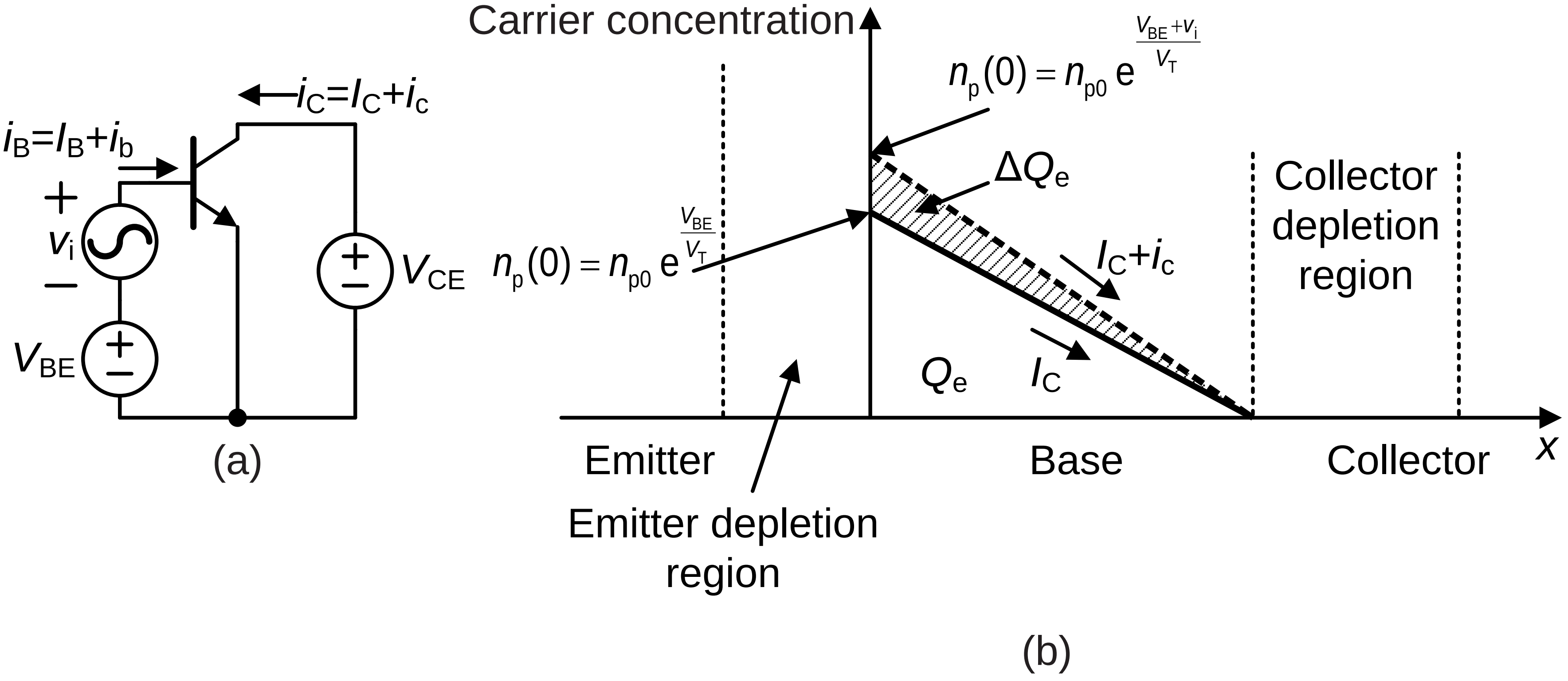
小信号模型



Effect of a small-signal input voltage applied to a bipolar transistor. (a) Circuit schematic. (b) Corresponding changes in carrier concentrations in the base when the device is in the forward-active region.

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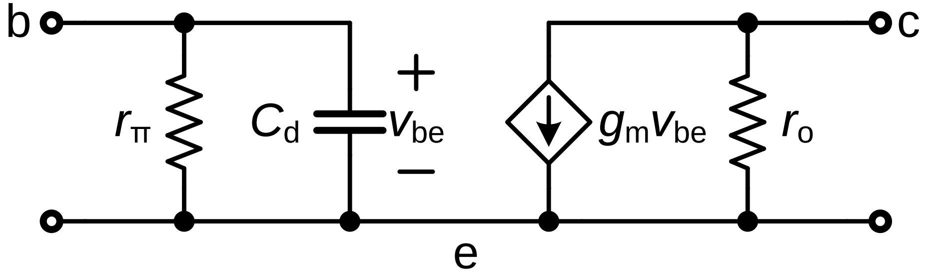
低频小信号模型：(1)



Basic bipolar transistor small-signal equivalent circuit.

跨导：
$$g_m = \frac{i_c}{v_{be}} = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_{CS}}{V_T} e^{\frac{V_{BE}}{V_T}} = \frac{I_C}{V_T} \qquad V_T = \frac{kT}{q}$$

小信号输入电阻:
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)^{-1} = \left(\frac{\partial (I_{CS} e^{V_{BE}/V_T} / \beta)}{\partial V_{BE}}\right)^{-1}$$

$$= \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$

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低频小信号模型：(2)

基区中少数载流子存储电荷:

$$Q_\mathrm{e}=\frac{1}{2}n_\mathrm{p}(0)W_\mathrm{B}qA=\frac{qAn_\mathrm{i}^2W_\mathrm{B}}{2N_\mathrm{B}}\mathrm{e}^{\frac{V_\mathrm{BE}}{V_\mathrm{T}}}\approx\frac{W_\mathrm{B}^2}{2D_\mathrm{n}}I_\mathrm{C}=\tau_\mathrm{b}I_\mathrm{C}$$

其中τ_b是基区渡越时间:

$$\tau_\mathrm{b}=\frac{W_\mathrm{B}^2}{2D_\mathrm{n}}$$

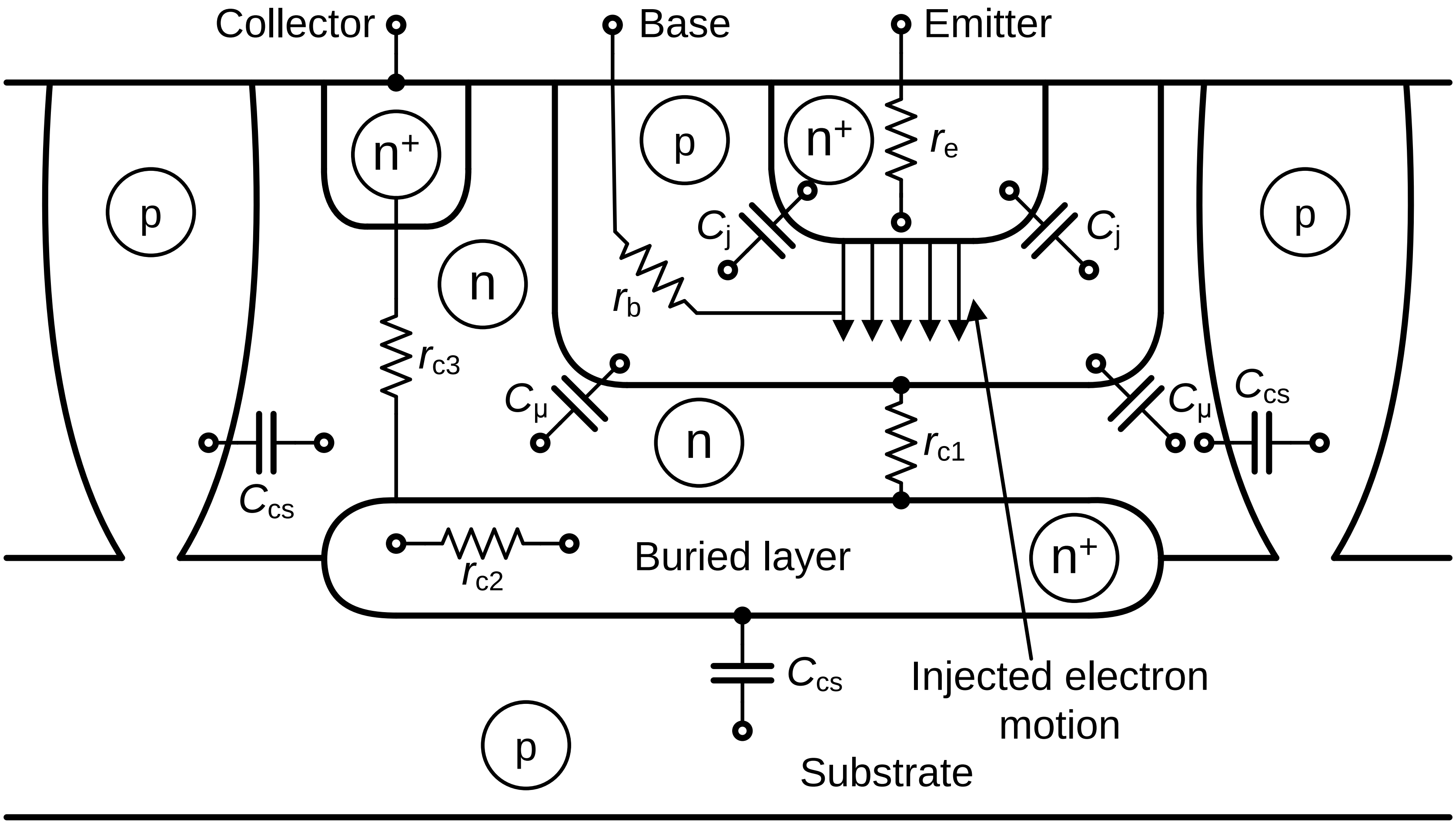
基区扩散电容:

$$C_\mathrm{d}\approx\frac{\mathrm{d}Q_\mathrm{e}}{\mathrm{d}V_\mathrm{BE}}=\frac{\mathrm{d}(\tau_\mathrm{b}I_\mathrm{C})}{\mathrm{d}V_\mathrm{BE}}\approx\tau_\mathrm{b}\frac{I_\mathrm{C}}{V_\mathrm{T}}=g_\mathrm{m}\tau_\mathrm{b}$$

输出阻抗:

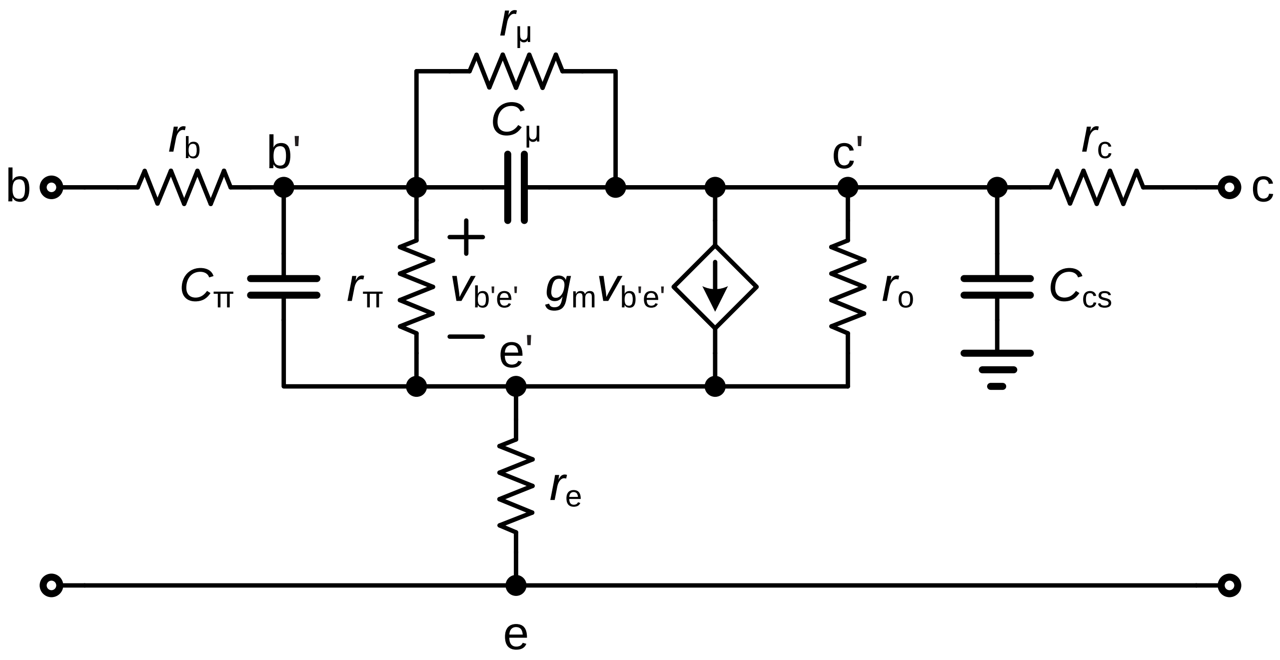
$$r_\mathrm{o}=\frac{\partial V_\mathrm{CE}}{\partial I_\mathrm{C}}=\left(\frac{\partial I_\mathrm{C}}{\partial V_\mathrm{CE}}\right)^{-1}=\left(\frac{\partial\left(I_\mathrm{CS}\mathrm{e}^{V_\mathrm{BE}/V_\mathrm{T}}\left(1+\frac{V_\mathrm{CE}}{V_\mathrm{A}}\right)\right)}{\partial V_\mathrm{CE}}\right)^{-1}\approx\frac{V_\mathrm{A}}{I_\mathrm{C}}$$

BJT晶体管的寄生参数



Integrated-circuit npn bipolar transistor structure showing parasitic elements.
(Not to scale.)

高频小信号模型：(1)



Complete bipolar transistor small-signal equivalent circuit.

集基电阻r_μ: 假设V_{BE}恒定，基极电流I_B=10I_{B1}

$$r_\mu = \frac{\partial V_{CE}}{\partial I_{B1}} = \frac{\partial V_{CE}}{\partial I_C}\frac{\partial I_C}{\partial I_{B1}} = 10 r_o \frac{\partial I_C}{\partial I_B} = 10\beta_0 r_o$$

高频小信号模型：(2)

正向偏置的基极-发射极结电容:

$$C_{\pi} = C_{j} + C_{d}$$

其中C_j是基极-发射极结的耗尽电容

$$C_{j} = \frac{C_{jbe0}}{\left(1 - \frac{V_{BE}}{\Phi_{0BE}}\right)^{1/2}}$$

基极-集电极结的耗尽电容: 缓变结

集电极-衬底结的耗尽电容:

$$C_{\mu} = \frac{C_{jbc0}}{\left(1 + \frac{V_{CB}}{\Phi_{0BC}}\right)^{1/3}} \qquad C_{cs} = \frac{C_{jsc0}}{\left(1 + \frac{V_{CS}}{\Phi_{0SC}}\right)^{1/3}}$$

寄生电阻: r_b， r_e和r_c

例题5:

计算双极型晶体管小信号参数。假设I_C=1 mA, V_{CB}=3 V, V_{CS}=5 V, C_{jbe0}=10 fF, n_{be}=0.5, Φ_{0BE}=0.9 V, C_{jbc0}=10 fF, n_{bc}=0.3, Φ_{0BC}=0.5 V, C_{jsc0}=20 fF, n_{sc}=0.3, Φ_{0SC}=0.65 V, β_0=100, τ_b=10 ps, V_A=20 V, r_b=300 Ω, r_c=50 Ω, r_e=5 Ω, $r_\mu=10\beta_0 r_0$。

解:

$$g_m = \frac{I_C}{V_T} = \frac{1\,\text{mA}}{26\text{ mV}} = 38\text{ mA/V} \qquad r_\pi = \frac{\beta}{g_m} = \frac{100}{38\text{ mA/V}} = 2.6\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20\text{ V}}{1\text{ mA}} = 20\text{ k}\Omega \qquad r_\mu = 10\beta_0 r_o = 10 \times 100 \times 20\text{ k}\Omega = 20\text{ M}\Omega$$

$$C_d = g_m \tau_b = 38\text{ mA/V} \times 10\text{ ps} = 0.38\text{ pF}$$

基极-发射极结正偏，耗尽电容很难计算，估算C_j=20 fF。

$$C_\pi = C_j + C_d = 0.4\text{ pF} \qquad C_\mu = 5.6\text{ fF} \qquad C_{cs} = 10.5\text{ fF}$$

模拟评价指标

固有增益：

$$A_\mathrm{i} = g_\mathrm{m} r_\mathrm{o} = \frac{I_\mathrm{C}}{V_\mathrm{T}} \frac{V_\mathrm{A}}{I_\mathrm{C}} = \frac{V_\mathrm{A}}{V_\mathrm{T}}$$

跨导电流比：

$$\frac{g_\mathrm{m}}{I_\mathrm{C}} = \frac{1}{V_\mathrm{T}}$$

特征频率：

$$f_\mathrm{T} = \frac{1}{2\pi} \frac{g_\mathrm{m}}{C_\pi + C_\mu}$$

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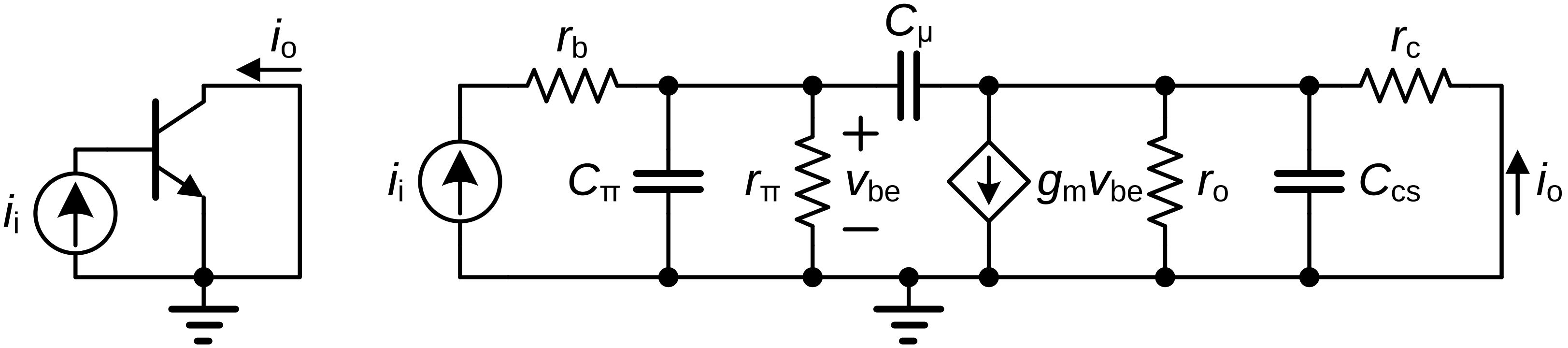
高频小信号电流增益



(a) (b)

(a)Schematic of ac circuit for measurement of f_T. (b) Small-signal equivalent circuit for the calculation of f_T.

忽略r_o、C_{cs}、r_c，忽略通过C_μ的电流，

$$v_{be} \approx \frac{r_\pi}{1+r_\pi(C_\pi+C_\mu)s} i_i \qquad i_o \approx g_m v_{be} = i_i \frac{g_m r_\pi}{1+r_\pi(C_\pi+C_\mu)s}$$

$$\beta(\mathrm{j}2\pi f) = \frac{i_o}{i_i} = \frac{\beta_0}{1+\beta_0 \dfrac{C_\pi+C_\mu}{g_m}\mathrm{j}2\pi f} \approx \frac{g_m}{\mathrm{j}2\pi f(C_\pi+C_\mu)}$$

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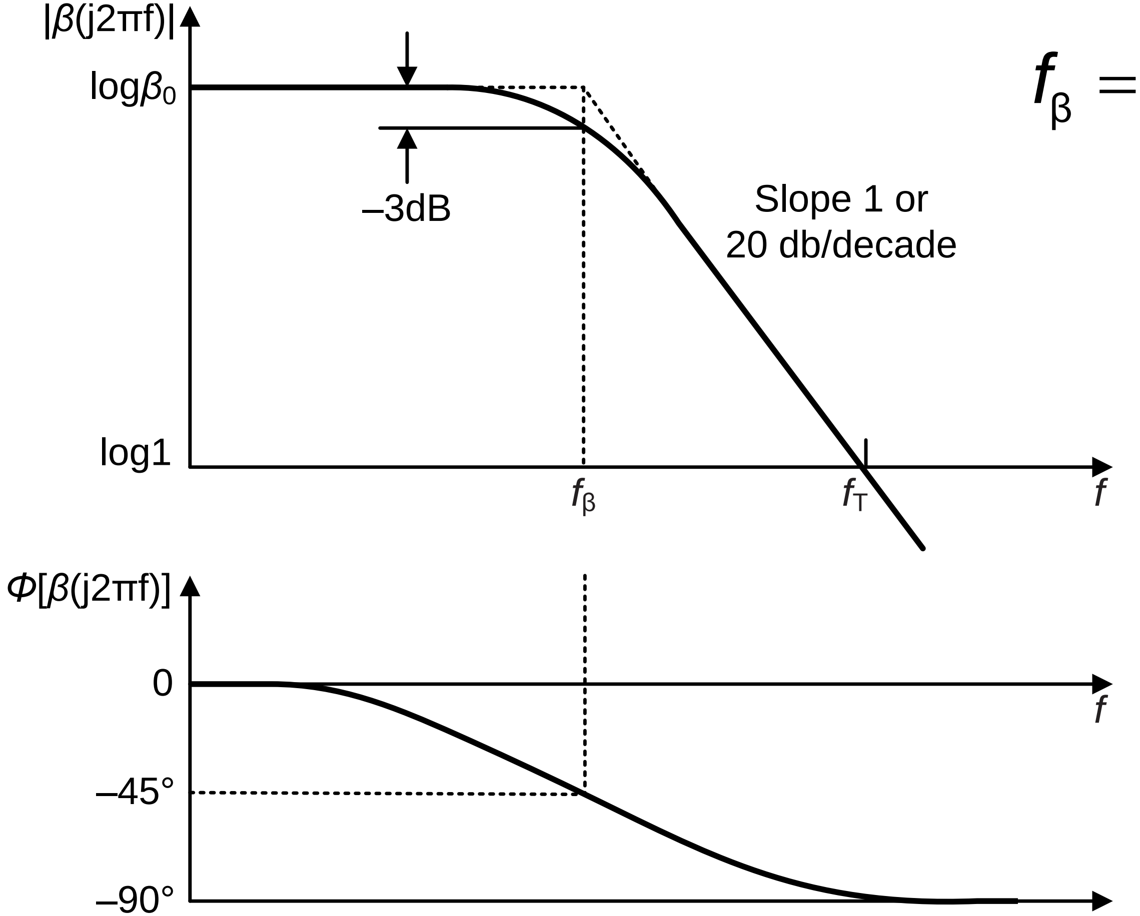
特征频率



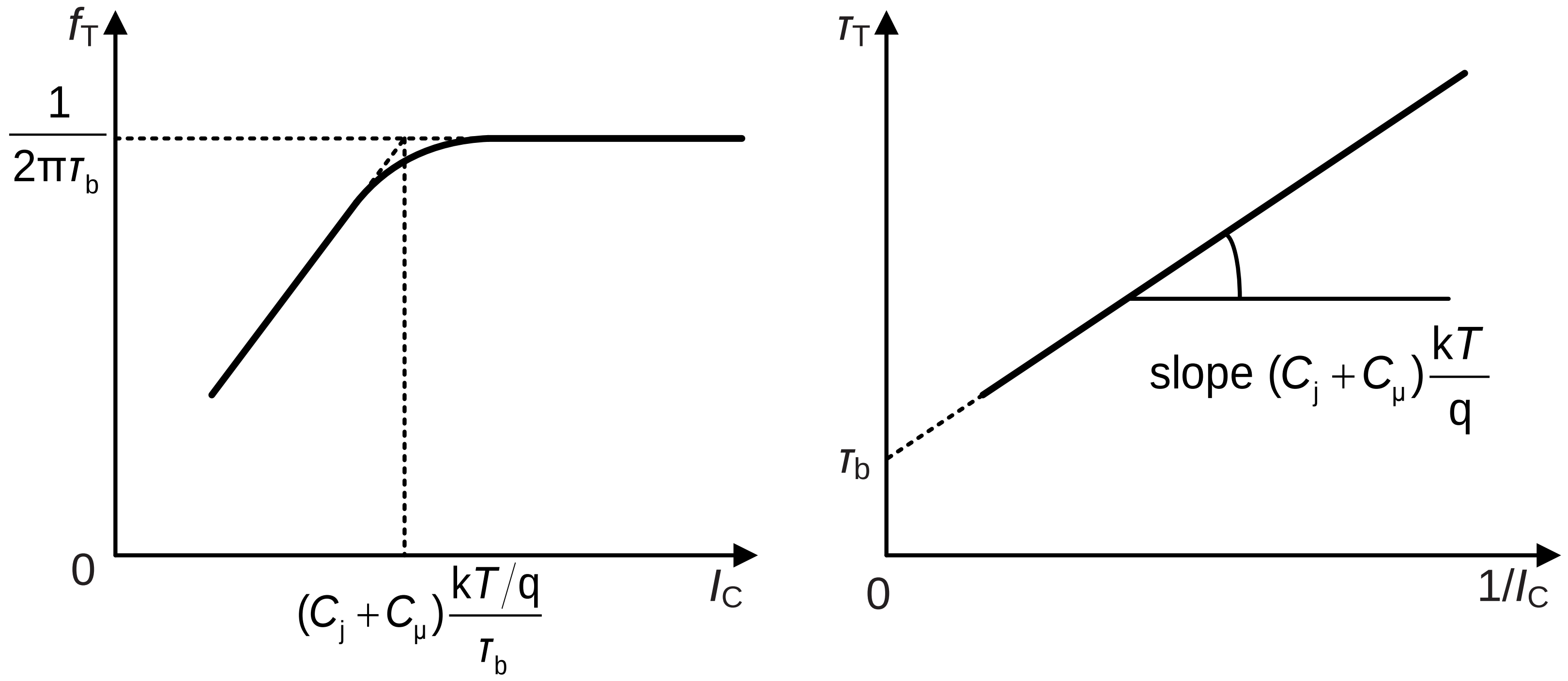
Bode diagrams of current driven CE bipolar transistor with shorted output.

$$f_\beta = \frac{1}{2\pi\beta_0}\frac{g_m}{C_\pi + C_\mu} = \frac{f_T}{\beta_0}$$

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_\pi + C_\mu}$$

$$\beta(j2\pi f) = \frac{\beta_0}{1 + jf/f_\beta}$$

f_T与I_{CE}的关系：理论上

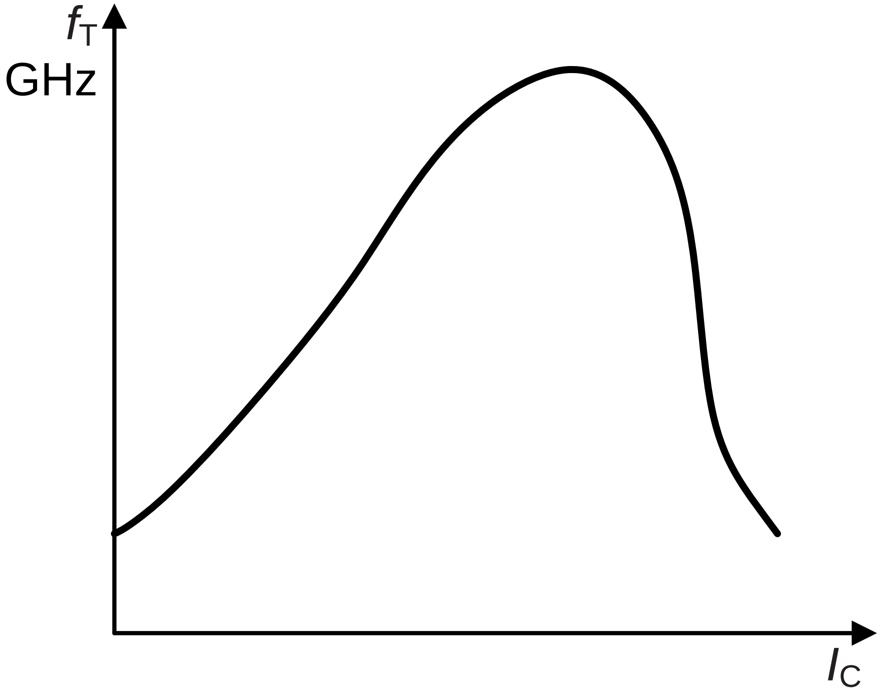


(a) (b)

Unity-gain frequency f_T versus current I_C.

$$f_T = \frac{1}{2\pi\tau_b}\frac{I_C}{I_C + I_{CfT}} \qquad I_{CfT} = (C_j + C_\mu)\frac{V_T}{\tau_b} \qquad \tau_T = \tau_b + (C_j + C_\mu)\frac{V_T}{I_C}$$

f_T与I_{CE}的关系：实际上



A sketch of transistor f_T versus collector current, I_C.

例题6:

某双极型晶体管在I_{C1}=0.25 mA和I_{C2}=1 mA两种工作情况下，在1 GHz频率点，输出短路的共射电流增益分别为8和9，电容C_μ的测试值为10 fF，假设C_j和τ_b是常数，计算它们的值。

解：在I_{C1}=0.25 mA和I_{C2}=1 mA两种工作情况下，特征频率分别为f_{T1}=8 GHz, f_{T2}= 9 GHz。

$$\begin{cases} \dfrac{1}{2\pi f_{T1}} = \tau_b + (C_j + C_\mu)\dfrac{kT/q}{I_{C1}} \\ \dfrac{1}{2\pi f_{T2}} = \tau_b + (C_j + C_\mu)\dfrac{kT/q}{I_{C2}} \end{cases}$$

$$\begin{cases} C_j = 18.2\ \text{fF} \\ \tau_b = 17\ \text{ps} \end{cases}$$

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器件模型概要：放大区直流参数

$I_{\mathrm{C}} = I_{\mathrm{S}} \mathrm{e}^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}}$	$V_{\mathrm{T}} \approx \frac{\mathrm{k}T}{\mathrm{q}} \approx 26\ \mathrm{mV\ at\ 300\ K}$
For more accuracy, $I_{\mathrm{C}} = I_{\mathrm{S}}\left(1+\frac{V_{\mathrm{CE}}}{V_{\mathrm{A}}}\right)\mathrm{e}^{\frac{V_{\mathrm{BE}}}{V_{\mathrm{T}}}}$	
$I_{\mathrm{S}} = \frac{\mathrm{q}A\overline{D_{\mathrm{n}}}n_{\mathrm{i}}^2}{Q_{\mathrm{B}}}$	$I_{\mathrm{B}} = \frac{I_{\mathrm{C}}}{\beta}$
$I_{\mathrm{E}} = -\left(I_{\mathrm{C}} + \frac{I_{\mathrm{C}}}{\beta}\right) = -\frac{I_{\mathrm{C}}}{\alpha} = -(\beta+1)I_{\mathrm{B}}$	$\beta = \frac{I_{\mathrm{C}}}{I_{\mathrm{B}}} \approx \frac{D_{\mathrm{n}}}{D_{\mathrm{p}}}\frac{L_{\mathrm{p}}}{W_{\mathrm{B}}}\frac{N_{\mathrm{D}}}{N_{\mathrm{A}}} \approx 2.5\frac{L_{\mathrm{p}}}{W_{\mathrm{B}}}\frac{N_{\mathrm{D}}}{N_{\mathrm{A}}}$
$\alpha = \frac{\beta}{1+\beta} = \frac{1}{1+\frac{1}{\beta}} = \frac{1}{1+\frac{W_{\mathrm{B}}^2}{2\tau_{\mathrm{b}}D_{\mathrm{n}}}+\frac{D_{\mathrm{p}}}{D_{\mathrm{n}}}\frac{W_{\mathrm{B}}}{L_{\mathrm{p}}}\frac{N_{\mathrm{A}}}{N_{\mathrm{D}}}} \approx \alpha_{\mathrm{T}}\gamma$	
$\alpha_{\mathrm{T}} = \frac{1}{1+\frac{W_{\mathrm{B}}^2}{2\tau_{\mathrm{b}}D_{\mathrm{n}}}}$	$\gamma = \frac{1}{1+\frac{D_{\mathrm{p}}}{D_{\mathrm{n}}}\frac{W_{\mathrm{B}}}{L_{\mathrm{p}}}\frac{N_{\mathrm{A}}}{N_{\mathrm{D}}}}$

器件模型概要：放大区小信号参数

$r_{\pi} = \frac{V_{\mathrm{T}}}{I_{\mathrm{B}}} = \frac{\beta}{g_{\mathrm{m}}}$	$r_{\mathrm{o}} = \frac{V_{\mathrm{A}}}{I_{\mathrm{C}}}$
$g_{\mathrm{m}} = \frac{I_{\mathrm{C}}}{V_{\mathrm{T}}}$	$g_{\mathrm{m}} r_{\mathrm{o}} = \frac{V_{\mathrm{A}}}{V_{\mathrm{T}}}$
$C_{\pi} = C_{\mathrm{j}} + C_{\mathrm{d}}$	
$C_{\mathrm{j}} = \frac{C_{\mathrm{jbe0}}}{\left(1 - \frac{V_{\mathrm{BE}}}{\Phi_{\mathrm{0BE}}}\right)^{1/2}}$	$C_{\mathrm{d}} = g_{\mathrm{m}} \tau_{\mathrm{b}}$
$C_{\mu} = \frac{C_{\mathrm{jbc0}}}{\left(1 + \frac{V_{\mathrm{CB}}}{\Phi_{\mathrm{0BC}}}\right)^{1/3}}$	$C_{\mathrm{cs}} = \frac{C_{\mathrm{jsc0}}}{\left(1 + \frac{V_{\mathrm{CS}}}{\Phi_{\mathrm{0SC}}}\right)^{1/3}}$

“单页” BJT型晶体管模型

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE}}{V_A}\right)$$

$I_S \approx 10^{-15}$ A

当k=300 K时，$V_T = kT/q = 26$ mV

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{V_A}{I_C}$$

$V_{Anpn} \approx 20$ V

$V_{Apnp} \approx 10$ V

$$f_T = \frac{1}{2\pi} \frac{1}{r_b + \dfrac{C_j + C_\mu}{g_m}}$$

或 $\approx \dfrac{v_{scl}}{2\pi W_B}$

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    - ❑MOS电容
    - ❑阈值电压
  - ❑大信号工作原理
    - ❑线性区
    - ❑强反型区
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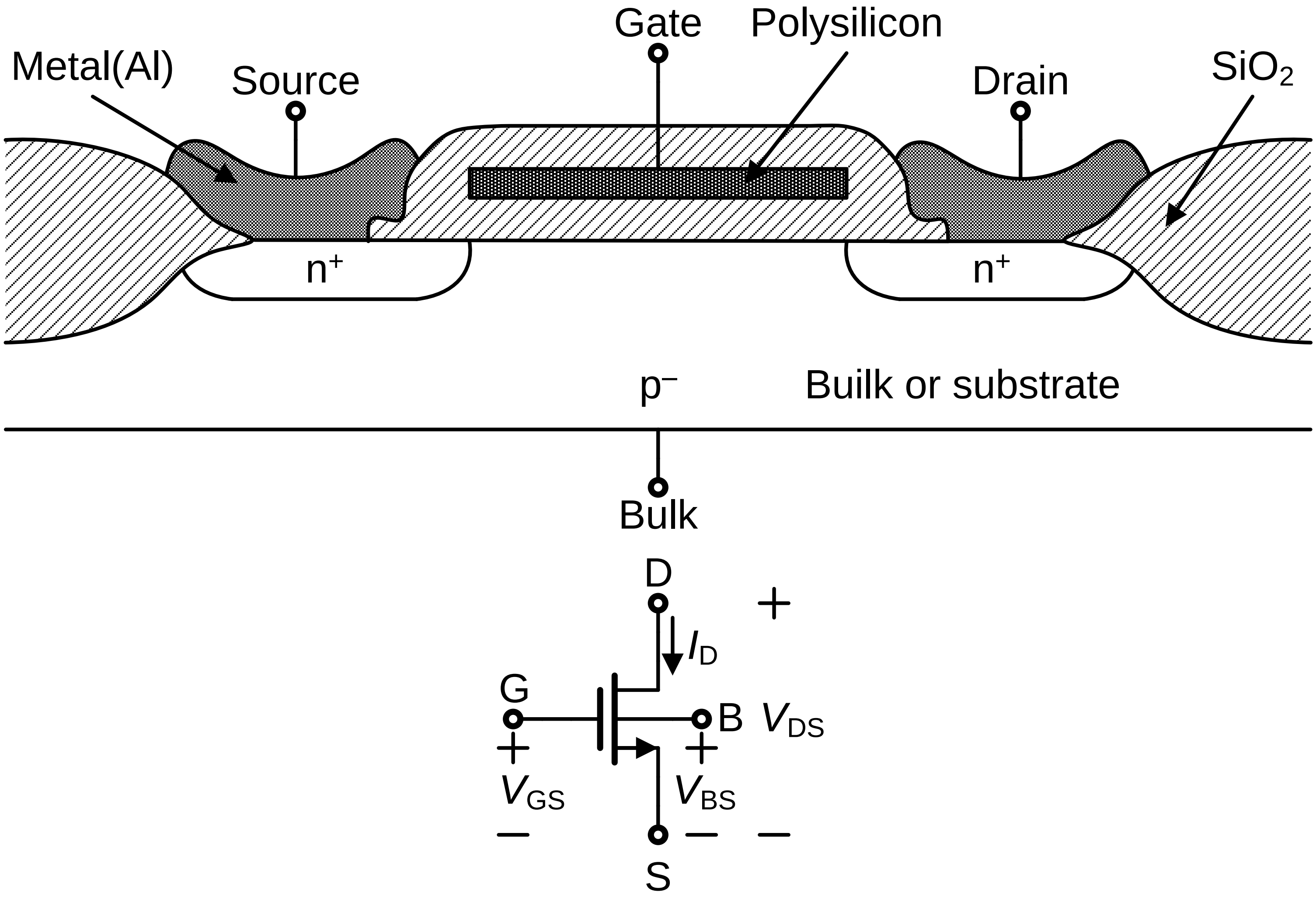
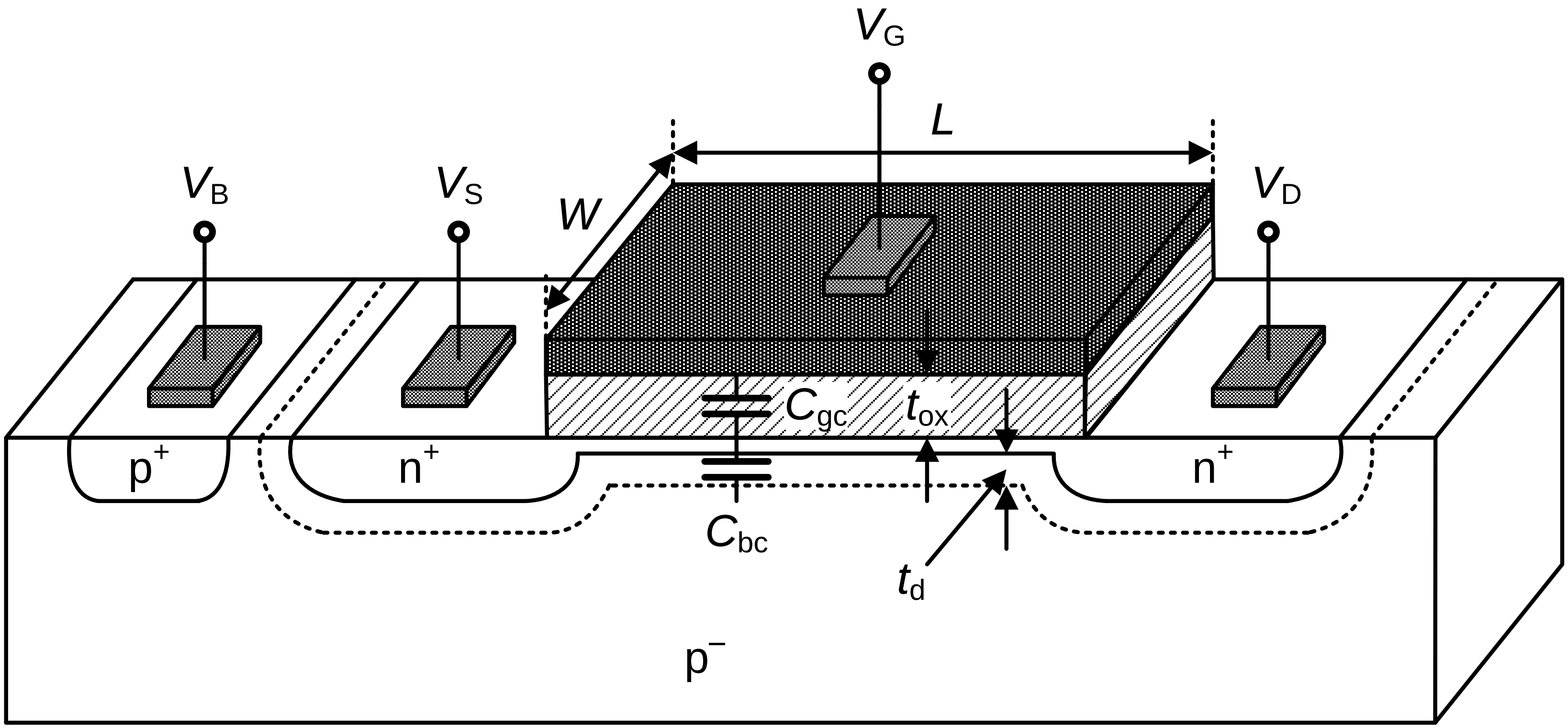
NMOS型晶体管的剖面图和符号



A cross section of a typical n-channel transistor.

MOS型晶体管的尺寸参数



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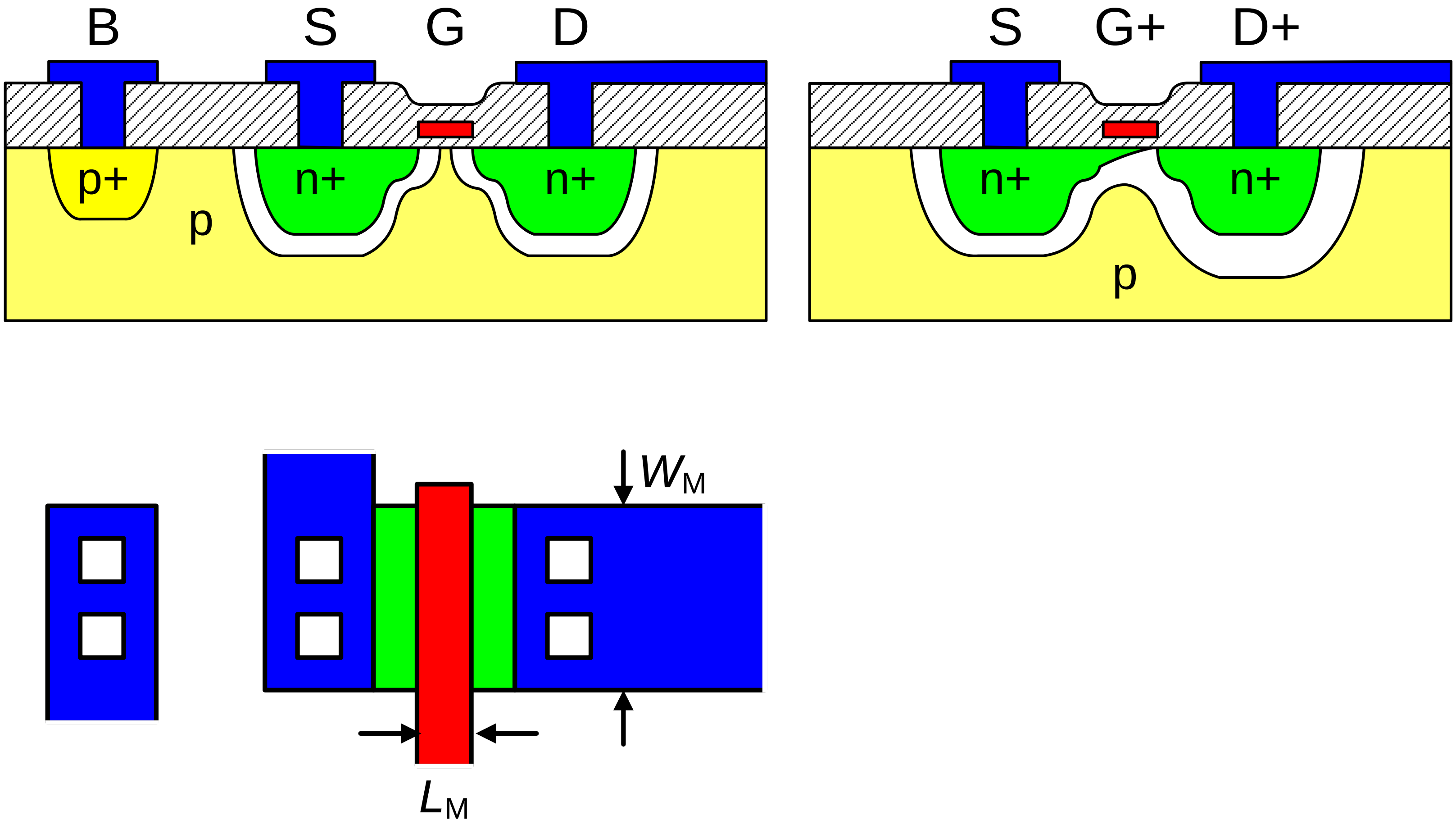
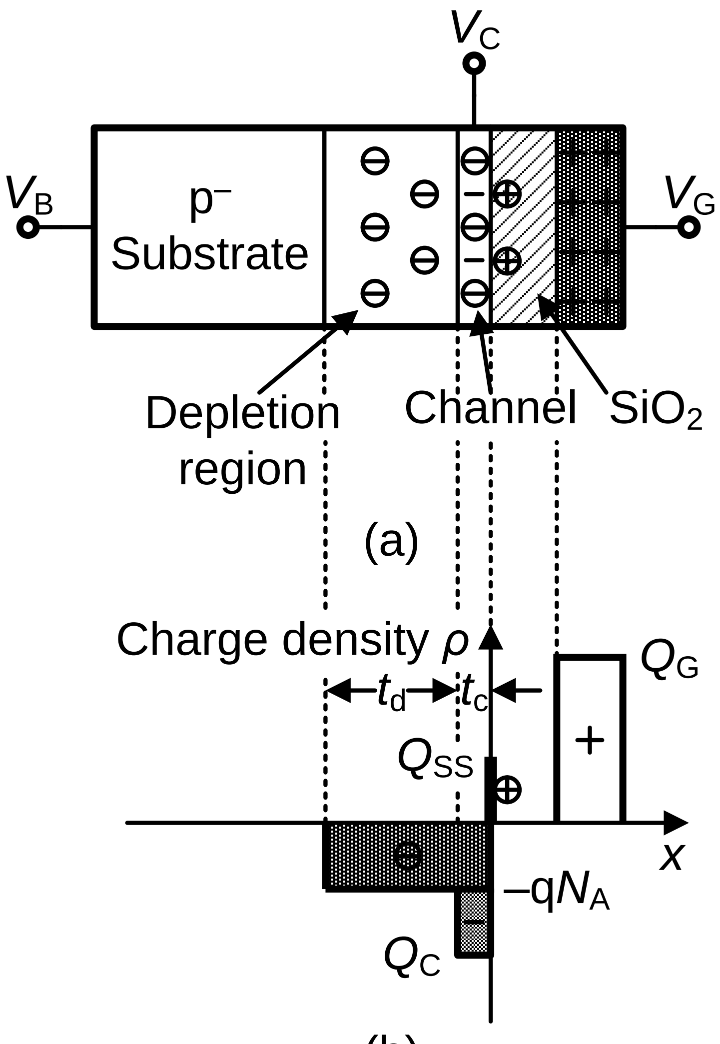
MOS型晶体管的版图



MIS结构：电荷密度



(a)

(b)

耗尽层中的电荷体密度：

$$\rho = -qN_B$$

沟道中电荷面密度：Q_C

沟道和栅氧之间的表面缺陷造成的电荷面密度：Q_{SS}

栅上电荷面密度：Q_G

电荷守恒：(面密度)

$$-qN_A t_d + Q_C + Q_{SS} + Q_G = 0$$

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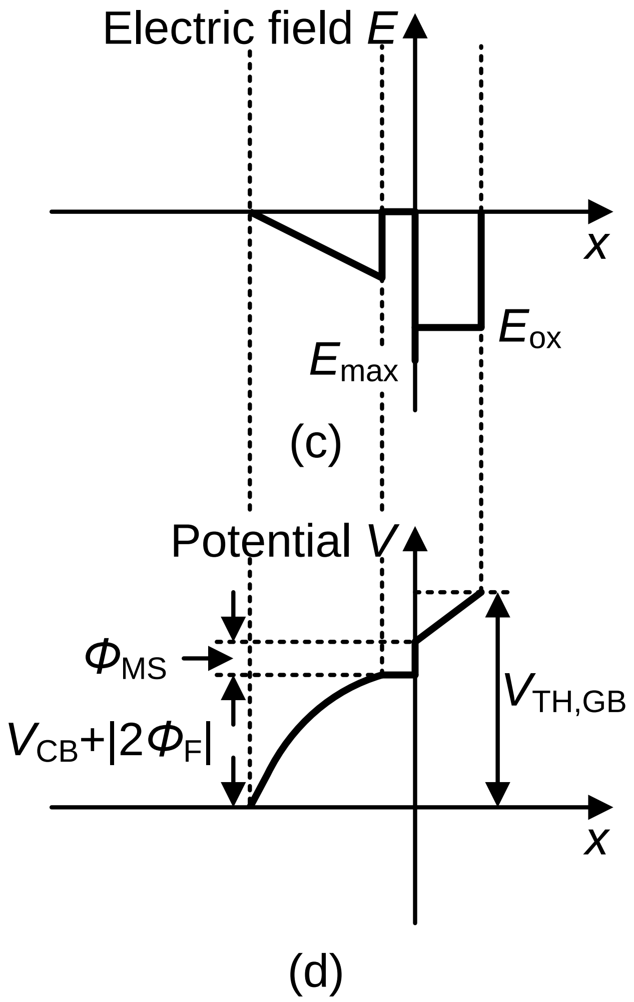
MIS结构：电场和静电势



在耗尽层中，由泊松方程：

$$\frac{d^2V(x)}{dx^2}=-\frac{\rho}{\varepsilon_{si}}=\frac{qN_B}{\varepsilon_{si}}$$

$$E(x)=-\frac{dV(x)}{dx}=-\left(\frac{qN_B}{\varepsilon_{si}}x+C_1\right)$$

边界条件：$x=-t_d-t_c$时，$E(x)=0$

$$E(x)=\frac{-qN_B}{\varepsilon_{si}}(x+t_d+t_c)$$

MIS结构：静电势

$$E(x) = -\frac{qN_B}{\varepsilon_{si}}(x + t_d + t_c) = -\frac{dV(x)}{dx}$$

$$V(x) = \frac{qN_B}{\varepsilon_{si}}\left(\frac{x^2}{2} + (t_d + t_c)x\right) + C_2$$

耗尽层静电势差：

$$V(-t_c) - V(-t_d - t_c) = \frac{qN_B}{\varepsilon_{si}}\frac{t_d^2}{2} = \left|\varPhi_S - \varPhi_F\right| + V_{CB}$$

其中$\varPhi_S$是反型层的表面势。

当$\varPhi_S$=–$\varPhi_F$时，衬底反型形成沟道。耗尽层的宽度：

$$\boxed{t_d = \left[\frac{2\varepsilon_{si}(\left|2\varPhi_F\right| + V_{CB})}{qN_B}\right]^{1/2}}$$

耗尽电容和栅氧电容

耗尽层中的电荷面密度： $Q_\mathrm{j} = \mathrm{q} N_\mathrm{B} t_\mathrm{d}$

耗尽电容： $$\boxed{C_\mathrm{bc} = WL \frac{\mathrm{d}Q_\mathrm{j}}{\mathrm{d}V_\mathrm{CB}} = WL \frac{\varepsilon_\mathrm{si}}{t_\mathrm{d}} = WLC_\mathrm{j}}$$

其中C_j为单位面积的耗尽电容： $$\boxed{C_\mathrm{j} = \frac{\varepsilon_\mathrm{si}}{t_\mathrm{d}}}$$

栅氧电容： $$\boxed{C_\mathrm{gc} = WLC_\mathrm{ox}}$$

其中C_ox为单位面积的栅氧电容： $$\boxed{C_\mathrm{ox} = \frac{\varepsilon_\mathrm{ox}}{t_\mathrm{ox}}}$$

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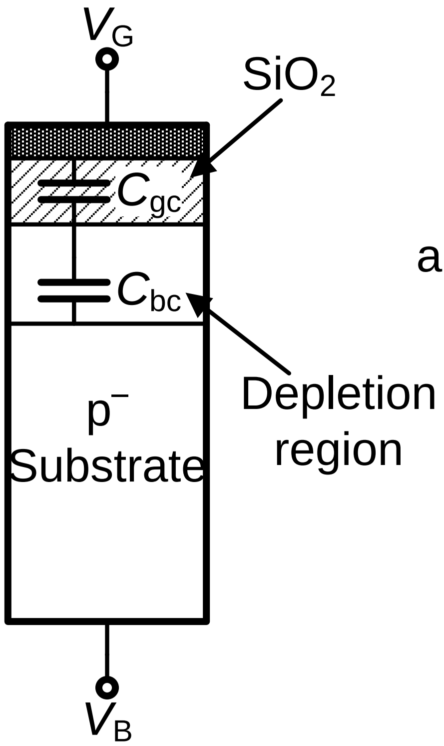
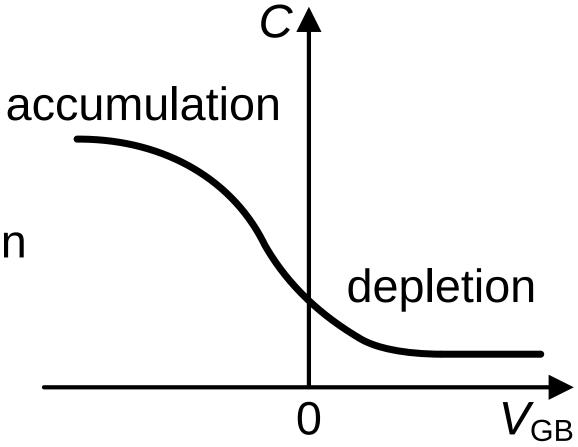
MIS结构：电容



(a)



(b)

$$C_{gb}=\frac{1}{\frac{1}{C_{gc}}+\frac{1}{C_{bc}}}=\frac{C_{gc}C_{bc}}{C_{gc}+C_{bc}}$$

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栅上的电荷面密度

耗尽区中的固定电荷面密度:

$$Q_{\mathrm{j}} = -qN_{\mathrm{B}}t_{\mathrm{d}} = -\sqrt{2qN_{\mathrm{B}}\varepsilon_{\mathrm{si}}(|2\Phi_{\mathrm{F}}| + V_{\mathrm{CB}})}$$

如果V_{C}浮空或者耗尽层偏压$V_{\mathrm{CB}}=0$，耗尽区中的固定电荷面密度:

$$Q_{\mathrm{j0}} = -qN_{\mathrm{B}}t_{\mathrm{d}} = -\sqrt{2qN_{\mathrm{B}}\varepsilon_{\mathrm{si}}(|2\Phi_{\mathrm{F}}|)}$$

当反型沟道层刚形成时，根据电荷守恒，栅上的电荷面密度:

$$Q_{\mathrm{G}} = -Q_{\mathrm{j}} - Q_{\mathrm{SS}}$$

V_{GB}的阈值电压

$$V_{TH,GB}=V_{CB}+\left|2\Phi_F\right|+\Phi_{MS}+\frac{Q_G}{C_{ox}}=V_{CB}+\left|2\Phi_F\right|+\Phi_{MS}+\frac{-Q_j-Q_{SS}}{C_{ox}}$$

$$=V_{CB}+\left|2\Phi_F\right|+\Phi_{MS}-\frac{Q_{j0}}{C_{ox}}-\frac{Q_{SS}}{C_{ox}}-\frac{Q_j-Q_{j0}}{C_{ox}}$$

$$=V_{CB}+V_{TH0}+\gamma(\sqrt{|2\Phi_F|+V_{CB}}-\sqrt{|2\Phi_F|})$$

$$\boxed{V_{TH0}=\left|2\Phi_F\right|+\Phi_{MS}-\frac{Q_{j0}}{C_{ox}}-\frac{Q_{SS}}{C_{ox}}}$$

体效应系数：

$$\boxed{\gamma=\frac{\sqrt{2qN_B\varepsilon_{si}}}{C_{ox}}=\frac{C_{bc}}{C_{gc}}2\sqrt{|2\Phi_F|+V_{CB}}}$$

V_{GC}的阈值电压

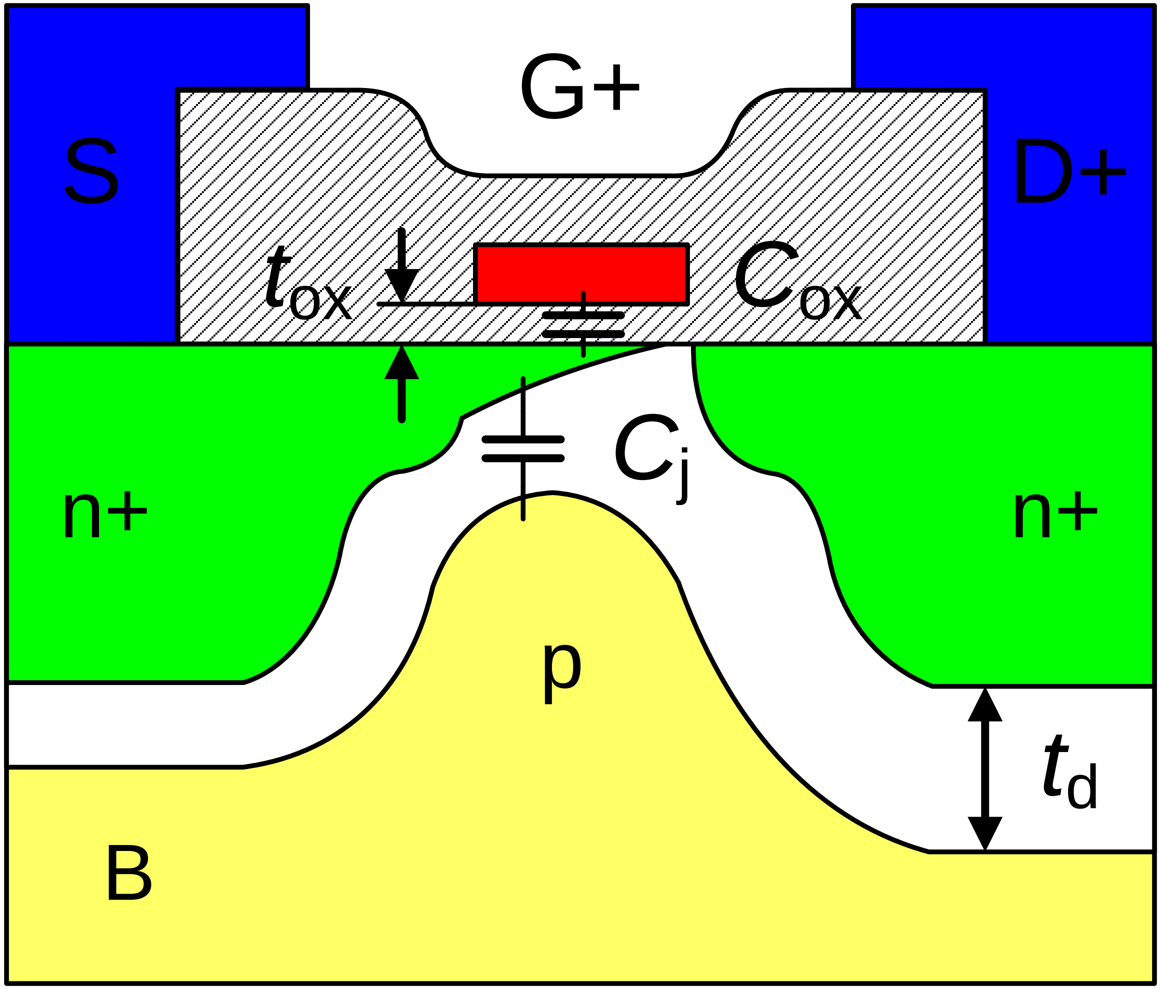
$$V_{\mathrm{TH,GC}} = V_{\mathrm{TH,GB}} - V_{\mathrm{CB}} = \left|2\Phi_{\mathrm{F}}\right| + \Phi_{\mathrm{MS}} - \frac{Q_{\mathrm{j0}}}{C_{\mathrm{ox}}} - \frac{Q_{\mathrm{SS}}}{C_{\mathrm{ox}}} - \frac{Q_{\mathrm{j}} - Q_{\mathrm{j0}}}{C_{\mathrm{ox}}}$$

$$= V_{\mathrm{TH0}} + \gamma\left(\sqrt{|2\Phi_{\mathrm{F}}| + V_{\mathrm{CB}}} - \sqrt{|2\Phi_{\mathrm{F}}|}\right)$$

耗尽电容与栅氧电容之比：

$$\boxed{\frac{C_{\mathrm{bc}}}{C_{\mathrm{gc}}} = \frac{C_{\mathrm{j}}}{C_{\mathrm{ox}}} = \frac{\gamma}{2\sqrt{|2\Phi_{\mathrm{F}}| + V_{\mathrm{CB}}}} = n - 1}$$

MOST版图：C_{ox}和C_j



$$C_j = \frac{\varepsilon_{si}}{t_d}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$\frac{C_j}{C_{ox}} = n-1$$

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MOST版图：C_{ox}和C_j的值

$$C_j = \frac{\varepsilon_{si}}{t_d} \qquad t_d = \left[\frac{2\varepsilon_{si}\left(\left|2\Phi_F\right| + V_{DB}\right)}{qN_B}\right]^{1/2}$$

$\varepsilon_{si} = 1\ \text{pF/cm}$

$\varepsilon_{ox} = 0.34\ \text{pF/cm}$

$N_B = 4\times10^{17}\ \text{cm}^{-3}$

$\left|2\Phi_F\right| = 0.6\ \text{V}$

$q = 1.6\times10^{-19}\ \text{C}$

例如：$L = 0.35\ \mu\text{m} \quad W/L = 8$

$V_{DB} = 3.3\ \text{V}$

$$t_d = 0.11\ \mu\text{m} \Rightarrow C_j \approx 1\times10^{-7}\ \text{F/cm}^2 = 1\ \text{fF/}\mu\text{m}^2$$

$$t_{ox} = \frac{L_{min}}{50} \Rightarrow t_{ox} = 7\ \text{nm} \Rightarrow C_{ox} = 5\times10^{-7}\ \text{F/cm}^2 = 5\ \text{fF/}\mu\text{m}^2$$

$$\frac{C_j}{C_{ox}} = n\text{-}1 \approx 0.2$$

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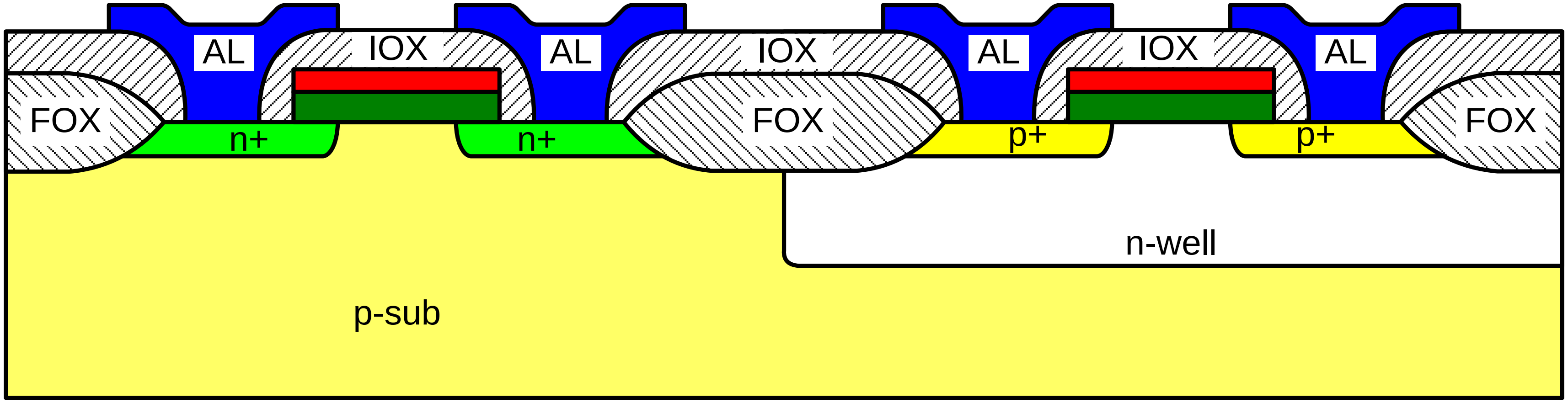
N阱CMOS工艺



🟥 多晶硅栅

🟩 栅氧

$N_{n\text{-well}} >> N_{p\text{-sub}}$，$n_{pmos}>n_{nmos}$!

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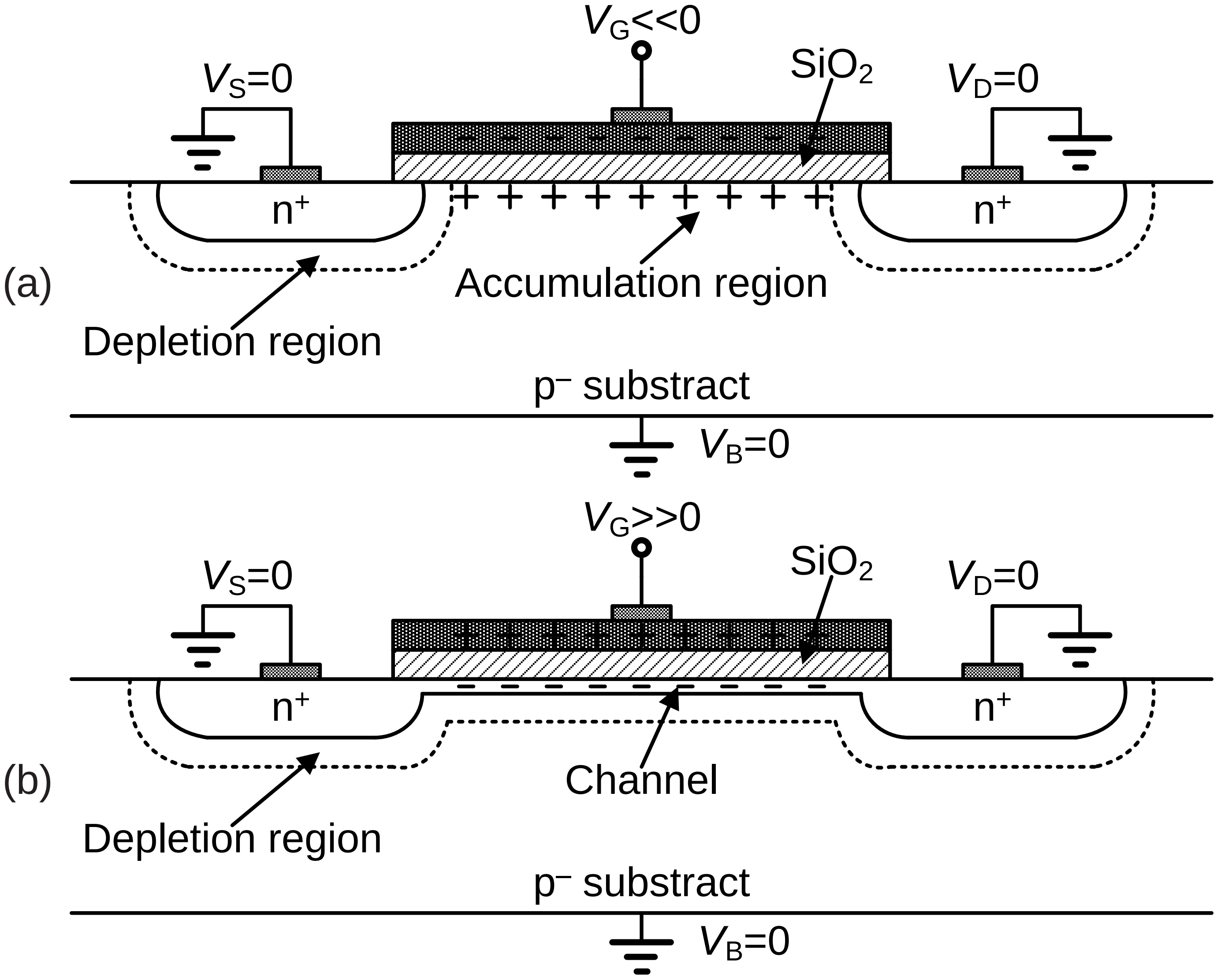
大信号工作原理：沟道的形成



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方块电阻：(1)

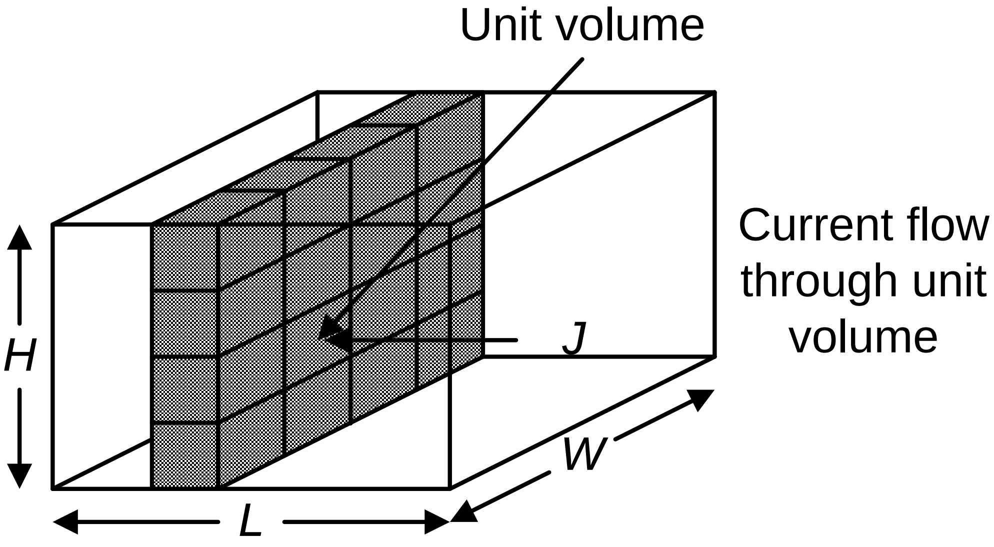
n型半导体的电流密度： $J = \sigma E$ $\sigma = -q n_n \mu_n$

n_n是单位体积中电子载流子的浓度，μ_n为电子迁移率。

沟道中的电流： $I = JWH = -q n_n \mu_n WHE$

其中W为沟道宽度，

H为沟道的厚度。



Current flowing through a unit volume.

方块电阻：(2)

沟道中的电场强度：

$$E = -\frac{\mathrm{d}V}{\mathrm{d}x} \qquad \mathrm{q}n_\mathrm{n}\mu_\mathrm{n}WH\mathrm{d}V = I\mathrm{d}x$$

沟道中的电流：

$$I = \mathrm{q}n_\mathrm{n}\mu_\mathrm{n}WH\frac{\mathrm{d}V}{\mathrm{d}x} \qquad \int_0^L I\mathrm{d}x = \int_0^V \mathrm{q}n_\mathrm{n}\mu_\mathrm{n}WH\mathrm{d}V \qquad I = \frac{\mathrm{q}n_\mathrm{n}\mu_\mathrm{n}WH}{L}V$$

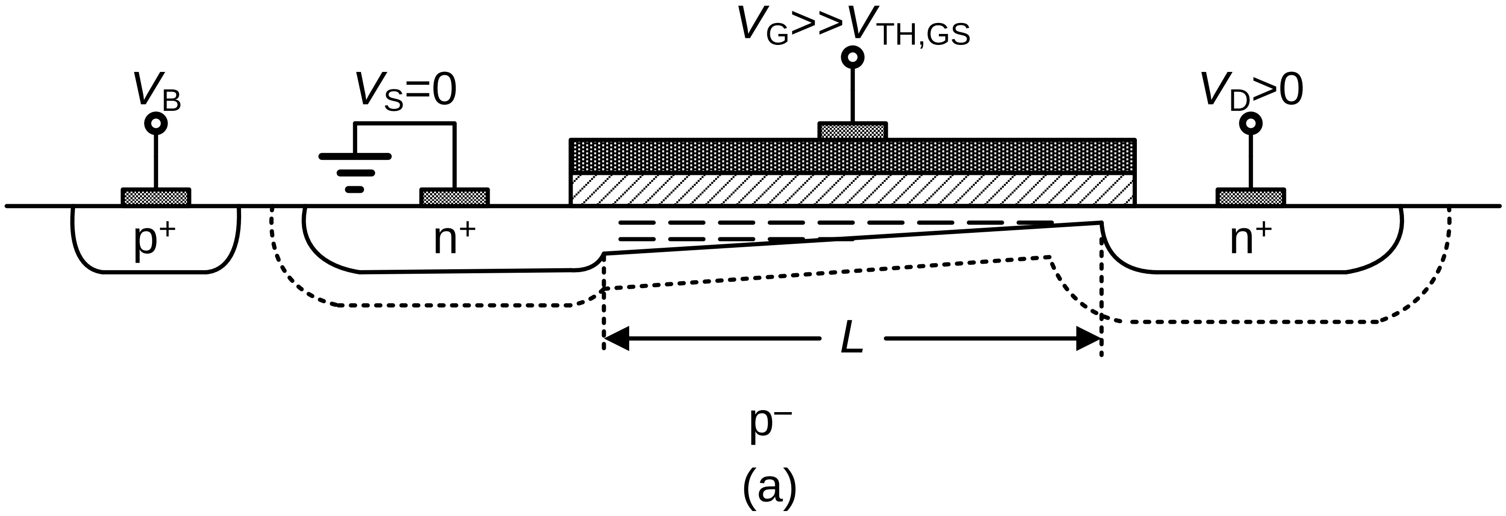
沟道总电阻：

$$R = \frac{V}{I} = \frac{L}{\mathrm{q}n_\mathrm{n}\mu_\mathrm{n}WH} = R_\square\frac{L}{W}$$

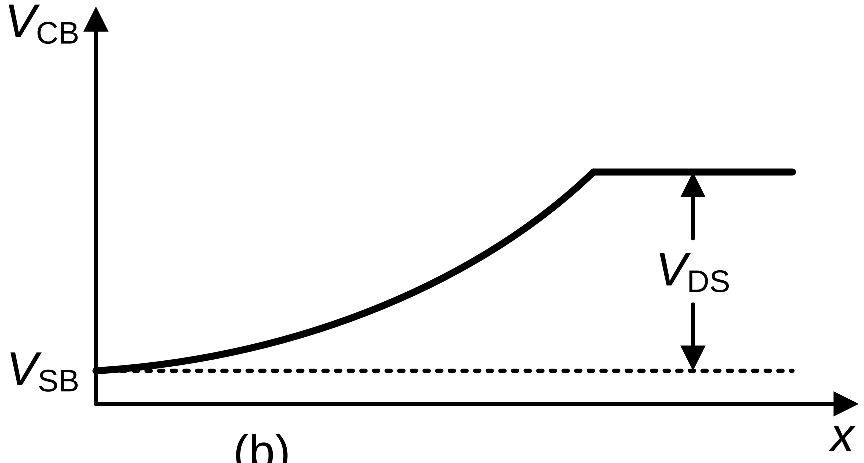
其中方块电阻：

$$\boxed{R_\square = \frac{1}{\mathrm{q}n_\mathrm{n}\mu_\mathrm{n}H} = \frac{1}{\sigma H}}$$

大信号工作原理：线性区



(a)



(b)

I-*V*关系式：(1)

在MOS管中，沟道电荷密度$n_\text{n}(x)$不是常数，单位面积的电荷密度：

$$Q_\text{n}(x)=\text{q}n_\text{n}(x)H=C_\text{ox}\{V_\text{GC}(x)-V_\text{TH,GC}[V_\text{CB}(x)]\}$$

$$I_\text{D}=W\mu_\text{n}Q_\text{n}(x)\frac{\text{d}V_\text{CS}(x)}{\text{d}x}$$

$$\int_0^L I_\text{D}\text{d}x=\int_0^{V_\text{DS}}W\mu_\text{n}C_\text{ox}\{V_\text{GC}(x)-V_\text{TH,GC}[V_\text{CB}(x)]\}\text{d}V_\text{CS}(x)$$

$$V_\text{GC}(x)=V_\text{GS}-V_\text{CS}(x)\qquad V_\text{CB}(x)=V_\text{CS}(x)+V_\text{SB}$$

$$V_\text{TH,GC}[V_\text{CB}(x)]=V_\text{TH0}+\gamma\left(\sqrt{|2\Phi_\text{F}|+V_\text{CB}(x)}-\sqrt{|2\Phi_\text{F}|}\right)$$

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*I-V*关系式：(2)

$$V_{\mathrm{TH,GC}}[V_{\mathrm{CB}}(x)]=V_{\mathrm{TH,GS}}+\gamma\left(\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{CB}}(x)}-\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}\right)$$

$$=V_{\mathrm{TH,GS}}+\gamma\frac{V_{\mathrm{CB}}(x)-V_{\mathrm{SB}}}{\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{CB}}(x)}+\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}}$$

$$\approx V_{\mathrm{TH,GS}}+\gamma\frac{V_{\mathrm{CS}}(x)}{2\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}}=V_{\mathrm{TH,GS}}+(n-1)V_{\mathrm{CS}}(x)$$

$$\int_0^L I_{\mathrm{D}}\mathrm{d}x=\int_0^{V_{\mathrm{DS}}}\mu_{\mathrm{n}}WC_{\mathrm{ox}}\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}\mathrm{d}V_{\mathrm{CS}}(x)$$

$$\boxed{I_{\mathrm{D}}=\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}\left[(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})V_{\mathrm{DS}}-\frac{nV_{\mathrm{DS}}^2}{2}\right]}$$

$$V_{\mathrm{TH,GS}}=V_{\mathrm{TH0}}+\gamma\left(\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}-\sqrt{|2\Phi_{\mathrm{F}}|}\right)$$

$$n=\frac{\gamma}{2\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}}+1$$

$$n=1.2\sim1.5$$

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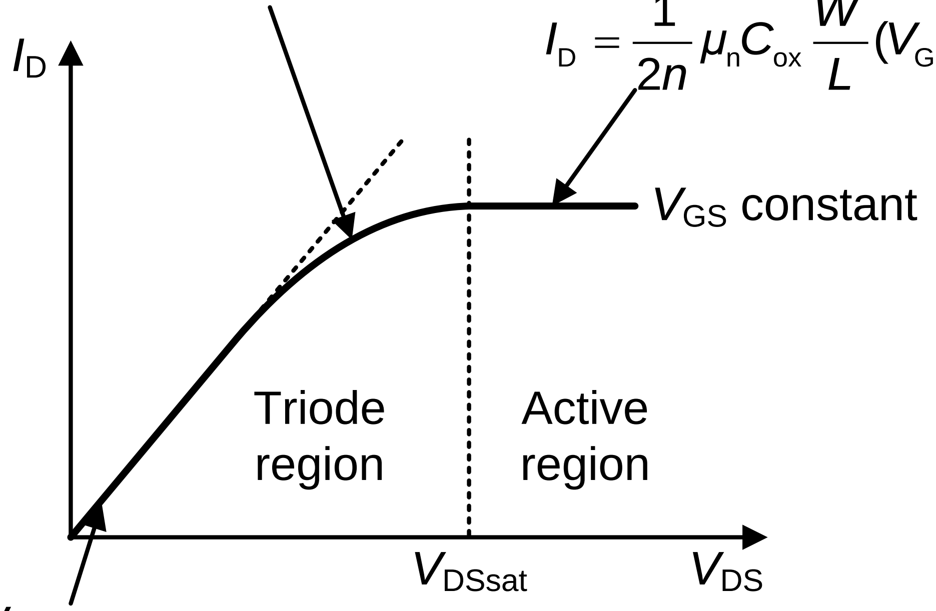
I-V关系式：(3)



$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{TH,GS})V_{DS} - \frac{nV_{DS}^2}{2}\right]$$

$$I_D = \frac{1}{2n}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH,GS})^2$$

$$I_D \approx \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH,GS})V_{DS}$$

The I_D versus V_{DS} curve for an ideal MOS transistor. For $V_{DS}>V_{DSsat}$, I_D is approximately constant.

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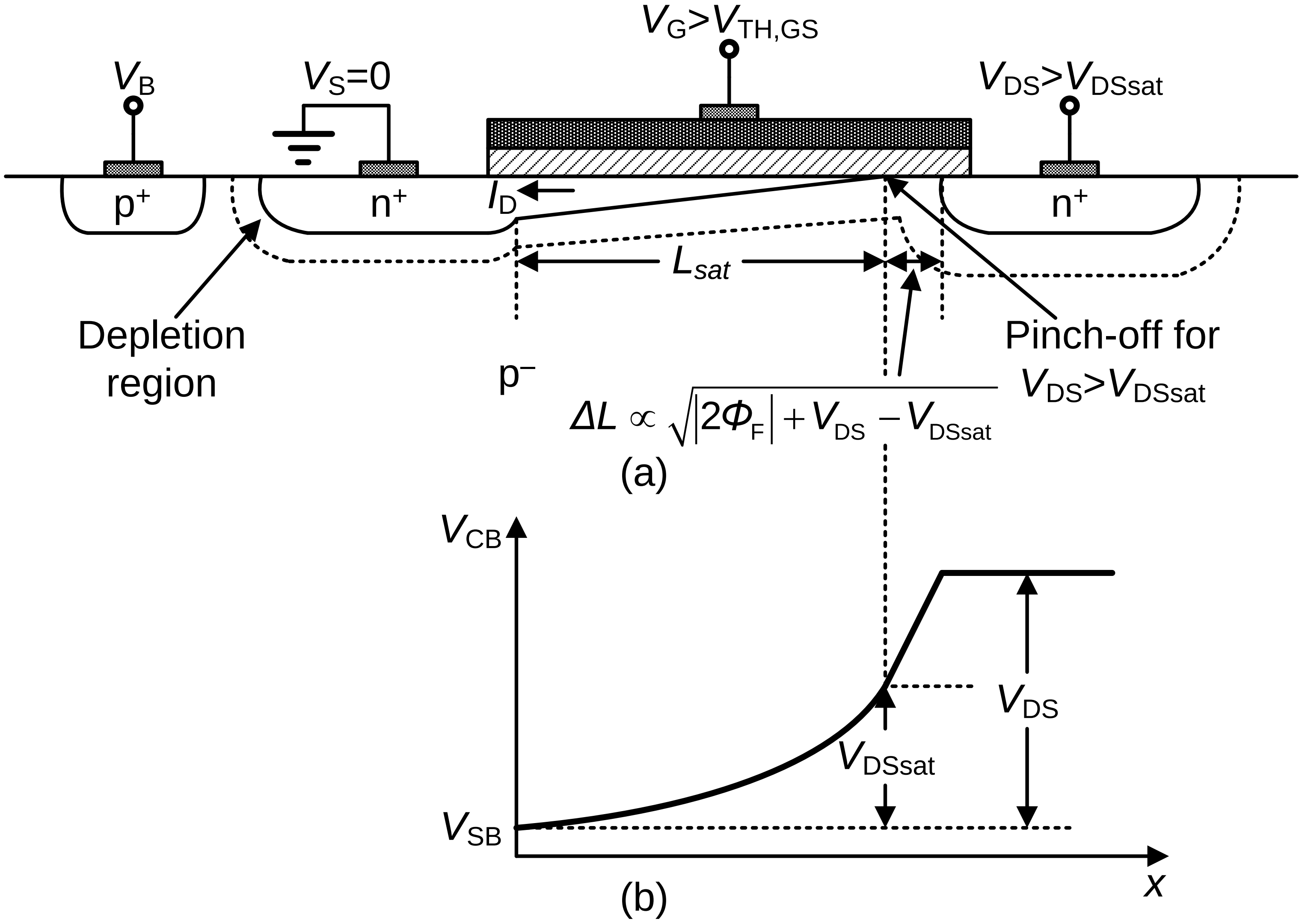
大信号工作原理：饱和区



$$\Delta L \propto \sqrt{\left|2\Phi_F\right| + V_{DS} - V_{DSsat}}$$

饱和条件和*I-V*关系式

饱和条件：

$$V_{GC}(x)-V_{TH,GC}[V_{CB}(x)]=V_{GS}-V_{TH,GS}-nV_{CS}(x)=0$$

$$V_{CS}=\frac{V_{GS}-V_{TH,GS}}{n}$$

$$\boxed{V_{DSsat}=\frac{V_{GS}-V_{TH,GS}}{n}=V_{eff}}$$

*I-V*关系式：

$$\boxed{I_D=\frac{1}{2n}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH,GS})^2}$$

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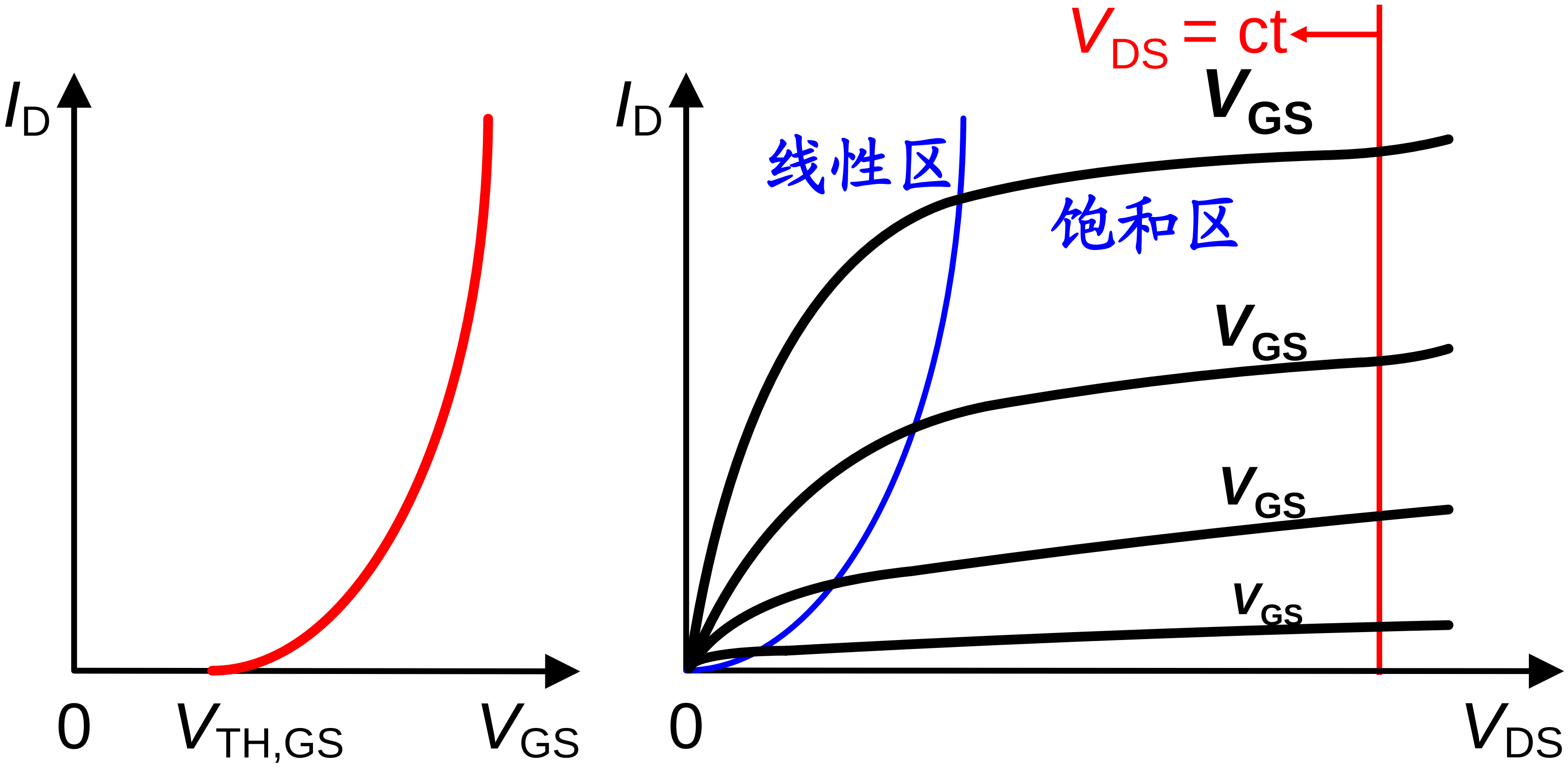
MOST的I_D与V_{GS}和V_{DS}的关系



高V_{DS}使MOST进入饱和区

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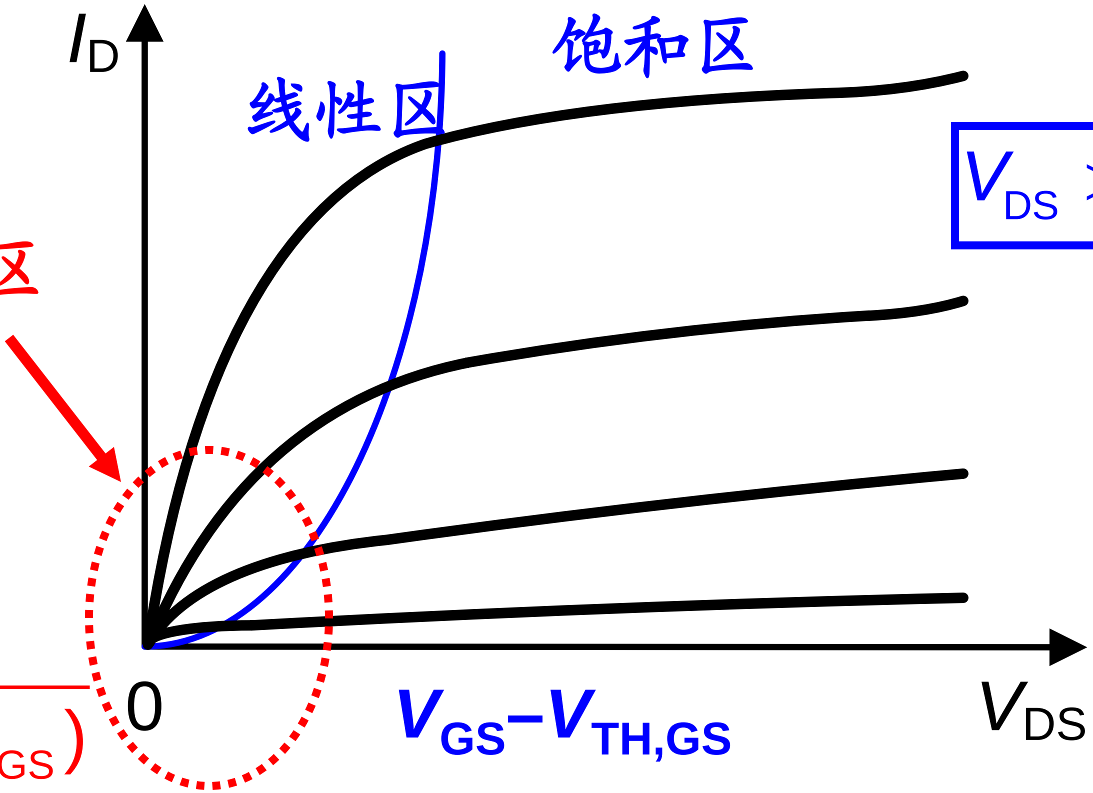
MOST的V_{DS}与I_D的关系

线性电阻区



$$R_{on}=\frac{1}{\beta(V_{GS}-V_{TH,GS})}$$

$$\beta=KP\frac{W}{L}$$

$$\boxed{V_{DS}>V_{GS}-V_{TH,GS}}$$

$$\frac{V_{GS}-V_{TH,GS}}{n}$$

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沟道长度调制效应：(1)

饱和时的沟道长度： $L_{\mathrm{sat}}=L-\Delta L$

$$\Delta L=\left[\frac{2\varepsilon_{\mathrm{si}}\left(\left|2\Phi_{\mathrm{F}}\right|+V_{\mathrm{DS}}-V_{\mathrm{DSsat}}\right)}{qN_{\mathrm{B}}}\right]^{1/2} \qquad I_{\mathrm{D}}=\frac{1}{2n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}\left(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}\right)^{2}$$

$$\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{DS}}}=\frac{\partial I_{\mathrm{D}}}{\partial L}\frac{\partial L}{\partial V_{\mathrm{DS}}}=-\frac{1}{2n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L^{2}}\left(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}\right)^{2}\frac{\partial L}{\partial V_{\mathrm{DS}}}=\frac{I_{\mathrm{D}}}{L}\frac{\partial L}{\partial V_{\mathrm{DS}}}$$

定义厄利电压 V_{A} 为： V_{E} 近似为常数

$$V_{\mathrm{A}}=\frac{I_{\mathrm{D}}}{\partial I_{\mathrm{D}}/\partial V_{\mathrm{DS}}}=L\left(\frac{\partial L}{\partial V_{\mathrm{DS}}}\right)^{-1}=\left[\frac{qN_{\mathrm{B}}\left(\left|2\Phi_{\mathrm{F}}\right|+V_{\mathrm{DS}}-V_{\mathrm{DSsat}}\right)}{\varepsilon_{\mathrm{si}}}\right]^{1/2}L$$

$$\boxed{=V_{\mathrm{E}}L}$$

沟道长度调制效应：(2)

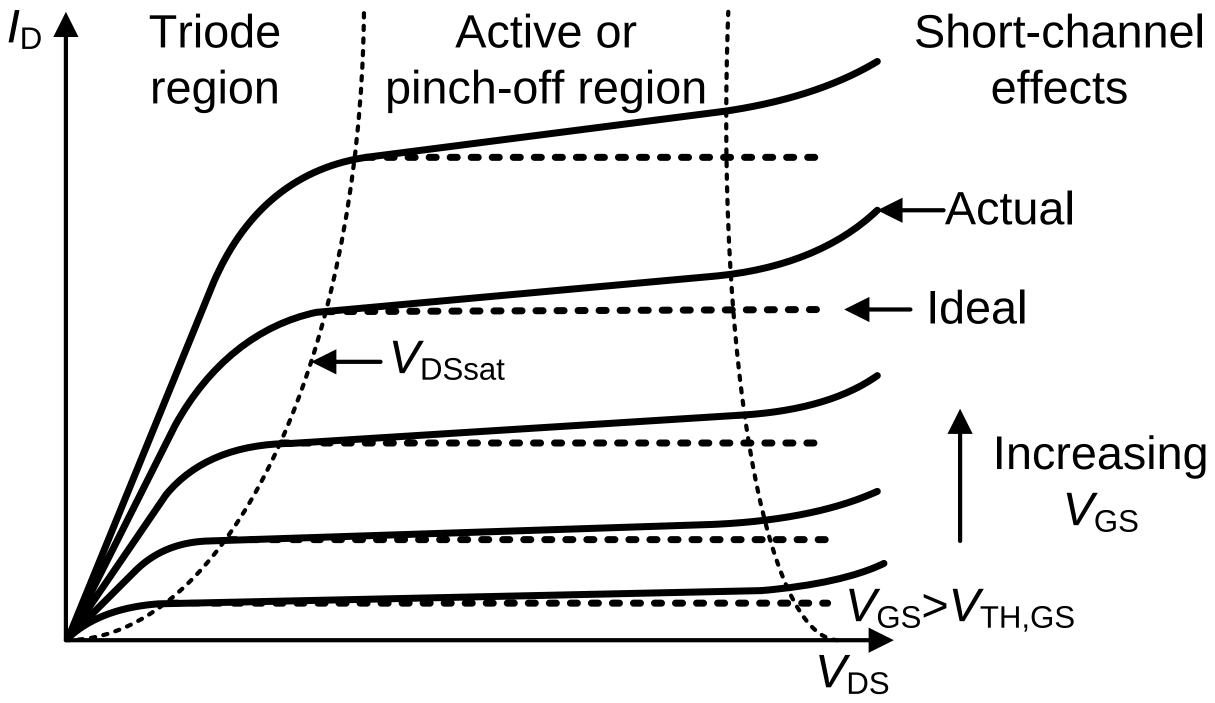
沟道长度调制系数:

$$\boxed{\lambda = \frac{1}{V_{\mathrm{A}}} = \frac{1}{V_{\mathrm{E}}L}}$$

沟道饱和电流：

$$\boxed{I_{\mathrm{D}} = \frac{1}{2n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})^2\left[1 + \lambda(V_{\mathrm{DS}} - V_{\mathrm{DSsat}})\right]}$$

沟道长度调制效应：(3)



I_D versus V_{DS} for different values of V_{GS}.

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MOST的参数β、*KP*、C_{ox}等

$$\beta = KP\frac{W}{L}$$

$$\Downarrow$$

$$KP = \mu C_{\text{ox}}$$

$$\Downarrow$$

$$C_{\text{ox}} = \frac{\varepsilon_{\text{ox}}}{t_{\text{ox}}}$$

$$\Downarrow$$

$$t_{\text{ox}} = \frac{L_{\text{min}}}{50}$$

$$KP_{\text{p}} \approx 125\ \mu\text{A/V}^2$$

$$KP_{\text{n}} \approx 300\ \mu\text{A/V}^2$$

$$C_{\text{ox}} \approx 5\times10^{-7}\ \text{F/cm}^2$$

$$\varepsilon_{\text{si}} = 1\ \text{pF/cm}$$

$$\varepsilon_{\text{ox}} = 0.34\ \text{pF/cm}$$

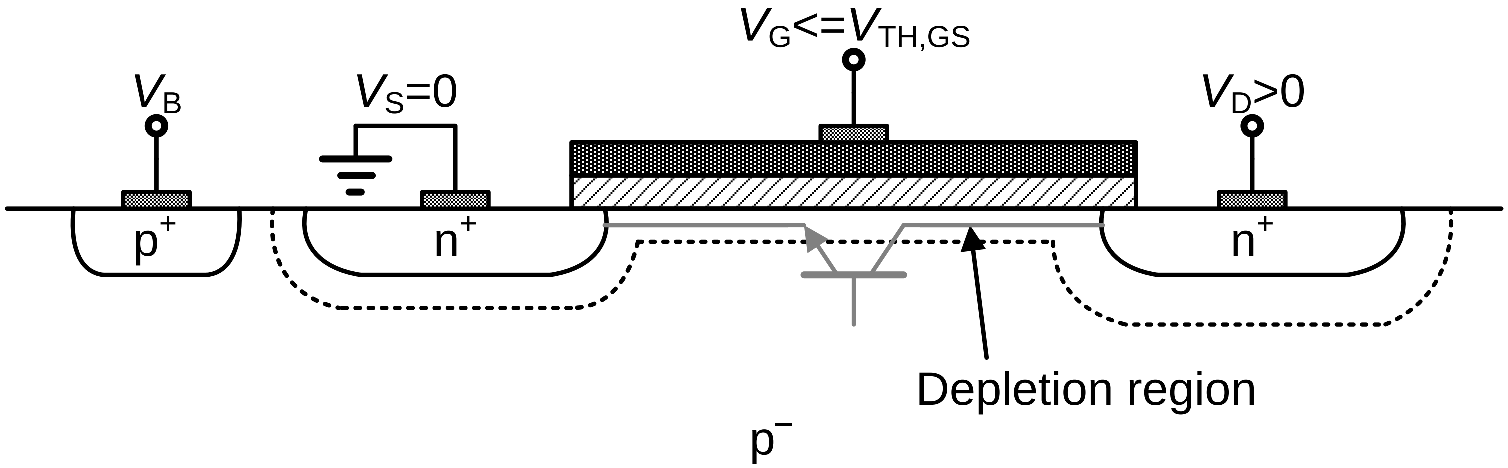
$$t_{\text{ox}} = 7\ \text{nm}$$

$$L = 0.35\ \mu\text{m}$$

$$\mu_{\text{p}} = 250\ \text{cm}^2\text{/Vs}$$

$$\mu_{\text{n}} = 600\ \text{cm}^2\text{/Vs}$$

大信号工作原理：弱反型(亚阈值)区



The depletion region of subthreshold operation

p型衬底中源端和漏端的少数载流子浓度：

$$n_p(0) = n_{p0} e^{\frac{\Phi_S}{V_T}} \qquad n_p(L) = n_{p0} e^{\frac{\Phi_S - V_{DS}}{V_T}}$$

其中，n_{p0}是衬底中的平衡载流子浓度，Φ_S是表面势。

I-*V*关系式：(1)

电子扩散产生的电流：

$$I_{\mathrm{D}} = \mathrm{q}AD_{\mathrm{n}}\frac{n_{\mathrm{p}}(0)-n_{\mathrm{p}}(L)}{L} = \frac{W}{L}\mathrm{q}t_{\mathrm{d}}D_{\mathrm{n}}n_{\mathrm{p0}}\mathrm{e}^{\frac{\Phi_{\mathrm{S}}}{V_{\mathrm{T}}}}(1-\mathrm{e}^{-\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}})$$

其中D_{n}是电子的扩散系数，A是电流流过区域的截面积，$A=W\times t_{\mathrm{d}}$，W为MOS管宽度，t_{d}为耗尽区深度。

在弱反型区，表面势正比于栅源电压：

$$\frac{\mathrm{d}\Phi_{\mathrm{S}}}{\mathrm{d}V_{\mathrm{GS}}} = \frac{C_{\mathrm{ox}}}{C_{\mathrm{j}}+C_{\mathrm{ox}}} = \frac{1}{n}$$

$$\Phi_{\mathrm{S}} = \frac{V_{\mathrm{GS}}}{n}+k_1 = \frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{n}+k_2$$

$$k_2 = k_1 + \frac{V_{\mathrm{TH,GS}}}{n}$$

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I-*V*关系式：(2)

漏极电流:

$$I_\mathrm{D}=\frac{W}{L}t_\mathrm{d}\mathrm{q}D_\mathrm{n}n_\mathrm{p0}\mathrm{e}^{\frac{k_2}{V_\mathrm{T}}}\mathrm{e}^{\frac{V_\mathrm{GS}-V_\mathrm{TH,GS}}{nV_\mathrm{T}}}(1-\mathrm{e}^{-\frac{V_\mathrm{DS}}{V_\mathrm{T}}})$$

$$\boxed{I_\mathrm{D}=\frac{W}{L}I_\mathrm{t}\mathrm{e}^{\frac{V_\mathrm{GS}-V_\mathrm{TH,GS}}{nV_\mathrm{T}}}(1-\mathrm{e}^{-\frac{V_\mathrm{DS}}{V_\mathrm{T}}})}$$

其中，

$$I_\mathrm{t}=t_\mathrm{d}\mathrm{q}D_\mathrm{n}n_\mathrm{p0}\mathrm{e}^{\frac{k_2}{V_\mathrm{T}}}$$

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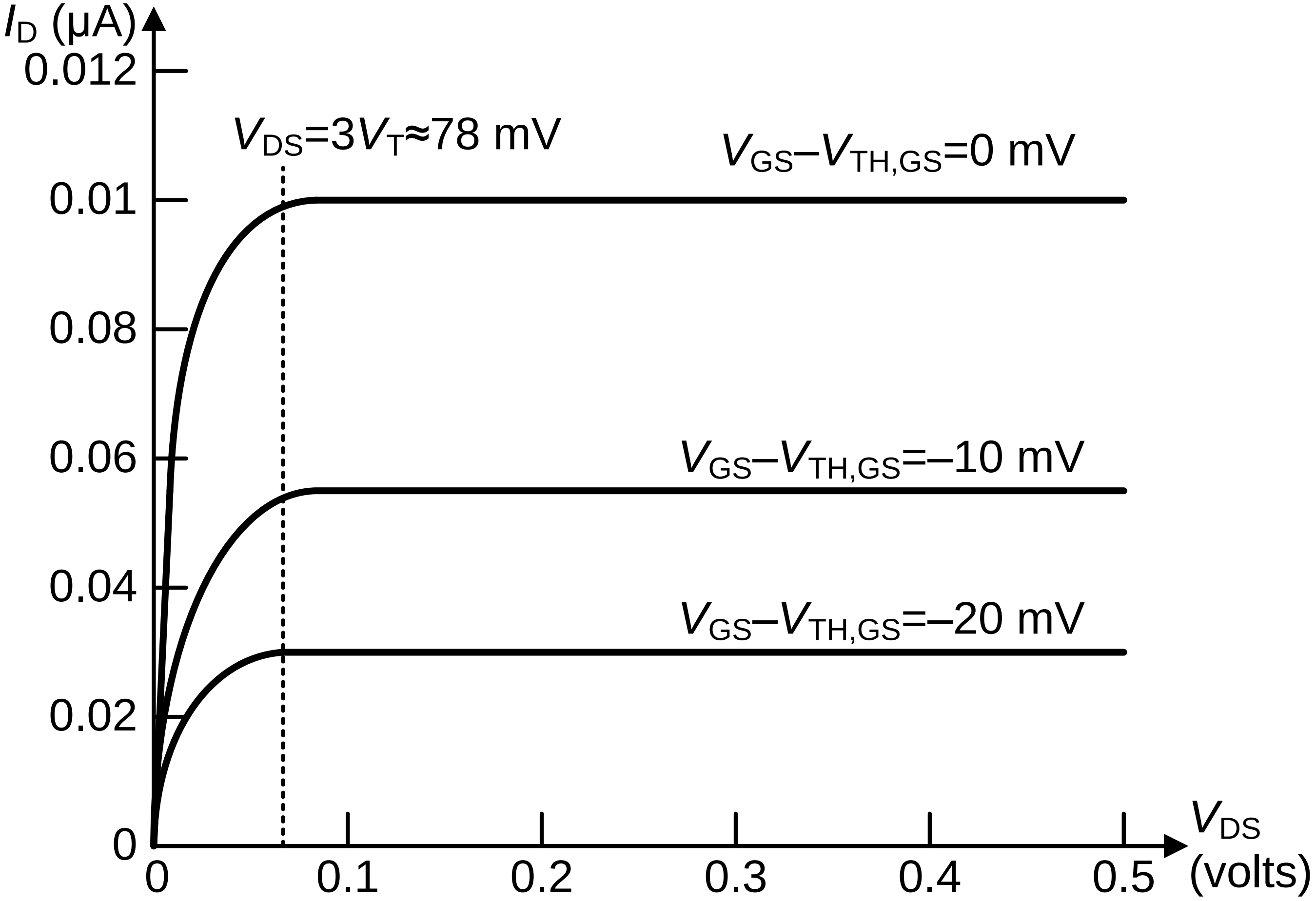
亚阈值区的典型*I-V*曲线



Drain current versus drain-source voltage in weak inversion.

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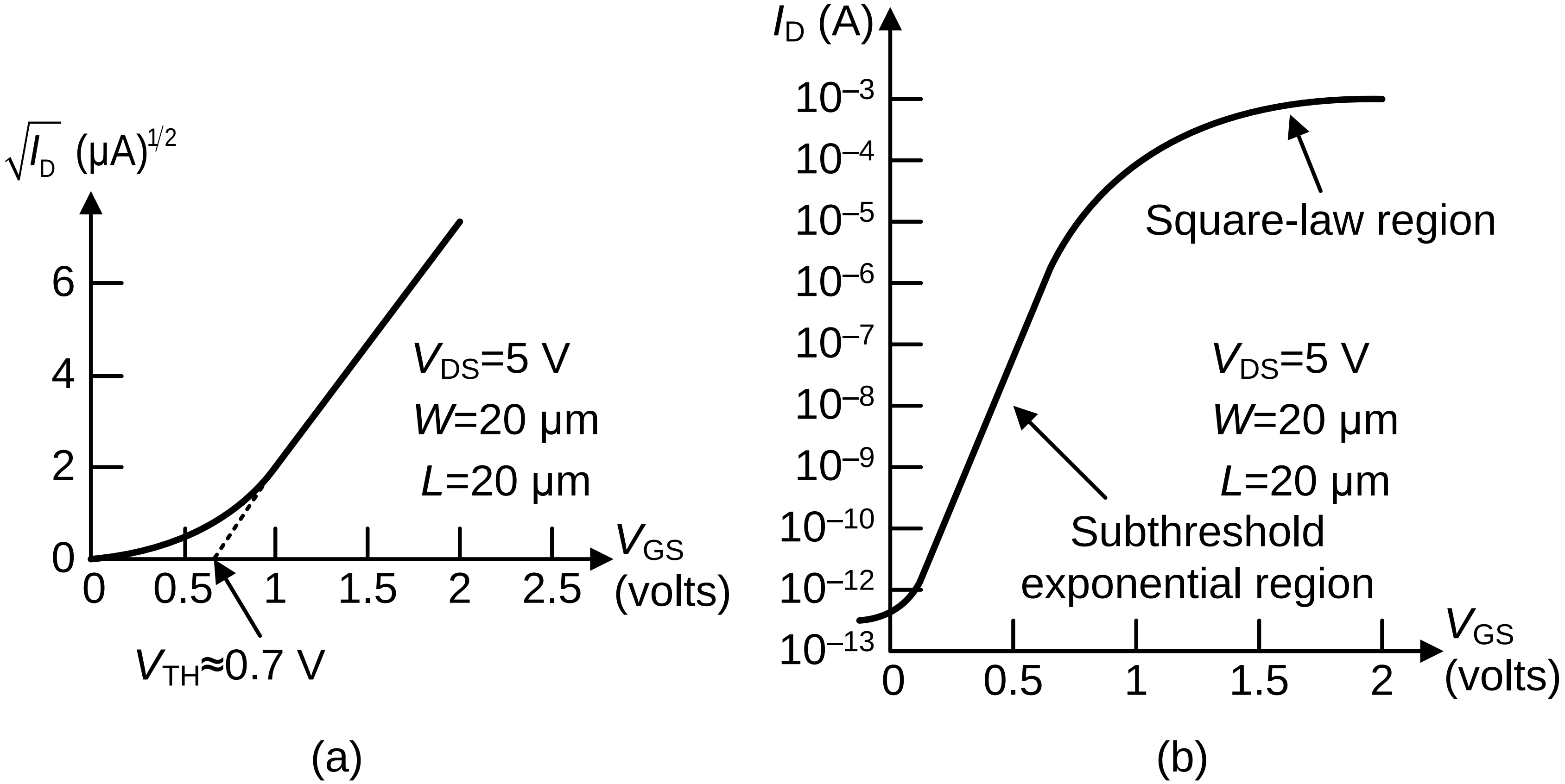
亚阈值区和饱和区的典型*I-V*曲线



Drain current versus gate-source voltage in weak inversion with a
(a) square root (b) logrithmic scale.

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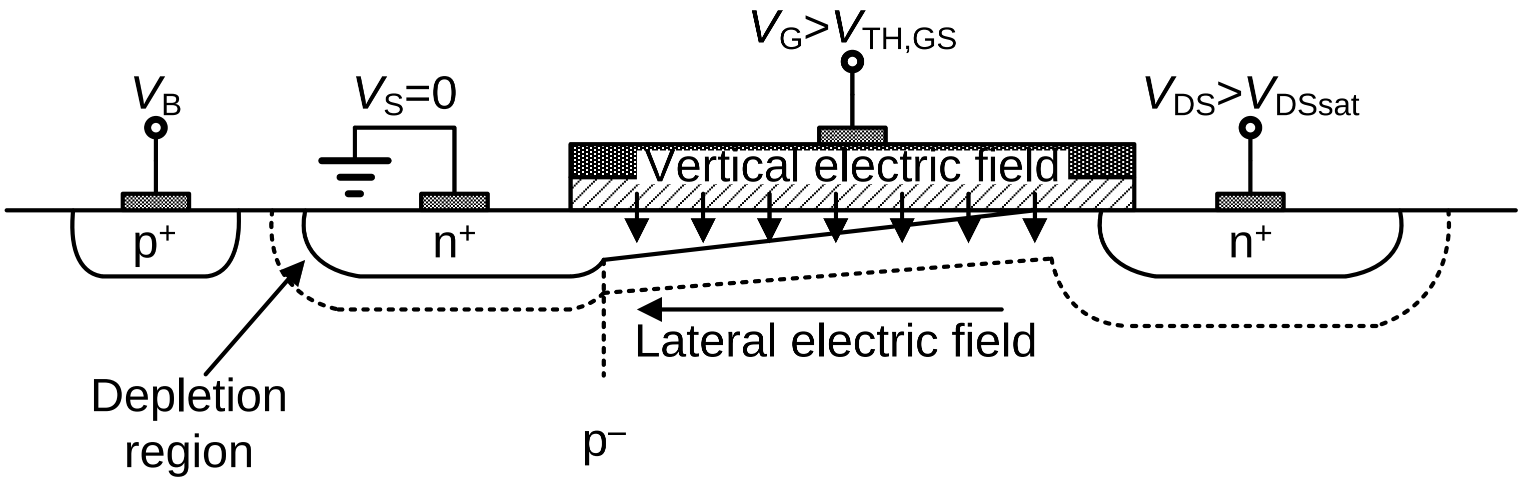
大信号工作原理：速度饱和区



An NMOS device in active mode (saturation) identifying the lateral and vertical electric field components.

在临界电场 E_{crit}下的速度定义为散射极限速度v_{scl}:

$$v_{scl} = \mu_n E_{crit}$$

速度饱和时的电流：

$$\boxed{I_D = WC_{ox}(V_{GS} - V_{TH,GS})v_{scl}}$$

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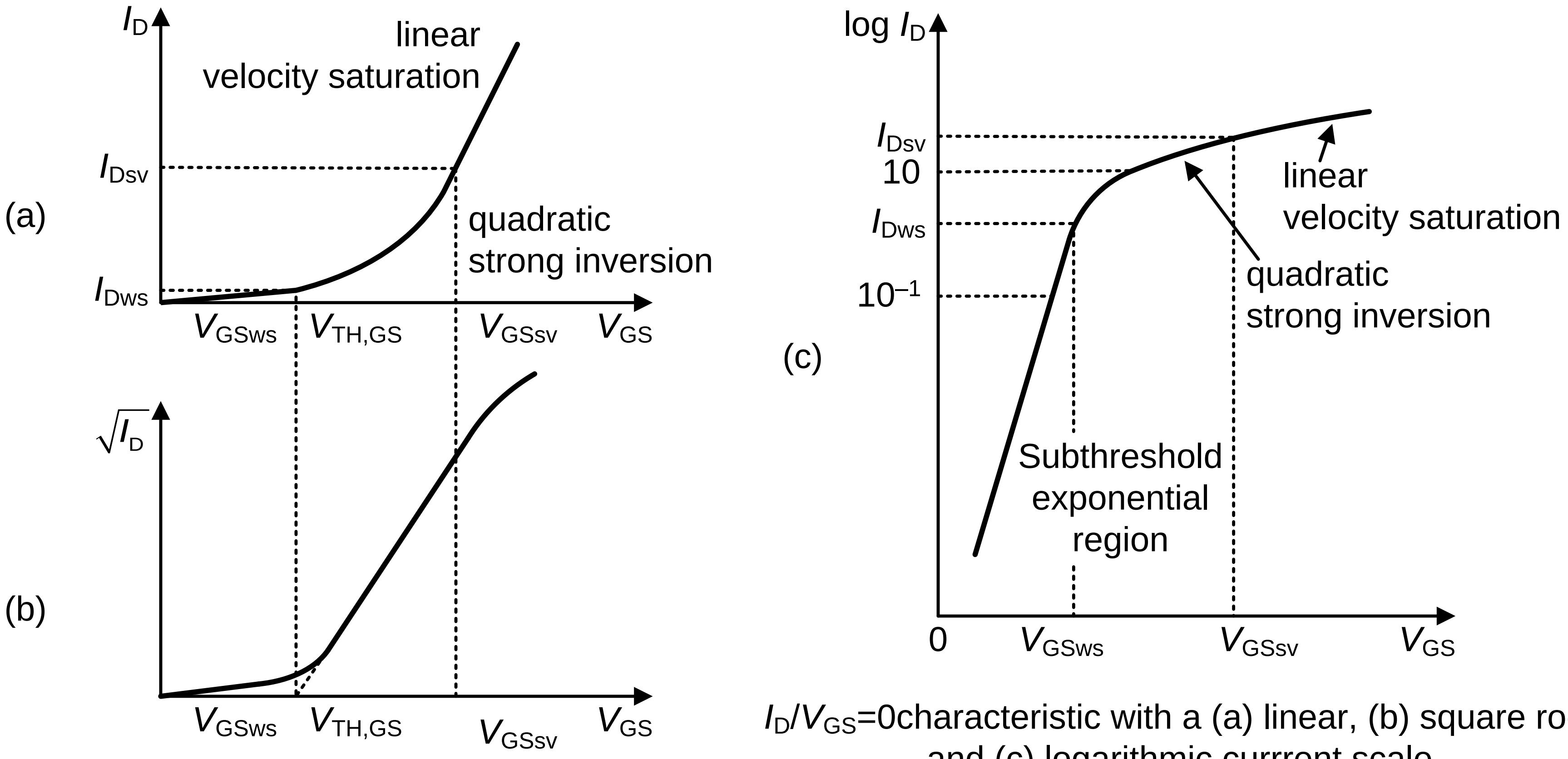
三个工作区域的*I-V*曲线



I_D/V_{GS}=0characteristic with a (a) linear, (b) square root, and (c) logarithmic currrent scale.

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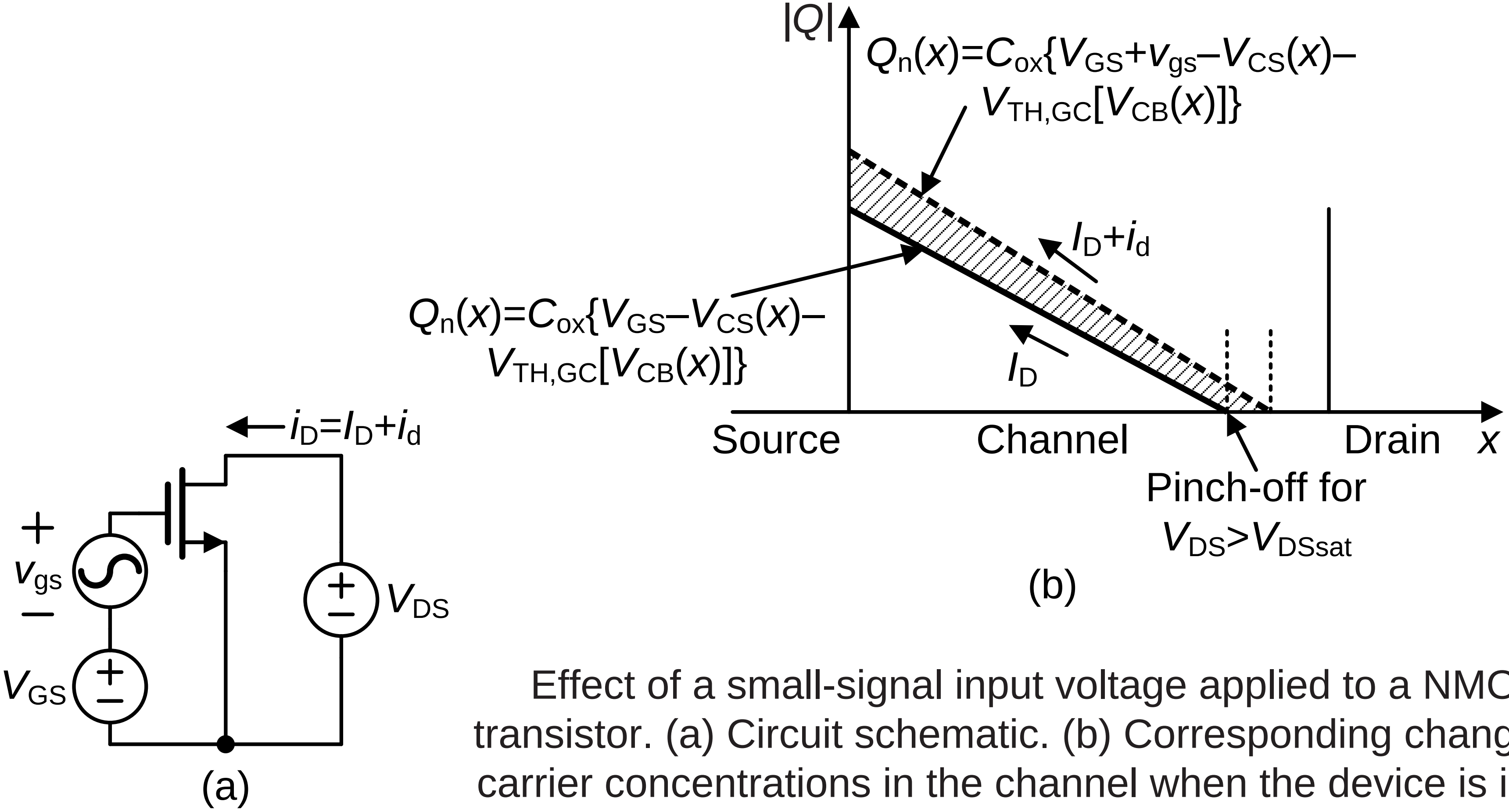
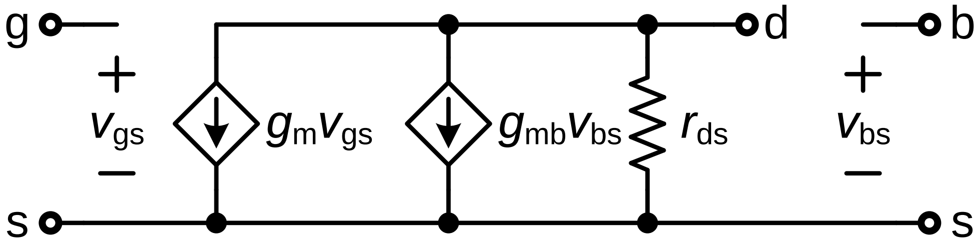
小信号模型



Effect of a small-signal input voltage applied to a NMOS transistor. (a) Circuit schematic. (b) Corresponding changes in carrier concentrations in the channel when the device is in the active region.

低频小信号模型：栅跨导(1)



The low-frequency, small-signal model for an active MOS transistor.

栅跨导：$g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

线性区栅跨导$g_{m,tri}$：

$$\boxed{g_{m,tri} = \mu_n C_{ox} \frac{W}{L} V_{DS}}$$

饱和区栅跨导$g_{m,si}$：

$$g_{m,si} = \frac{1}{n} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH,GS}) \left[1 + \lambda (V_{DS} - V_{DSsat})\right]$$

低频小信号模型：栅跨导(2)

若$\lambda(V_{DS}-V_{DSsat})<<1$

$$\boxed{g_{m,si}=\frac{1}{n}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH,GS})=\sqrt{\frac{2}{n}\mu_n C_{ox}\frac{W}{L}I_D}=\frac{2I_D}{V_{GS}-V_{TH,GS}}}$$

弱反型区栅跨导$g_{m,wi}$:

$$\boxed{g_{m,wi}=\frac{W}{L}\frac{I_t}{nV_T}e^{\frac{V_{GS}-V_{TH,GS}}{nV_T}}(1-e^{-\frac{V_{DS}}{V_T}})=\frac{I_D}{nV_T}=\frac{I_D}{V_T}\frac{C_{ox}}{C_j+C_{ox}}}$$

速度饱和区栅跨导$g_{m,vs}$：

$$\boxed{g_{m,vs}=WC_{ox}v_{scl}}$$

体跨导：(1)

$$g_{\mathrm{mb}}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{BS}}}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{TH,GS}}}\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}}=-\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{GS}}}\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}}=g_{\mathrm{m}}(n-1)$$

$$\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}}=-\frac{\gamma}{2\sqrt{|2\Phi_{\mathrm{F}}|+V_{\mathrm{SB}}}}=-(n-1)$$

线性区体跨导$g_{\mathrm{mb,tri}}$：

$$g_{\mathrm{mb,tri}}=-\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}V_{\mathrm{DS}}\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}}=(n-1)\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}V_{\mathrm{DS}}=(n-1)g_{\mathrm{m,tri}}$$

饱和区体跨导$g_{\mathrm{mb,si}}$

$$g_{\mathrm{mb,si}}=-\frac{1}{n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})\left[1+\lambda(V_{\mathrm{DS}}-V_{\mathrm{DSsat}})\right]\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}}$$
$$=(n-1)g_{\mathrm{m,si}}$$

体跨导：(2)

弱反型区体跨导$g_{\mathrm{mb,wi}}$：

$$g_{\mathrm{mb,wi}} = -\frac{1}{nV_{\mathrm{T}}}\frac{W}{L}I_{\mathrm{t}}\mathrm{e}^{\frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{nV_{\mathrm{T}}}}(1-\mathrm{e}^{-\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}})\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}} = \frac{(n-1)I_{\mathrm{D}}}{nV_{\mathrm{T}}}$$

$$= (n-1)g_{\mathrm{m,wi}}$$

速度饱和区体跨导$g_{\mathrm{mb,vs}}$

$$g_{\mathrm{mb,vs}} = -WC_{\mathrm{ox}}v_{\mathrm{scl}}\frac{\partial V_{\mathrm{TH,GS}}}{\partial V_{\mathrm{BS}}} = (n-1)g_{\mathrm{m,vs}}$$

输出阻抗

$$r_{\mathrm{ds}} = \frac{\partial V_{\mathrm{DS}}}{\partial I_{\mathrm{D}}} = \left(\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{DS}}} \right)^{-1}$$

线性区输出阻抗$r_{\mathrm{ds,tri}}$： $$r_{\mathrm{ds,tri}} = \frac{1}{\mu_{\mathrm{n}} C_{\mathrm{ox}} \frac{W}{L} (V_{\mathrm{GS}} - V_{\mathrm{TH,GS}} - nV_{\mathrm{DS}})}$$

饱和区输出阻抗$r_{\mathrm{ds,si}}$： $$r_{\mathrm{ds,si}} \approx \frac{V_{\mathrm{A}}}{I_{\mathrm{D}}} = \frac{1}{\lambda I_{\mathrm{D}}} = \frac{V_{\mathrm{E}} L}{I_{\mathrm{D}}}$$

弱反型区输出阻抗$r_{\mathrm{ds,wi}}$：

$$r_{\mathrm{ds,wi}} = \frac{V_{\mathrm{T}}}{\frac{W}{L} I_{\mathrm{t}} \mathrm{e}^{\frac{V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}}{nV_{\mathrm{T}}}} \mathrm{e}^{-\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}}} = \frac{V_{\mathrm{T}}}{I_{\mathrm{D}}} \left(\mathrm{e}^{\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}} - 1 \right)$$

速度饱和区输出阻抗$r_{\mathrm{ds,vs}}$： $$\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{DS}}} = 0 \qquad r_{\mathrm{ds,vs}} \to \infty$$

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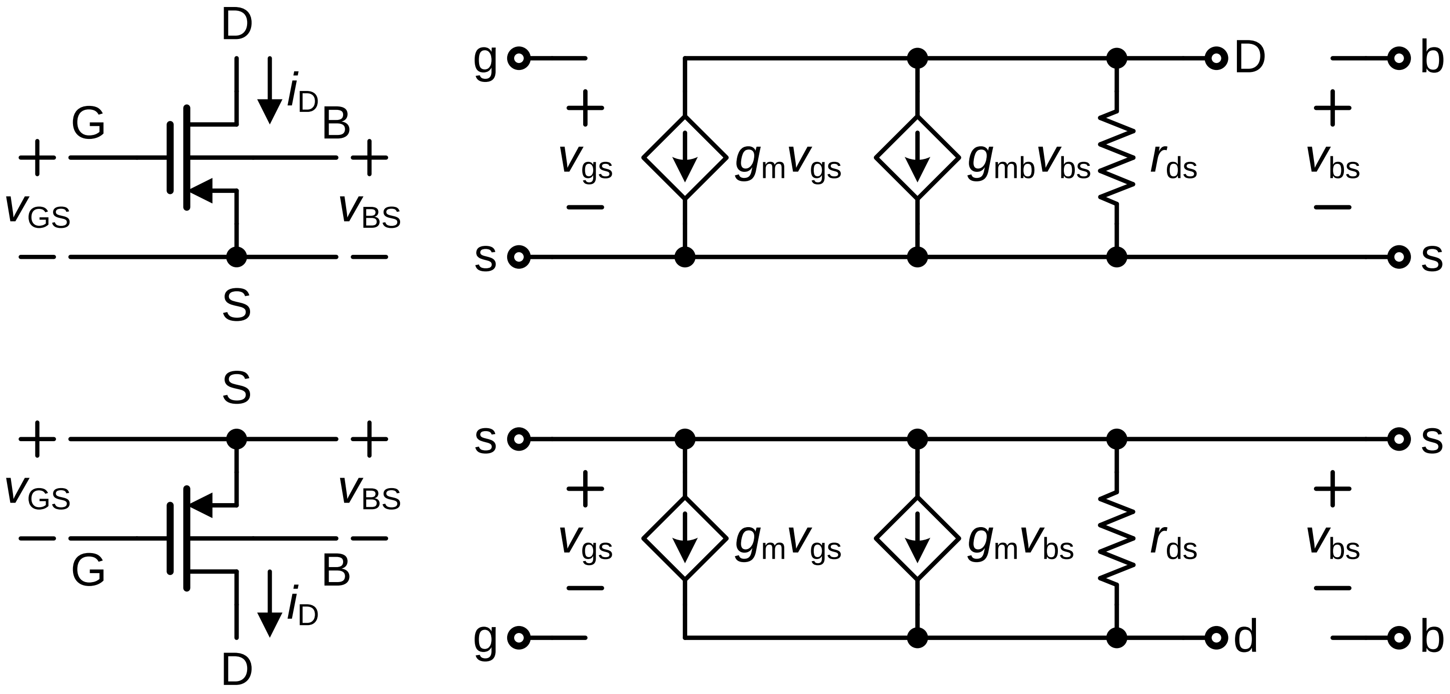
PMOST的小信号模型



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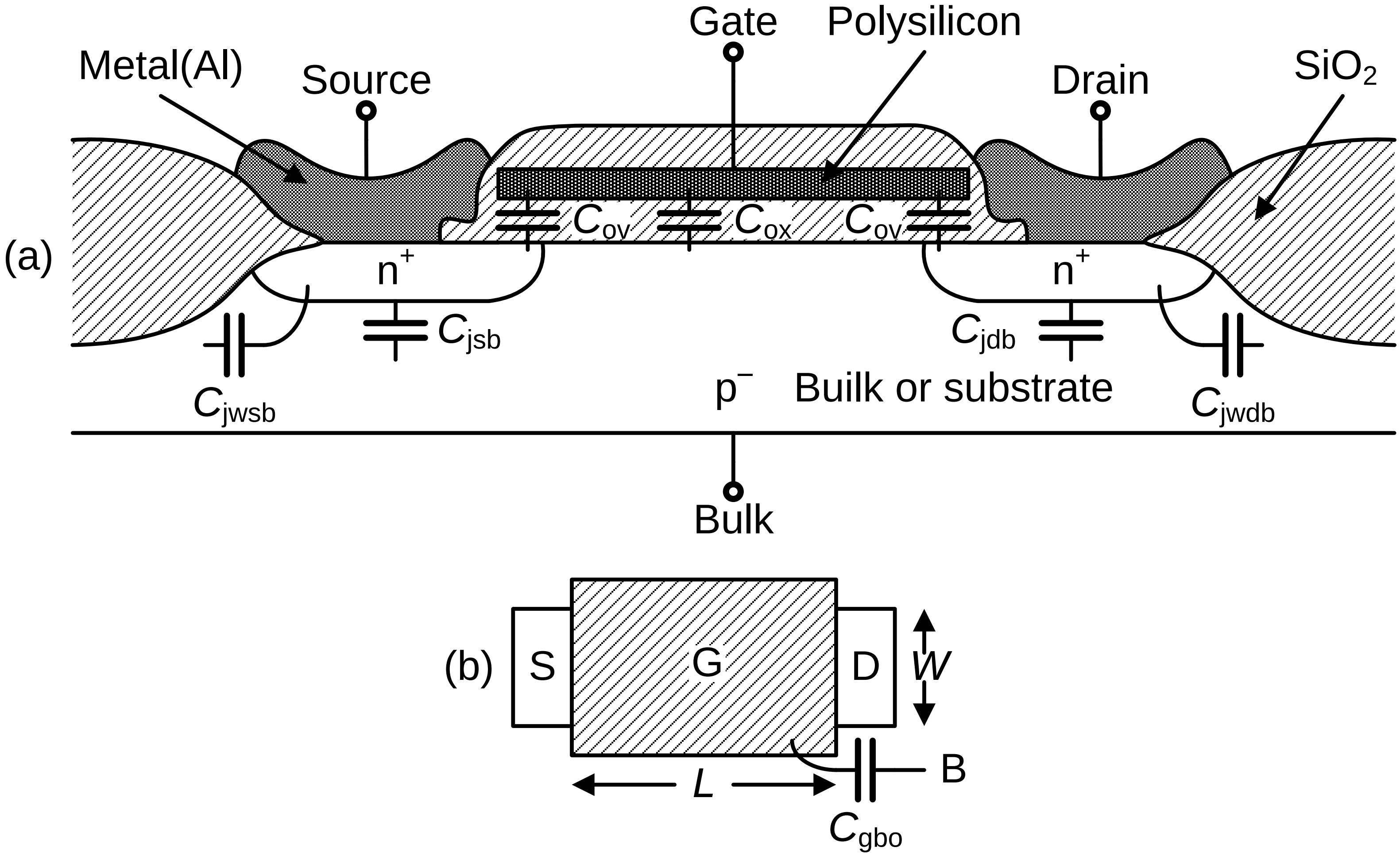
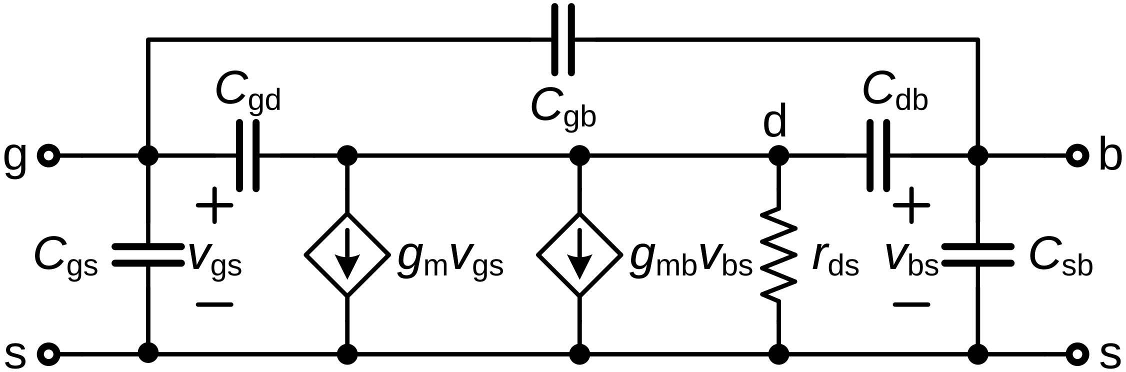
高频小信号模型：(1)



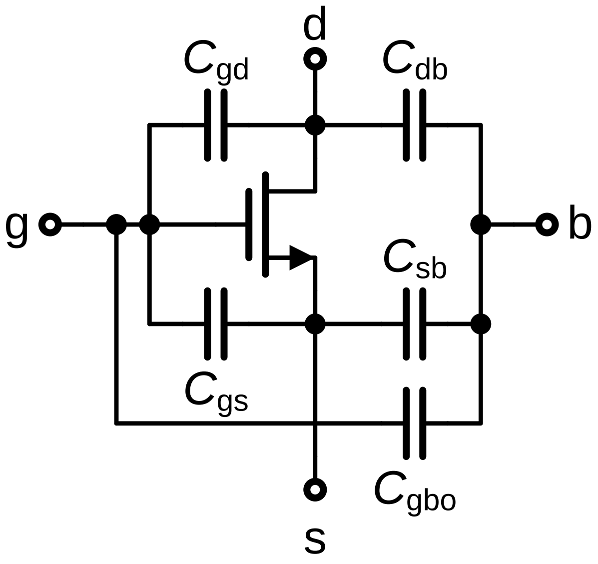
(a) A cross section of an n-channel MOS transistor showing the small-signal capacitances.(b) Top view of an NMOS

高频小信号模型：(2)



The high-frequency, small-signal model for a MOS transistor.

栅寄生电容C_{gs}和C_{gd}：(1)



Terminal capacitance in a NMOS

线性区：

$$C_{gs}=C_{gd}=\frac{C_{ox}WL}{2}+C_{ov}$$

其中C_{ov}为栅与源(漏)的交叠电容，

$$C_{ov}=WL_{ov}C_{ox}$$

弱反型区：

$$C_{gs}=C_{gd}=C_{ov}=WL_{ov}C_{ox}$$

交叠电容： $C_{gbo}=2W_{ov}LC_{ox}$

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栅寄生电容C_{gs}和C_{gd}：(2)

饱和区和速度饱和区，沟道中的总电荷和沟道电流：

$$Q_{\mathrm{T}}=WC_{\mathrm{ox}}\int_0^L\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}\mathrm{d}x$$

$$I_{\mathrm{D}}=W\mu_{\mathrm{n}}C_{\mathrm{ox}}[V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)]\frac{\mathrm{d}V_{\mathrm{CS}}(x)}{\mathrm{d}x}$$

$$Q_{\mathrm{T}}=\frac{W^2C_{\mathrm{ox}}^2\mu_{\mathrm{n}}}{I_{\mathrm{D}}}\int_0^{\frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{n}}\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}^2\mathrm{d}V_{\mathrm{CS}}(x)$$

$$=\frac{2}{3}WLC_{\mathrm{ox}}(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})$$

$$\boxed{C_{\mathrm{gs}}^{'}=\frac{\partial Q_{\mathrm{T}}}{\partial V_{\mathrm{GS}}}=\frac{2}{3}WLC_{\mathrm{ox}}}$$

$$C_{\mathrm{gs}}=C_{\mathrm{gs}}^{'}+C_{\mathrm{ov}}=\frac{2}{3}C_{\mathrm{ox}}WL+C_{\mathrm{ov}}\qquad C_{\mathrm{gd}}=C_{\mathrm{ov}}$$

源/漏寄生结电容

$$C_{\mathrm{jsbt}} = A_{\mathrm{S}} C_{\mathrm{jsb}} + P_{\mathrm{S}} C_{\mathrm{jwsb}} \qquad C_{\mathrm{jdbt}} = A_{\mathrm{D}} C_{\mathrm{jdb}} + P_{\mathrm{D}} C_{\mathrm{jwdb}}$$

其中A_{S}和A_{D}分别为源和漏的底部面积，P_{S}和P_{D}分别为源和漏的侧壁周长。

源衬和漏衬的面结电容：

$$C_{\mathrm{jsb}} = \frac{C_{\mathrm{j}}}{\sqrt{1 + \dfrac{V_{\mathrm{sb}}}{\Phi_0}}} \qquad C_{\mathrm{jdb}} = \frac{C_{\mathrm{j}}}{\sqrt{1 + \dfrac{V_{\mathrm{db}}}{\Phi_0}}}$$

源衬和漏衬的边墙结电容：

其中m=1/3~1/2

$$C_{\mathrm{jwsb}} = \frac{C_{\mathrm{jw}}}{\left(1 + \dfrac{V_{\mathrm{sb}}}{\Phi_0}\right)^m} \qquad C_{\mathrm{jwdb}} = \frac{C_{\mathrm{jw}}}{\left(1 + \dfrac{V_{\mathrm{db}}}{\Phi_0}\right)^m}$$

背栅寄生电容C_{bs}和C_{bd}：(1)

线性区：

$$C_{sb}=\frac{1}{2}C_{bc}+C_{jsbt}\qquad C_{db}=\frac{1}{2}C_{bc}+C_{jdbt}$$

饱和区和速度饱和区：

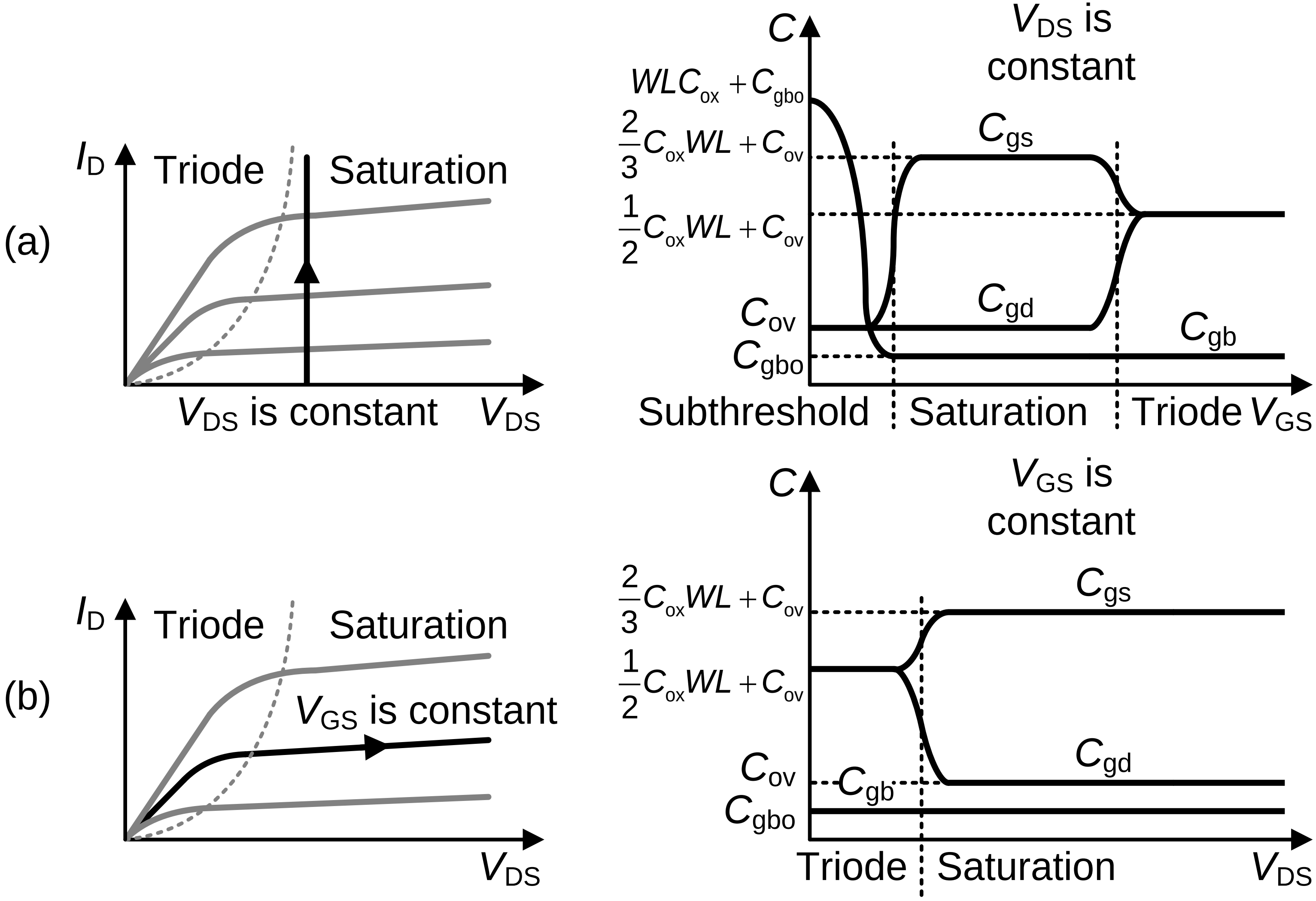
$$C_{sb}=\frac{2}{3}C_{bc}+C_{jsbt}\qquad C_{db}=C_{jdbt}$$

弱反型区：

$$C_{sb}=C_{jsbt}\qquad C_{db}=C_{jdbt}$$

$$C_{gb}=\frac{C_jC_{ox}}{C_j+C_{ox}}+C_{gbo}$$

寄生电容与V_{GS}和V_{DS}的关系



Voltage dependence of C_{gs} and C_{gd} as a function of V_{GS} and V_{DS}.

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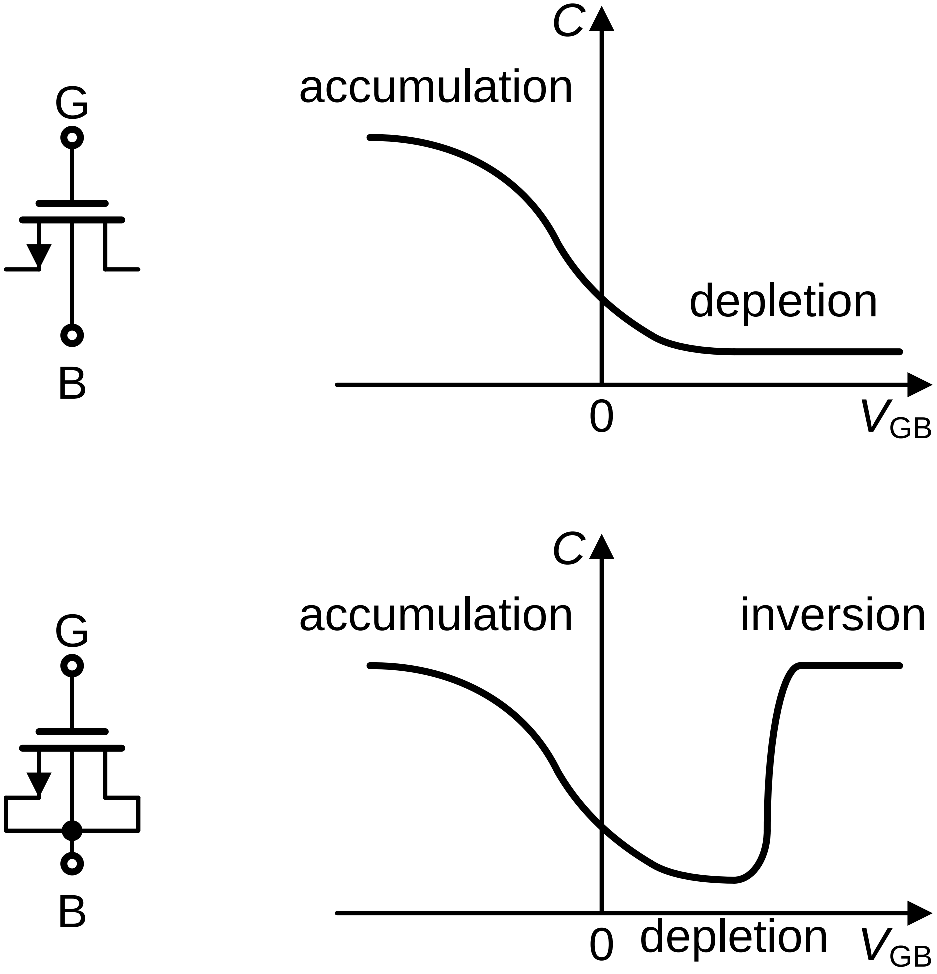
NMOS管可变电容



NMOS varactor

模拟评价指标：固有增益

$$A_\mathrm{i} = g_\mathrm{m} r_\mathrm{ds}$$

线性区：

$$A_\mathrm{i} = \frac{V_\mathrm{DS}}{V_\mathrm{GS} - V_\mathrm{TH,GS} - nV_\mathrm{DS}}$$

饱和区：

$$A_\mathrm{i} = \frac{2I_\mathrm{D}}{V_\mathrm{GS} - V_\mathrm{TH,GS}} \frac{V_\mathrm{E} L}{I_\mathrm{D}} = \frac{2V_\mathrm{E} L}{(V_\mathrm{GS} - V_\mathrm{TH,GS})}$$

弱反型区：

$$A_\mathrm{i} = \frac{I_\mathrm{D}}{nV_\mathrm{T}} \frac{V_\mathrm{T}}{I_\mathrm{D}} (\mathrm{e}^{\frac{V_\mathrm{DS}}{V_\mathrm{T}}} - 1) = \frac{\mathrm{e}^{\frac{V_\mathrm{DS}}{V_\mathrm{T}}} - 1}{n}$$

速度饱和区：$A_\mathrm{i} \to \infty$!?

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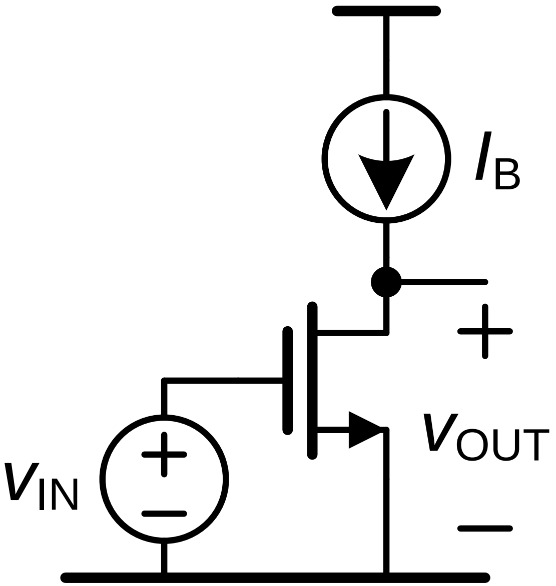
单MOS管增益A_V



$$A_V = g_m r_{ds} = \frac{2I_D}{V_{GS}-V_{TH,GS}} \cdot \frac{V_E L}{I_D} = \frac{2V_E L}{V_{GS}-V_{TH,GS}}$$

如果$V_{GS}-V_{TH,GS}=0.2\ \mathrm{V}$、$V_E L\approx 10\ \mathrm{V}$，则$A_V\approx 100$。

高增益设计

	高增益	高速
$V_{GS}-V_{TH,GS}$	低(0.2 V)	
L	大	

$V_{GS}-V_{TH,GS}$ 决定 g_m/I_D 值，能效比!

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例题7：单管放大器

用三个单管串联的结构，实现总增益为10,000的三级放大器。$V_{GS} - V_{TH,GS} = 0.2\ \mathrm{V}$。

使用先进的65 nm CMOS工艺($V_{En} = 4\ \mathrm{V/\mu m}$)，求最小栅长。

模拟评价指标：跨导电流比g_m/I_D

线性区：

$$\frac{g_\mathrm{m}}{I_\mathrm{D}}=\frac{1}{V_\mathrm{GS}-V_\mathrm{TH,GS}-\frac{n}{2}V_{DS}}$$

饱和区：

$$\frac{g_\mathrm{m}}{I_\mathrm{D}}=\frac{2}{V_\mathrm{GS}-V_\mathrm{TH,GS}}$$

弱反型区：

$$\frac{g_\mathrm{m}}{I_\mathrm{D}}=\frac{1}{nV_\mathrm{T}}$$

速度饱和区：

$$\frac{g_\mathrm{m}}{I_\mathrm{D}}=\frac{1}{V_\mathrm{GS}-V_\mathrm{TH,GS}}$$

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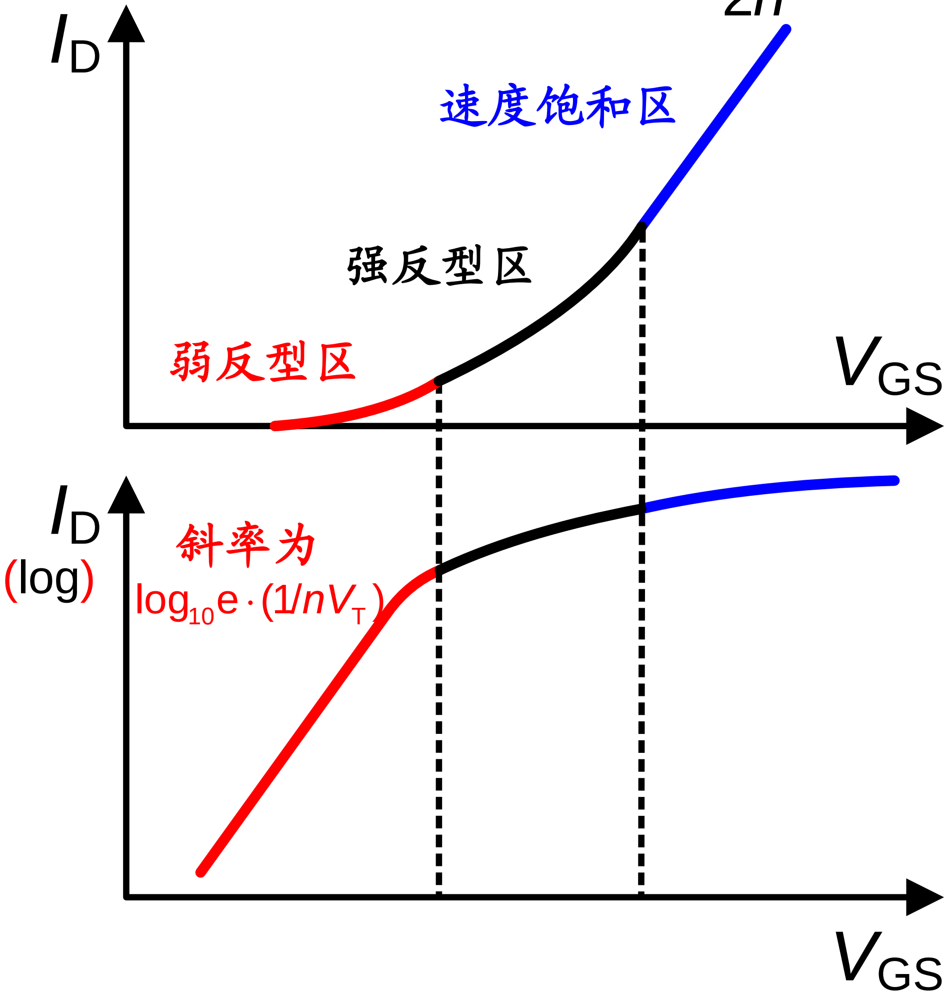
I_D与V_{GS}的关系：尺寸W/L固定

$$I_{D,si}=\frac{1}{2n}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH,GS}\right)^2\left[1+\lambda\left(V_{DS}-V_{DSsat}\right)\right]$$



$$I_{D,wi}=I_t\frac{W}{L}e^{\frac{V_{GS}-V_{TH,GS}}{nV_T}}\left(1-e^{-\frac{V_{DS}}{V_T}}\right)$$

$$I_{D,vs}=C_{ox}W\left(V_{GS}-V_{TH,GS}\right)v_{scl}$$

$$k=1.38\times10^{-23}\ \text{J/K}$$

$$q=1.6\times10^{-19}\ \text{C}$$

$$V_T=kT/q=26\ \text{mV}@T=300\ \text{K}$$

$$v_{scl}\approx10^7\ \text{cm/s}$$

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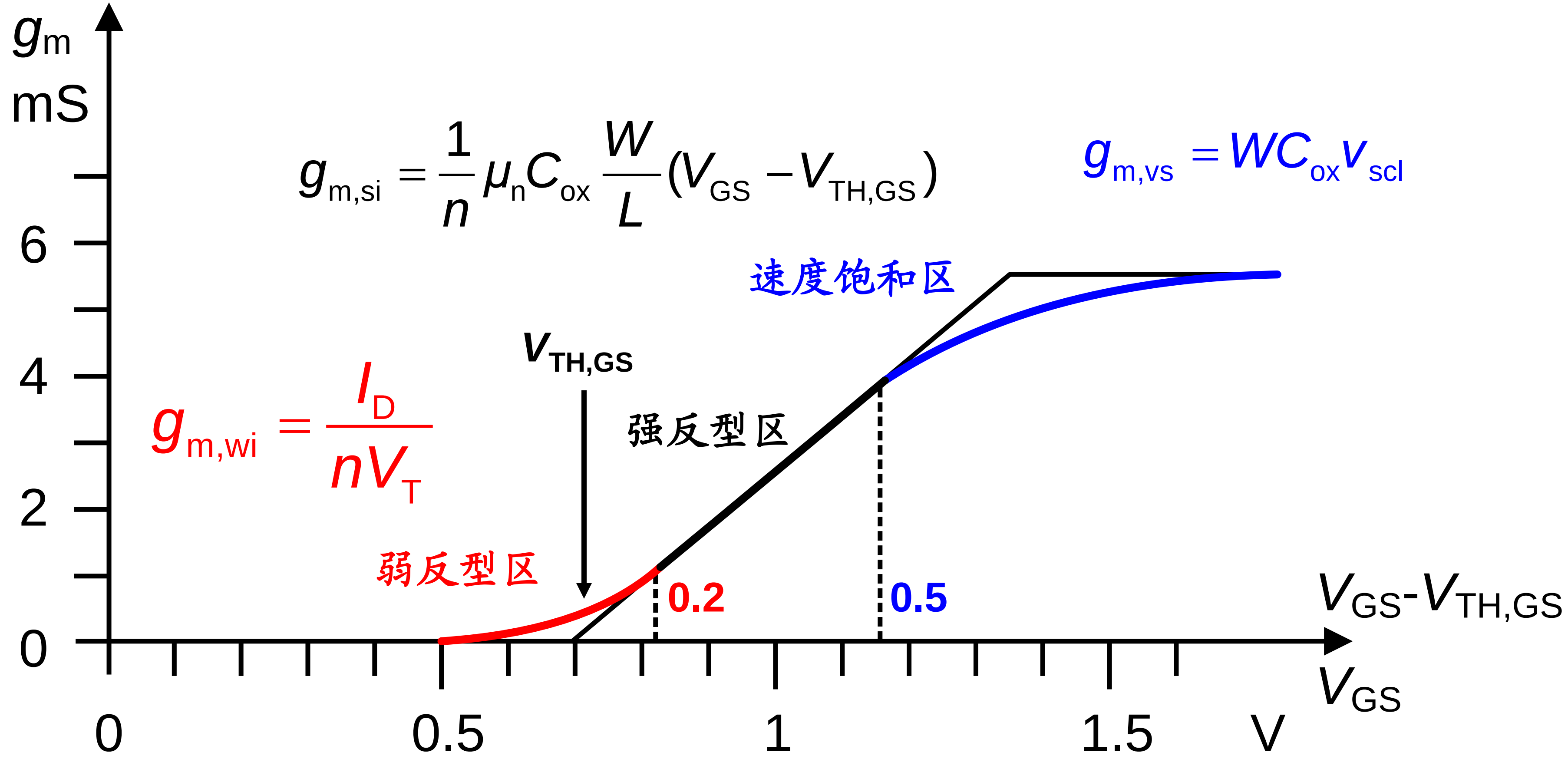
跨导g_m与V_{GS}的关系：尺寸*W*/*L*固定



$$g_{m,si}=\frac{1}{n}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH,GS})$$

$$g_{m,vs}=WC_{ox}v_{scl}$$

$$g_{m,wi}=\frac{I_D}{nV_T}$$

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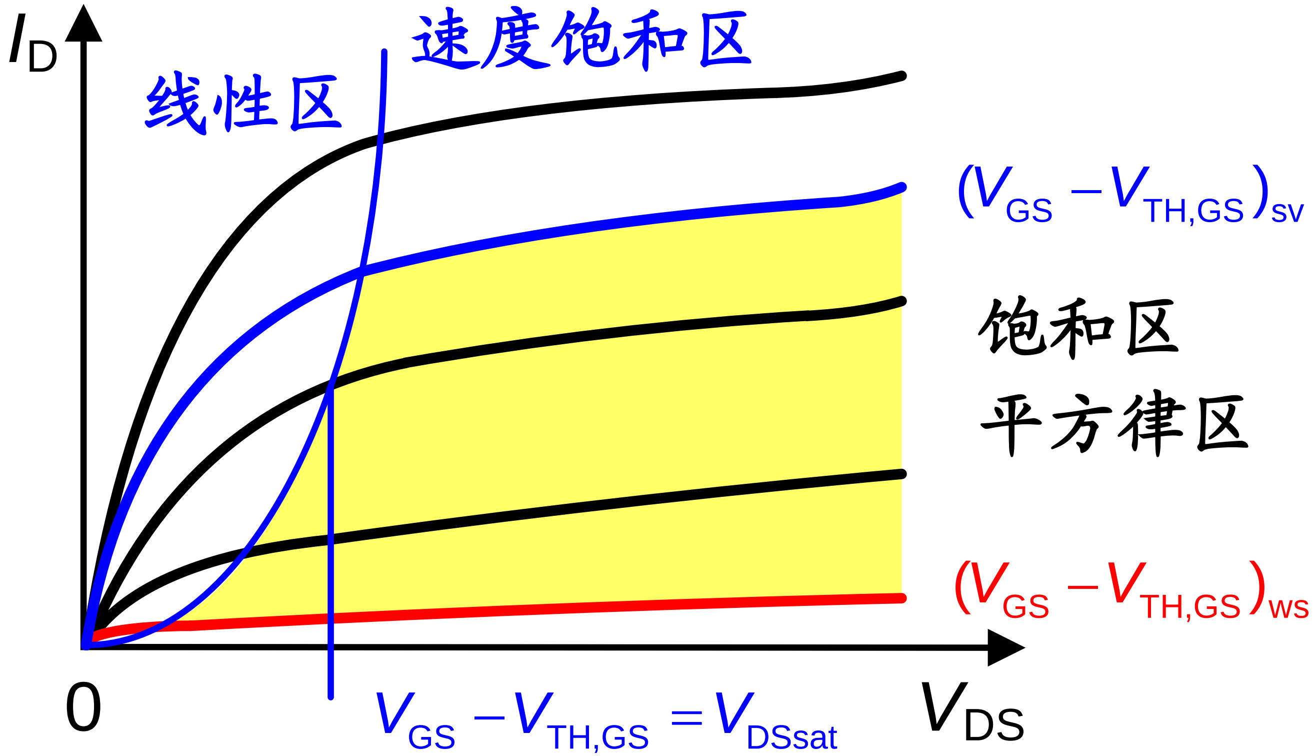
饱和区和速度饱和区



wi与si转换点电压$V_{\mathrm{GST,ws}}$

$$I_{\mathrm{D,wi}} = I_{\mathrm{t}} \frac{W}{L} e^{\frac{V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}}{nV_{\mathrm{T}}}} \left(1 - e^{-\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}}\right)$$

$$I_{\mathrm{D,si}} = \frac{1}{2n} \mu_{\mathrm{n}} C_{\mathrm{ox}} \frac{W}{L} (V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})^2$$

$$g_{\mathrm{m,wi}} = \frac{I_{\mathrm{D,wi}}}{nV_{\mathrm{T}}}$$

$$g_{\mathrm{m,si}} = \frac{2I_{\mathrm{D,si}}}{V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}}$$

$$\frac{g_{\mathrm{m,wi}}}{I_{\mathrm{D,wi}}} = \frac{1}{nV_{\mathrm{T}}}$$

$$\frac{g_{\mathrm{m,si}}}{I_{\mathrm{D,si}}} = \frac{2}{V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}}$$

$$\boxed{(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})_{\mathrm{ws}} = 2nV_{\mathrm{T}}}$$

转换点电压$V_{\mathrm{GST,ws}}$：与L无关

$$\boxed{(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})_{\mathrm{ws}}=2nV_{\mathrm{T}}}$$

$$I_{\mathrm{Dws}}=\frac{1}{2n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\frac{W}{L}(2nV_{\mathrm{T}})^2$$

$$(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})_{\mathrm{ws}}=2nV_{\mathrm{T}}\approx 70\ \mathrm{mV}$$

$$\frac{1}{2n}\mu_{\mathrm{n}}C_{\mathrm{ox}}\approx 100\ \mu\mathrm{A/V}^2$$

$$\frac{1}{2n}\mu_{\mathrm{p}}C_{\mathrm{ox}}\approx 40\ \mu\mathrm{A/V}^2$$

与沟道长度L无关，

很长时间内仍然遵循这一规律！

例如，当$\frac{W}{L}=10$时，NMOS的$I_{\mathrm{D,ws}}\approx 5\ \mu\mathrm{A}$；

PMOS的$I_{\mathrm{D,ws}}\approx 2\ \mu\mathrm{A}$。

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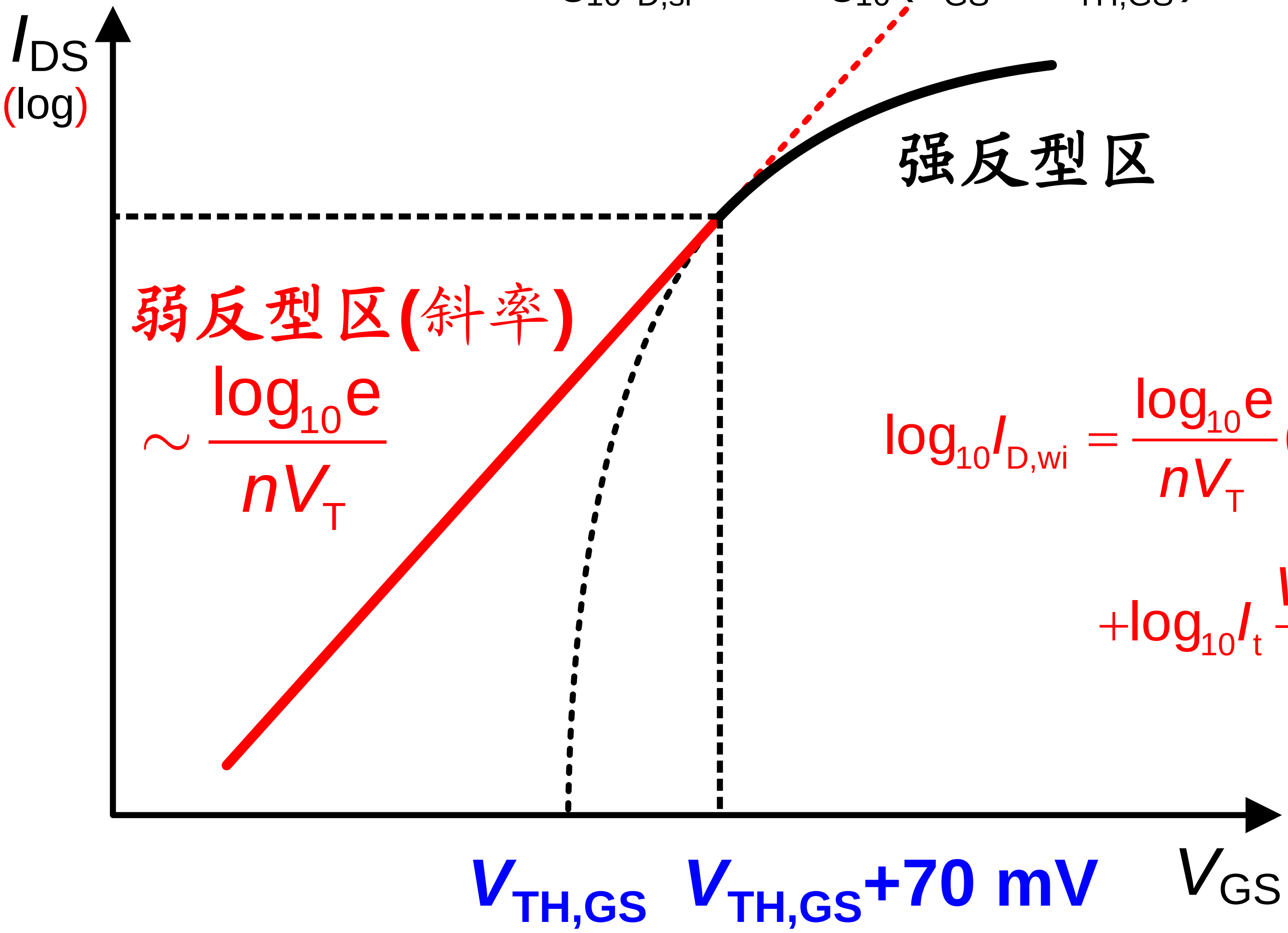
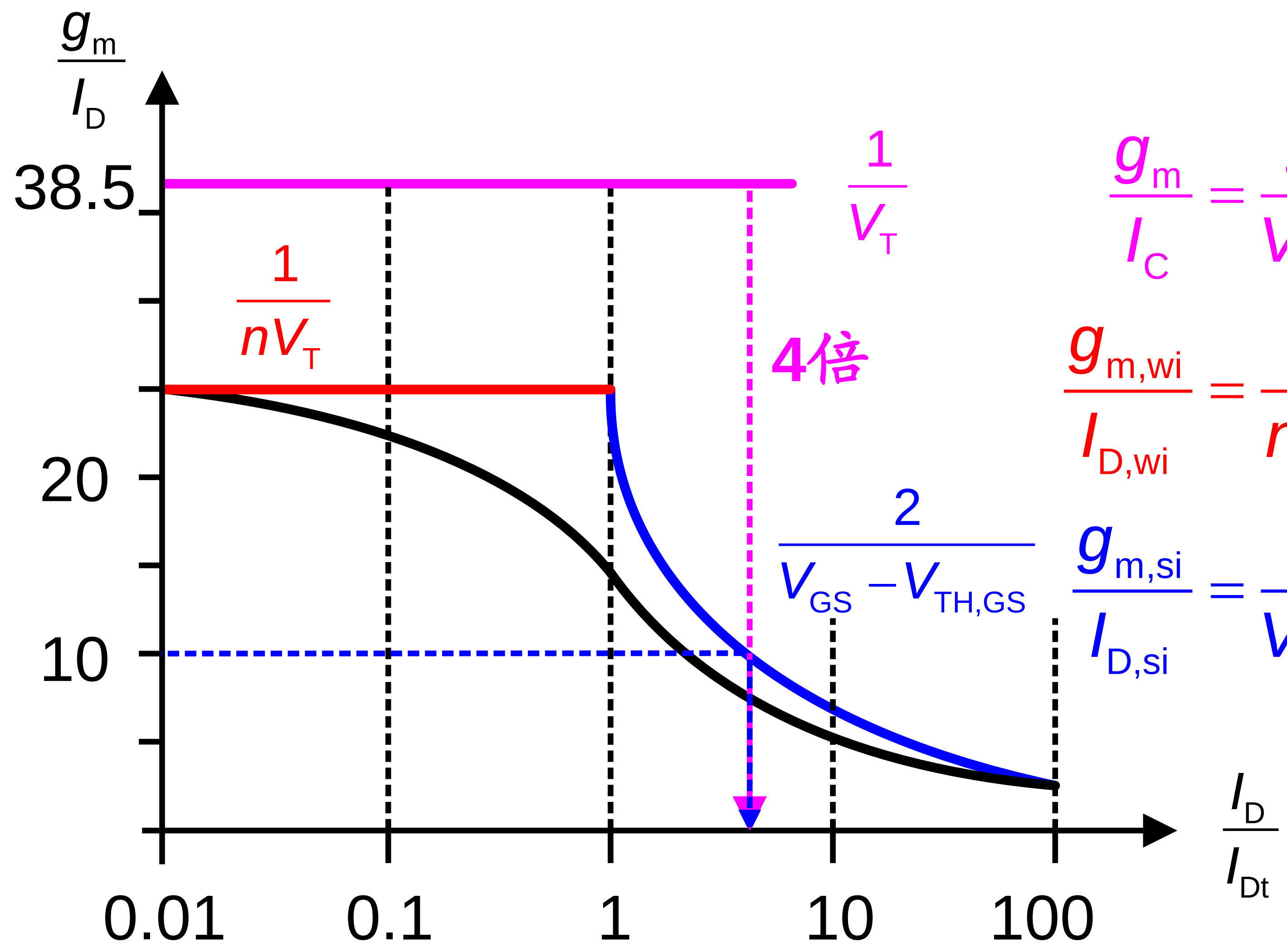
wi与si的转换点

$$\log_{10} I_{\mathrm{D,si}} = 2\log_{10}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}) + \log_{10}\frac{1}{2n}\mu_{\mathrm{n}} C_{\mathrm{ox}}\frac{W}{L}$$



$$\log_{10} I_{\mathrm{D,wi}} = \frac{\log_{10} e}{nV_{\mathrm{T}}}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}) + \log_{10} I_{\mathrm{t}}\frac{W}{L}\left(1 - e^{-\frac{V_{\mathrm{DS}}}{V_{\mathrm{T}}}}\right)$$

wi与si转换点的能效比g_m/I_D



$$\frac{g_m}{I_C}=\frac{1}{V_T}$$

$$\frac{g_{m,wi}}{I_{D,wi}}=\frac{1}{nV_T}$$

$$\frac{g_{m,si}}{I_{D,si}}=\frac{2}{V_{GS}-V_{TH,GS}}$$

wi与si转换点电流$I_{\mathrm{D,ws}}$

$$\boxed{I_{\mathrm{D,ws}} = K'\frac{W}{L}V_{\mathrm{GST,ws}}^2}$$

$$i = \frac{I_{\mathrm{D}}}{I_{\mathrm{D,ws}}} = [\ln(1+\mathrm{e}^{v})]^2$$ 反型系数

$$v = \ln(\mathrm{e}^{\sqrt{i}} - 1)$$

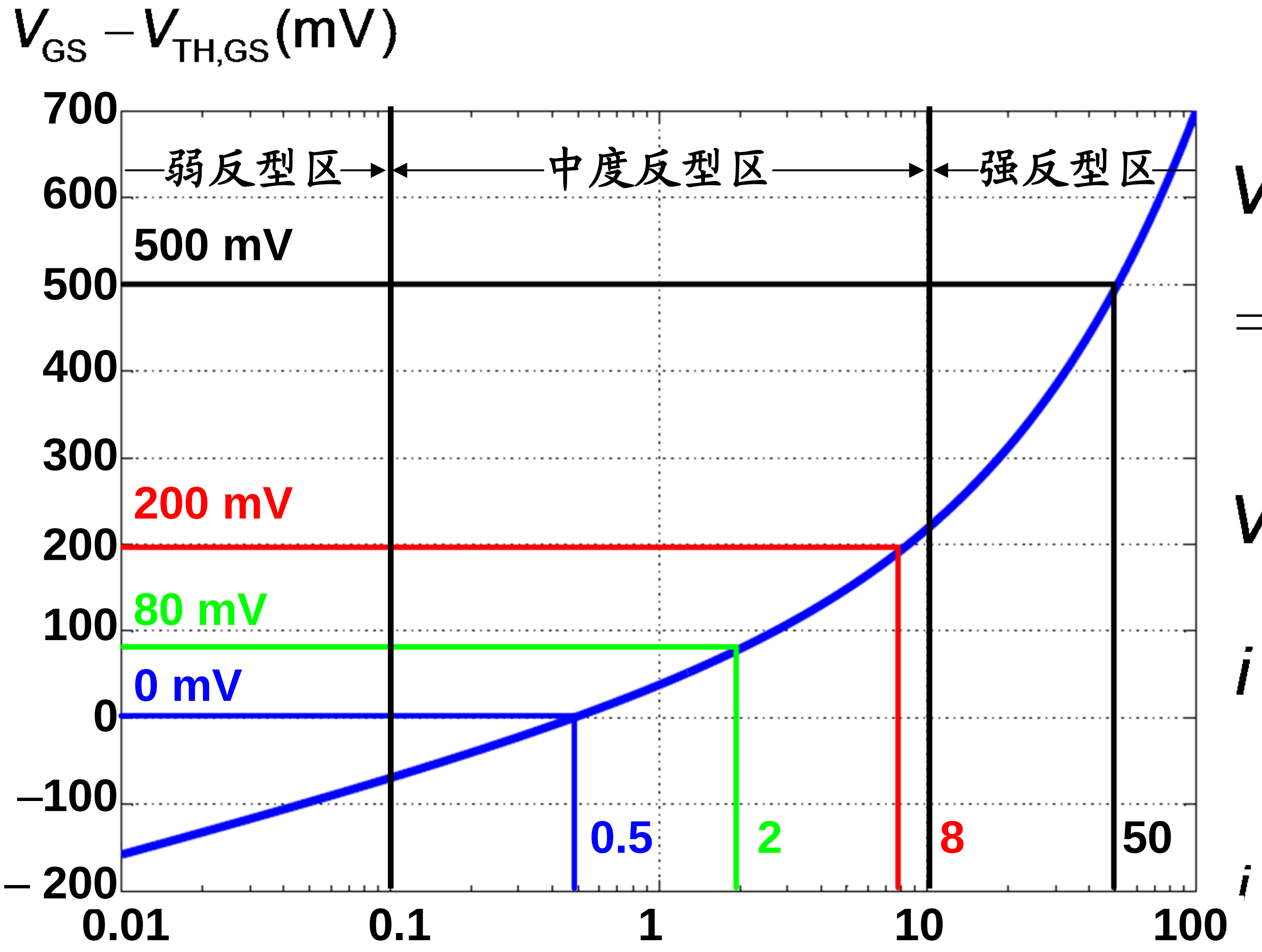
$$V_{\mathrm{GS}} - V_{\mathrm{TH,GS}} = V_{\mathrm{GST,ws}}\ln(\mathrm{e}^{\sqrt{i}} - 1) \qquad V_{\mathrm{GST,ws}} = 2nV_{\mathrm{T}} \approx 70\ \mathrm{mV}$$

$$i = 1 \quad v = \ln(\mathrm{e}^{\sqrt{1}} - 1) = 0.54 \qquad V_{\mathrm{GS}} - V_{\mathrm{TH,GS}} \approx 38\ \mathrm{mV}$$

$$v = 1 \quad i = [\ln(1+\mathrm{e}^{1})]^2 = 1.72$$

$$v = 0 \quad i = [\ln(1+\mathrm{e}^{0})]^2 = 0.48$$

$V_{GS}-V_{TH,GS}$与反型系数 i 的关系



$$V_{GS}-V_{TH,GS}$$

$$=V_{GST.ws}\ln(e^{\sqrt{i}}-1)$$

$$V_{GST,ws}=2nV_T$$

$$i=\frac{I_D}{I_{D,ws}}$$

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wi与si之间的跨导g_m：归一化GM

$$i=\frac{I_\mathrm{D}}{I_\mathrm{D,ws}}=[\ln(1+\mathrm{e}^{v})]^2 \qquad g_\mathrm{m}\approx\dots$$

$$\boxed{GM=\frac{\dfrac{g_\mathrm{m}}{I_\mathrm{D}}}{\dfrac{1}{nV_\mathrm{T}}}=\frac{g_\mathrm{m}}{I_\mathrm{DS}}nV_\mathrm{T}}$$

$$(1)GM=\frac{1-\mathrm{e}^{-\sqrt{i}}}{\sqrt{i}} \qquad 大\ i:GM=\frac{1}{\sqrt{i}} \quad 小\ i:GM=1-\frac{\sqrt{i}}{2}$$

$$(2)GM=\frac{1}{\sqrt{1+0.5\sqrt{i}+i}} \qquad 大\ i:GM=\frac{1}{\sqrt{i}} \quad 小\ i:GM=1-\frac{\sqrt{i}}{4}$$

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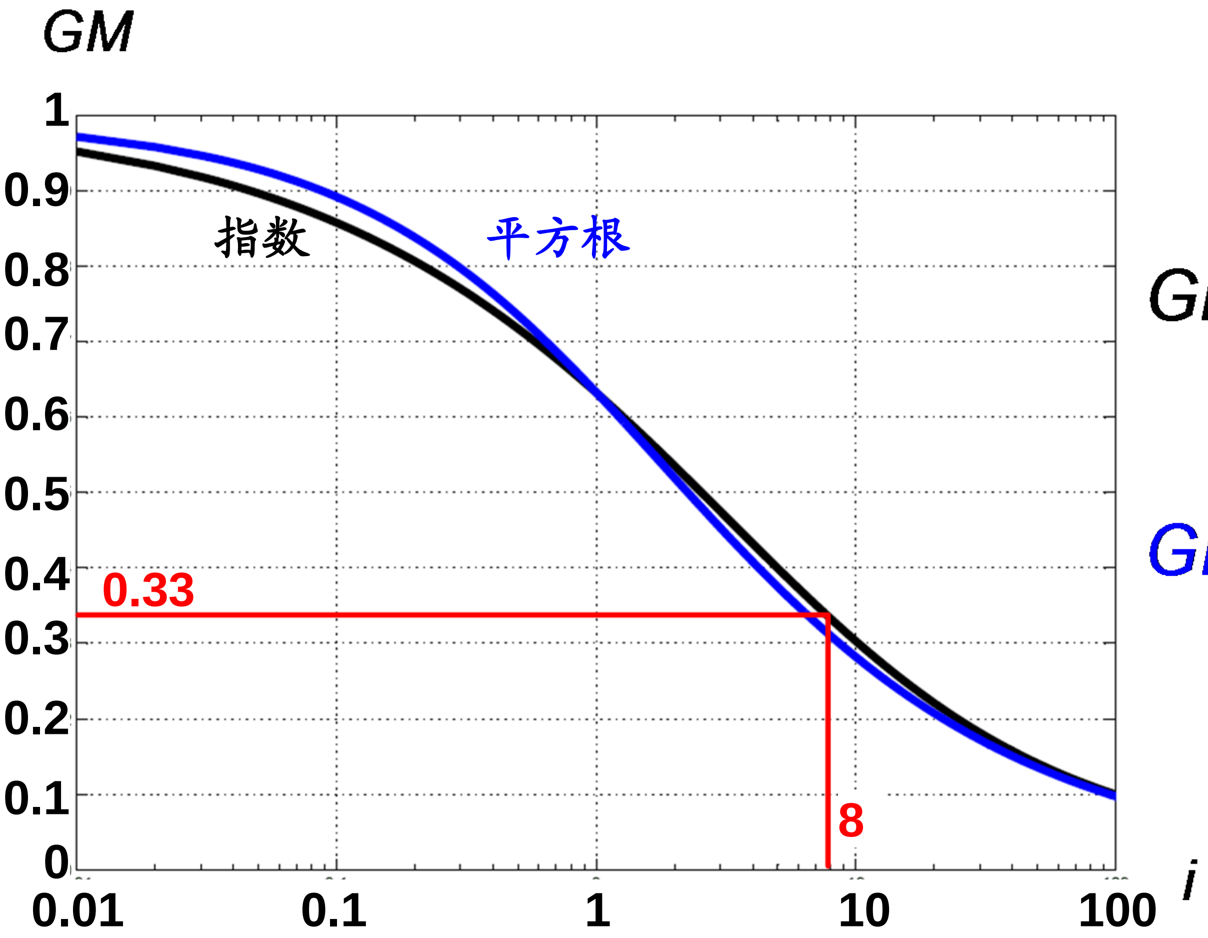
归一化跨导GM与反型系数 *i* 的关系



$$GM = \frac{g_\text{m}}{I_\text{D}} n V_\text{T}$$

$$GM = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

(指数)

$$GM = \frac{1}{\sqrt{1 + 0.5\sqrt{i} + i}}$$

(平方根)

$$i = \frac{I_\text{D}}{I_\text{D,ws}}$$

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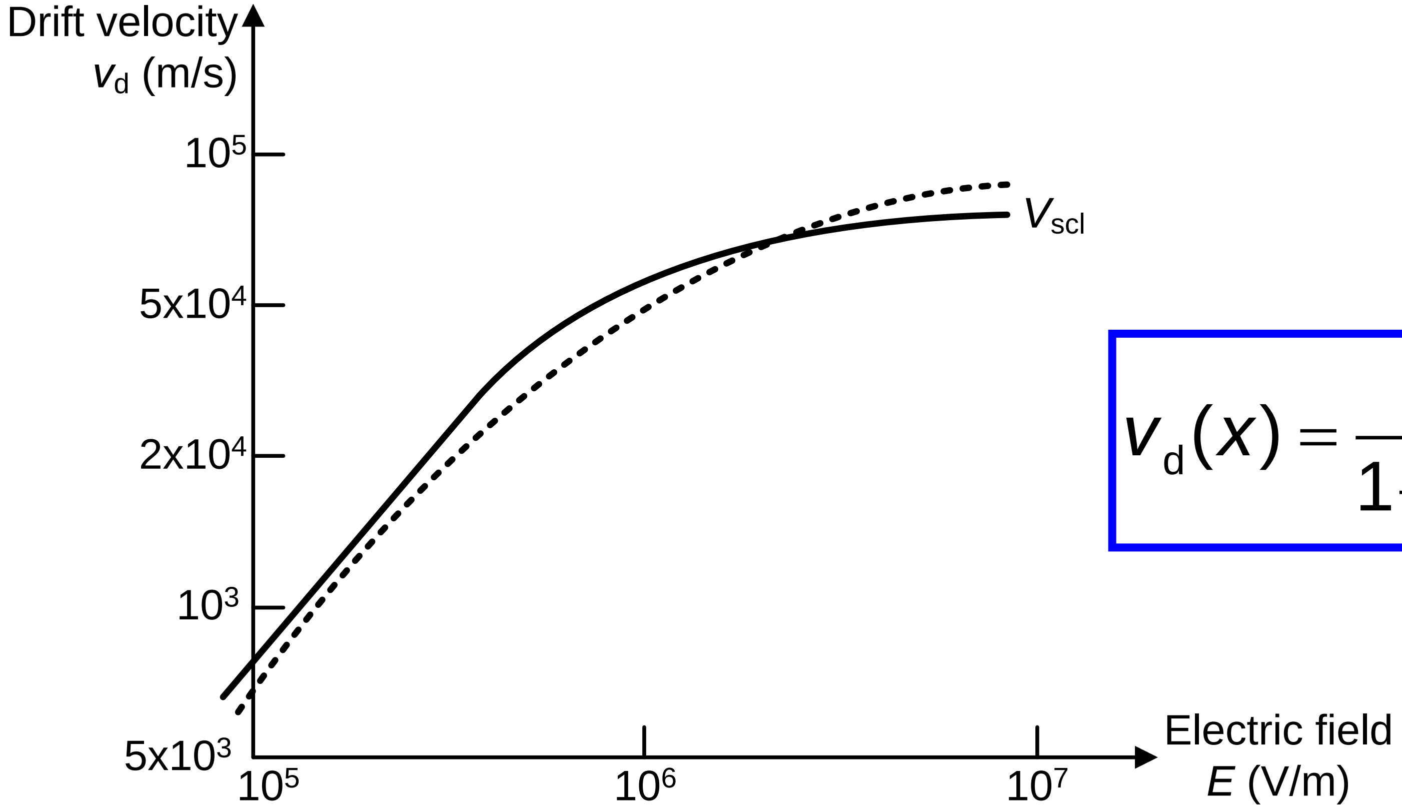
电子漂移速度与横向电场的关系



$$\boxed{v_d(x) = \frac{\mu_n E(x)}{1 + E(x)/E_{crit}}}$$

Typical measured electron drift velocity v_d versus horizontal electric field E in an MOS surface channel (solid plot). Also shown (dashed plot) is the analytical approximation with $E_{crit} = 1.5 \times 10^6$ V/m and $\mu_n = 0.07$ m^2/Vs.

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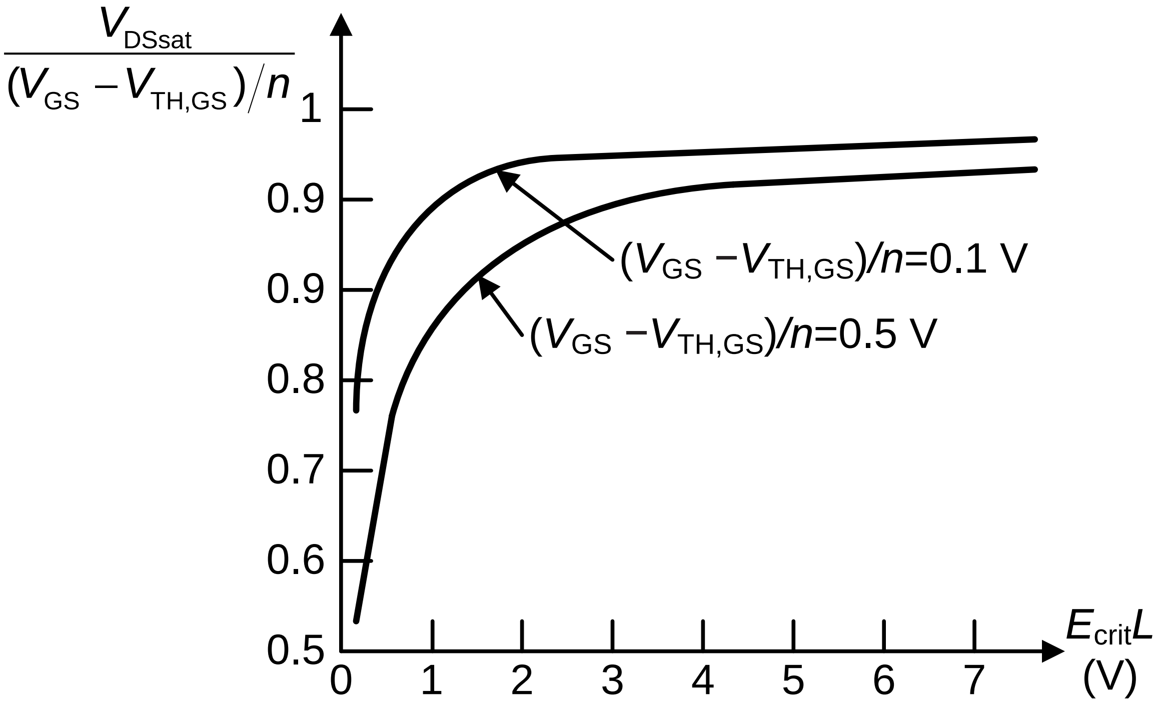
漏源饱和电压V_{DSsat}



Ratio of the minimum drain-source voltage required for operation in the active region to the overdrive versus the product of the critical field and the channel length. When $E_{crit}\to\infty$, velocity saturation is not a factor, and $V_{DSsat}\to(V_{GS}-V_{TH,GS})/n$, as expected. When velocity saturation is significant, $V_{DSsat}<(V_{GS}-V_{TH,GS})/n$.

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I-V关系式：(1)

单位面积的电荷密度：$Q_{\mathrm{n}}(x)=C_{\mathrm{ox}}\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}$

$$I_{\mathrm{D}}=WQ_{\mathrm{n}}(x)v_{\mathrm{d}}(x)\qquad v_{\mathrm{d}}(x)=\frac{\mu_{\mathrm{n}}E(x)}{1+E(x)/E_{\mathrm{crit}}}\qquad E(x)=-\frac{\mathrm{d}V_{\mathrm{CS}}(x)}{\mathrm{d}x}$$

$$I_{\mathrm{D}}\left(1+\frac{1}{E_{\mathrm{crit}}}\frac{\mathrm{d}V_{\mathrm{CS}}(x)}{\mathrm{d}x}\right)=W\mu_{\mathrm{n}}C_{\mathrm{ox}}\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}\frac{\mathrm{d}V_{\mathrm{CS}}(x)}{\mathrm{d}x}$$

$$\int_0^L I_{\mathrm{D}}\left(1+\frac{1}{E_{\mathrm{crit}}}\frac{\mathrm{d}V_{\mathrm{CS}}(x)}{\mathrm{d}x}\right)\mathrm{d}x=\int_0^{V_{\mathrm{DS}}}W\mu_{\mathrm{n}}C_{\mathrm{ox}}\{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}-nV_{\mathrm{CS}}(x)\}\mathrm{d}V_{\mathrm{CS}}(x)$$

$$\boxed{I_{\mathrm{D}}=\frac{\mu_{\mathrm{n}}C_{\mathrm{ox}}}{1+\dfrac{V_{\mathrm{DS}}}{E_{\mathrm{crit}}L}}\frac{W}{L}\left[(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})V_{\mathrm{DS}}-\frac{nV_{\mathrm{DS}}^2}{2}\right]}$$

饱和条件(1)

饱和条件:

$$\frac{\partial I_D}{\partial V_{DS}}=\mu_n C_{ox}\left[\frac{\left(1+\frac{V_{DS}}{E_{crit}L}\right)\left(V_{GS}-V_{TH,GS}-\frac{nV_{DS}}{2}\right)-\frac{(V_{GS}-V_{TH,GS})V_{DS}-\frac{nV_{DS}^2}{2}}{E_{crit}L}}{\left(1+\frac{V_{DS}}{E_{crit}L}\right)^2}\right]=0$$

$$\frac{nV_{DS}^2}{E_{crit}L}+2nV_{DS}-2(V_{GS}-V_{TH,GS})=0$$

$$V_{DSsat}=E_{crit}L\left(\sqrt{1+\frac{2(V_{GS}-V_{TH,GS})}{nE_{crit}L}}-1\right)$$

饱和条件(2)

$$x=\frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{nE_{\mathrm{crit}}L}\qquad \sqrt{1+2x}=1+x-\frac{x^2}{2}+\cdots$$

$$\boxed{V_{\mathrm{DSsat}}=\frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{n}\left(1-\frac{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}{2nE_{\mathrm{crit}}L}+\cdots\right)}$$

$$\boxed{\frac{2(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})}{n}=E_{\mathrm{crit}}L\left[\left(\frac{V_{\mathrm{DSsat}}}{E_{\mathrm{crit}}L}+1\right)^2-1\right]}$$

$$\boxed{I_{\mathrm{D}}=\frac{\mu_{\mathrm{n}}C_{\mathrm{ox}}}{2n}\frac{W}{L}(nV_{\mathrm{DSsat}})^2}$$

I-V关系式(2)

$$I_D = \frac{\mu_n C_{ox}}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS})^2\left(1-\frac{V_{GS}-V_{TH,GS}}{2nE_{crit}L}+\cdots\right)^2$$

$$= \frac{\mu_n C_{ox}}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS})^2(1-\frac{V_{GS}-V_{TH,GS}}{nE_{crit}L}+\cdots)$$

$$\boxed{I_D \approx \frac{\mu_n C_{ox}}{2n\left(1+\frac{V_{GS}-V_{TH,GS}}{nE_{crit}L}\right)}\frac{W}{L}(V_{GS}-V_{TH,GS})^2}$$

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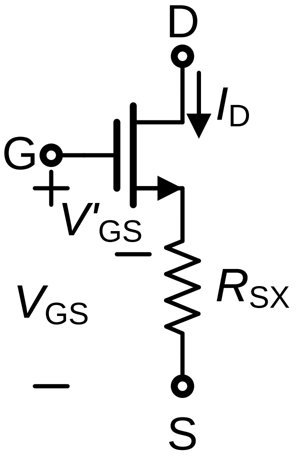
速度饱和区：R_{SX}

$$I_D = \frac{1}{2n}\mu_n C_{ox}\frac{W}{L}(V'_{GS}-V_{TH,GS})^2 \qquad V_{GS}=V'_{GS}+I_D R_{SX}$$

$$I_D=\frac{1}{2n}\mu_n C_{ox}\frac{W}{L}[(V_{GS}-V_{TH,GS})^2-2(V_{GS}-V_{TH,GS})I_D R_{SX}+(I_D R_{SX})^2]$$

$$=\frac{\mu_n C_{ox}}{2n\left[1+\mu_n C_{ox}\frac{W}{L}R_{SX}(V_{GS}-V_{TH,GS})\right]}\frac{W}{L}(V_{GS}-V_{TH,GS})^2$$



$$\mu_n C_{ox}\frac{W}{L}R_{SX}=\frac{1}{nE_{crit}L}$$

$$\boxed{R_{SX}=\frac{1}{n\mu_n C_{ox}WE_{crit}}}$$

si与vs转换点电压$V_{\mathrm{GST,sv}}$

$$I_{\mathrm{D,si}} = \frac{\mu_n C_{\mathrm{ox}}}{2n}\frac{W}{L}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})^2 \qquad I_{\mathrm{D,vs}} = WC_{\mathrm{ox}}v_{\mathrm{scl}}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})$$

$$g_{\mathrm{m,si}} = 2\frac{\mu_n C_{\mathrm{ox}}}{2n}\frac{W}{L}(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}) \qquad g_{\mathrm{m,vs}} = WC_{\mathrm{ox}}v_{\mathrm{scl}}$$

$$\boxed{I_{\mathrm{D,si}} = I_{\mathrm{D,vs}} \quad \text{or} \quad \frac{g_{\mathrm{m,si}}}{I_{\mathrm{D,si}}} = \frac{g_{\mathrm{m,vs}}}{I_{\mathrm{D,vs}}}}$$

$$\boxed{(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})_{\mathrm{sv}} = 2nL\frac{v_{\mathrm{scl}}}{\mu_n} \approx 5L}$$

$$I_{\mathrm{D,sv}} = \frac{2nWLC_{\mathrm{ox}}v_{\mathrm{scl}}^2}{\mu_n}$$

$$v_{\mathrm{scl}} = 10^7\ \mathrm{cm/s}$$

$$n = 1.4$$

正比于沟道长度*L*！！！

当$L = 0.13\ \mu\mathrm{m}$时，$(V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})_{\mathrm{sv}} \approx 0.65\ \mathrm{V}$ $\quad \mu_n = 500\ \mathrm{cm^2/Vs}$

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si与vs转换点电流$I_{\mathrm{D,sv}}$

$$I_{\mathrm{D,sv}} \approx \frac{\mu_{\mathrm{n}} C_{\mathrm{ox}}}{2n} \frac{W}{L} \left(2nL \frac{v_{\mathrm{scl}}}{\mu_{\mathrm{n}}}\right)^2 \approx 100 n \varepsilon_{\mathrm{ox}} W \frac{v_{\mathrm{scl}}^2}{\mu_{\mathrm{n}}}$$

$$C_{\mathrm{ox}} = \frac{\varepsilon_{\mathrm{ox}}}{t_{\mathrm{ox}}} \quad t_{\mathrm{ox}} = \frac{L_{\mathrm{min}}}{50}$$

$$\frac{I_{\mathrm{D,sv}}}{W} \approx 10\ \mathrm{A/cm}$$

$$\varepsilon_{\mathrm{ox}} = 0.34\ \mathrm{pF/cm}$$

$$v_{\mathrm{sat}} = 10^7\ \mathrm{cm/s}$$

$$n = 1.4$$

$$\mu_{\mathrm{n}} = 500\ \mathrm{cm^2/Vs}$$

当 $W = 1\ \mu\mathrm{m}$,$L_{\mathrm{min}} = 0.13\ \mu\mathrm{m}$ 时，$I_{\mathrm{D,sv}} \approx 1\ \mathrm{mA}$ 。

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si与vs转换点电压$V_{GST,sv}$

$$I_{D,si}=\frac{\mu_n C_{ox}}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS})^2 \qquad I_{D,vs}=WC_{ox}v_{scl}(V_{GS}-V_{TH,GS})$$

$$g_{m,si}=2\frac{\mu_n C_{ox}}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS}) \qquad g_{m,vs}=WC_{ox}v_{scl}$$

$$\boxed{g_{m,si}=g_{m,vs}}$$

$$\boxed{(V_{GS}-V_{TH,GS})_{sv}=nL\frac{v_{scl}}{\mu_n}\approx 2.5L}$$

$$I_{D,sv}=\frac{nWLC_{ox}v_{scl}^2}{2\mu_n}$$

正比于沟道长度L!！！

$$v_{scl}=10^7\ \text{cm/s}$$

$$n=1.4$$

当$L=0.13\ \mu\text{m}$时，$(V_{GS}-V_{TH,GS})_{sv}\approx 0.325\ \text{V}$ $\mu_n=500\ \text{cm}^2/\text{Vs}$

si与vs转换点跨导g_m

$$g_{m,vs} = WC_{ox}v_{scl} = 50\varepsilon_{ox}v_{scl}\frac{W}{L_{min}} = 17\times10^{-5}W/L_{min}\quad(S)\Big|_{cm}$$

$$g_{m,si} = 2K_n^{'}\frac{W}{L_{min}}(V_{GS} - V_{TH,GS})$$

$$= \frac{50\mu_n\varepsilon_{ox}}{n}\frac{W}{L_{min}^2}V_{GST} = 6\times10^{-9}V_{GST}W/L_{min}^2\quad(S)\Big|_{cm}$$

$$\frac{1}{g_m} = \frac{1}{g_{m,vs}} + \frac{1}{g_{m,si}}$$

$$\boxed{g_m \approx \frac{W}{L_{min}}\frac{17\times10^{-5}}{1+2.8\times10^4 L_{min}/V_{GST}}}$$

L_{min}的单位为cm

$$g_{m,vs} = g_{m,si}\quad L_{min} = \frac{\mu_n V_{GST,sv}}{nv_{scl}} = 0.4V_{GST,sv}(\mu m)$$

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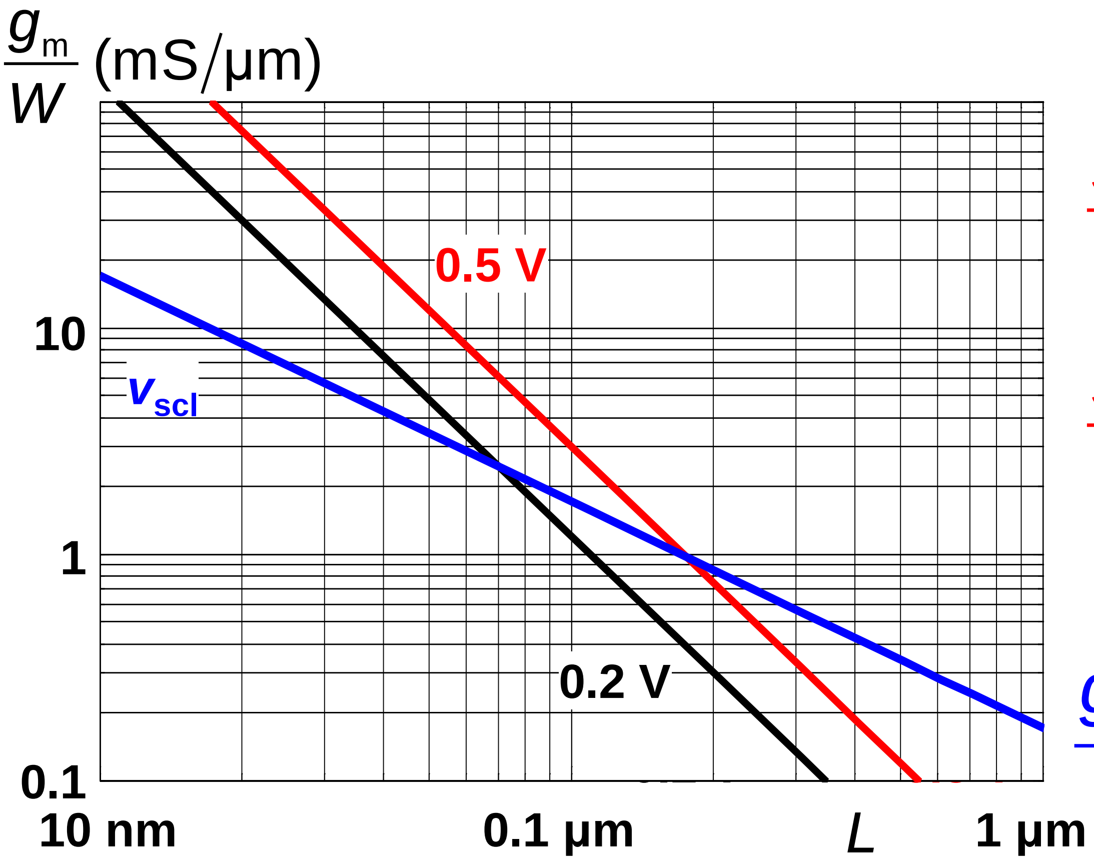
速度饱和区？饱和区？



$V_{GS}-V_{TH,GS}\approx 0.2$ V

(0.5 V)

$$\frac{g_{m,si}}{W}=\frac{2K'_n}{L_{min}}(V_{GS}-V_{TH,GS})$$

$$\frac{g_{m,si}}{W}\approx\frac{0.06V_{GST}}{L_{min}^2}$$

L的单位为μm

$$\frac{g_{m,vs}}{W}=C_{ox}v_{scl}$$

$$\frac{g_{m,vs}}{W}\approx\frac{0.17}{L_{min}}$$

L的单位为μm

当$V_{GST,sv}$=0.2 V，L_{min}<65 nm;

或$V_{GST,sv}$=0.5 V，L_{min}<0.18 μm 时，晶体管进入速度饱和区

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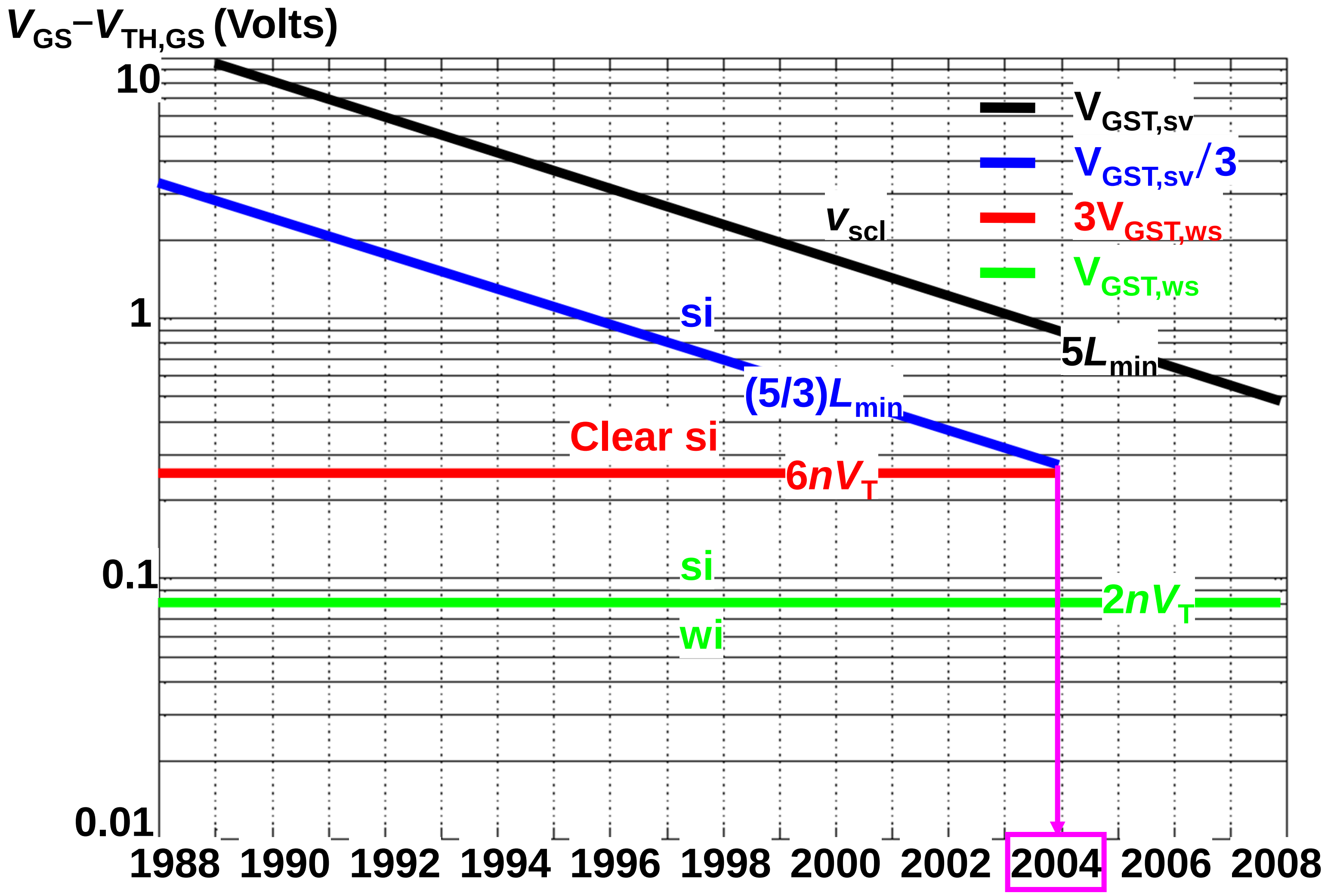
平方率饱和区V_{GST}的有效范围



L_{min}=0.18 μm

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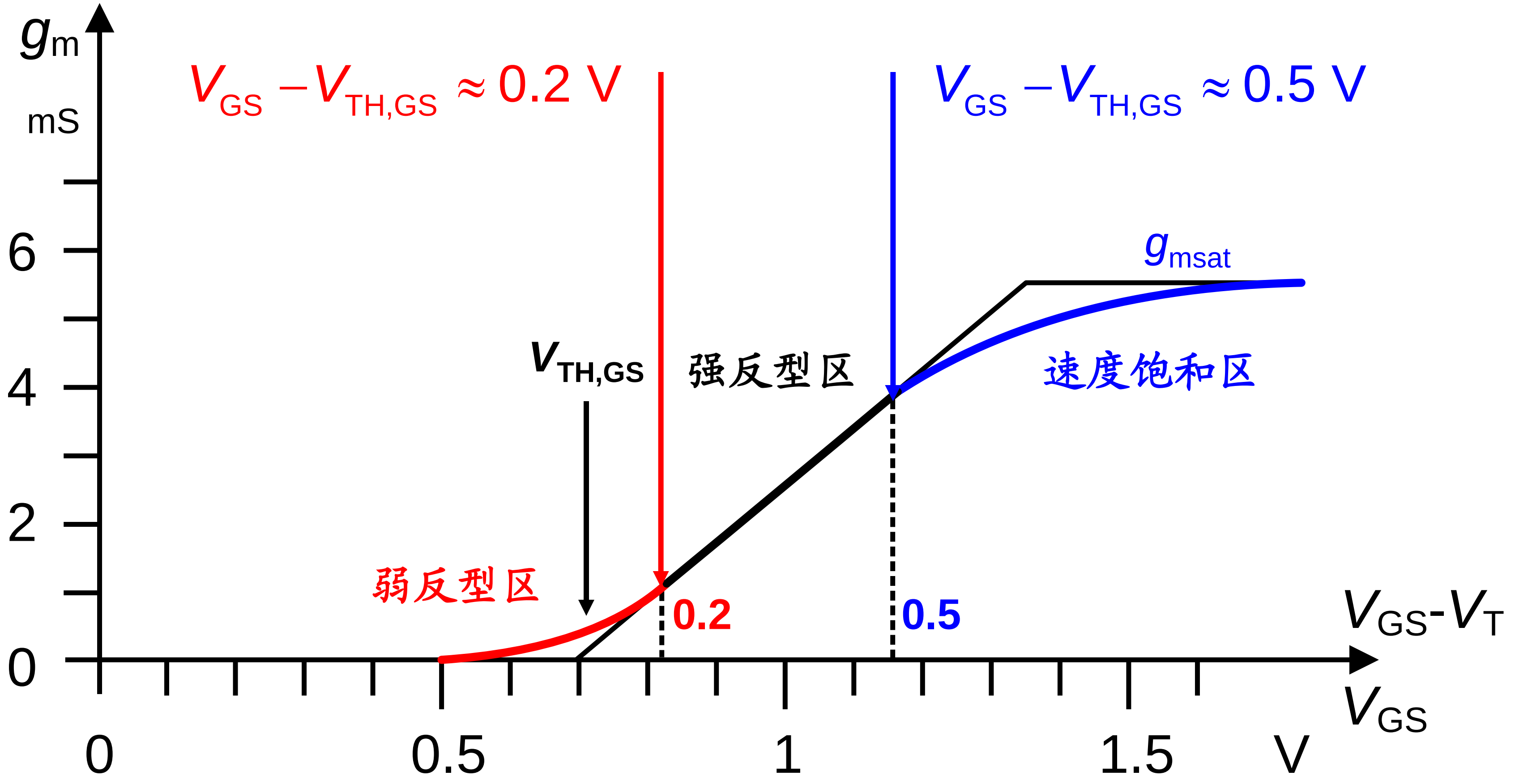
在饱和区MOST的工作范围



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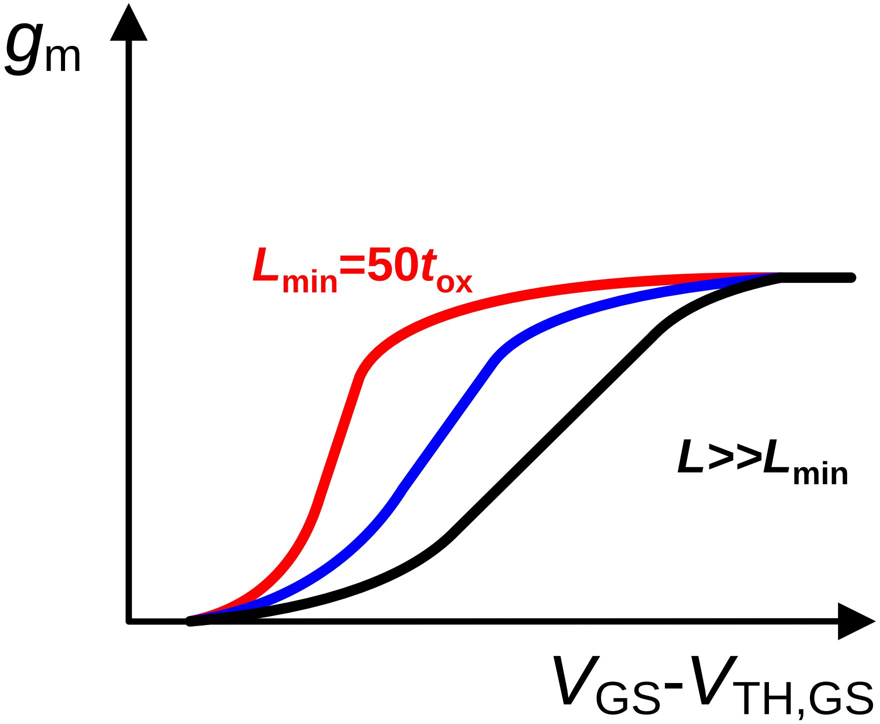
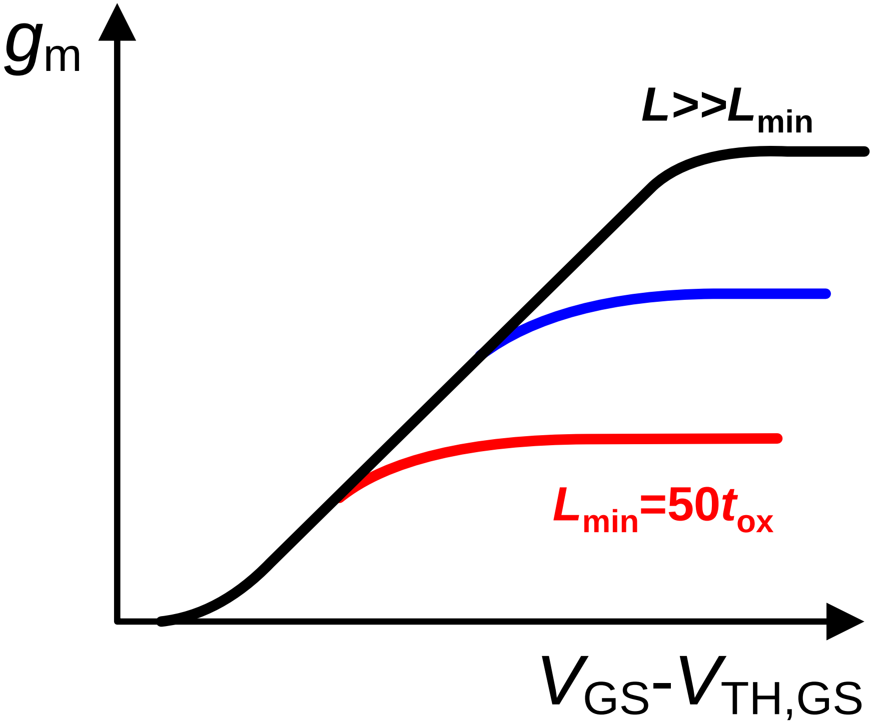
练习2：相同工艺下，不同L值

相同t_{ox}值！



$$\boxed{W = \mathrm{ct}}$$



$$\boxed{\frac{W}{L} = \mathrm{ct}}$$

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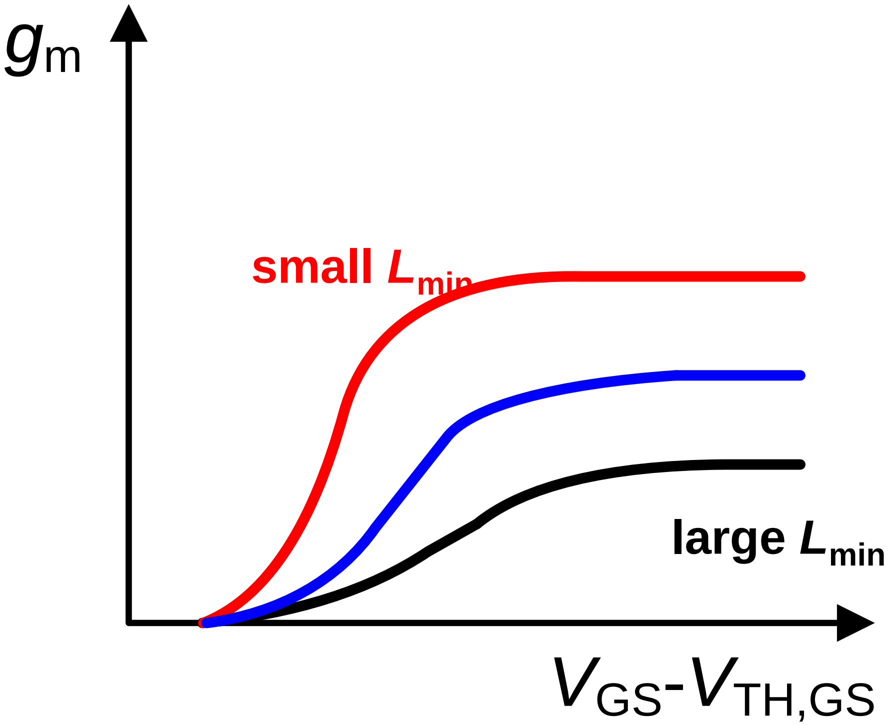
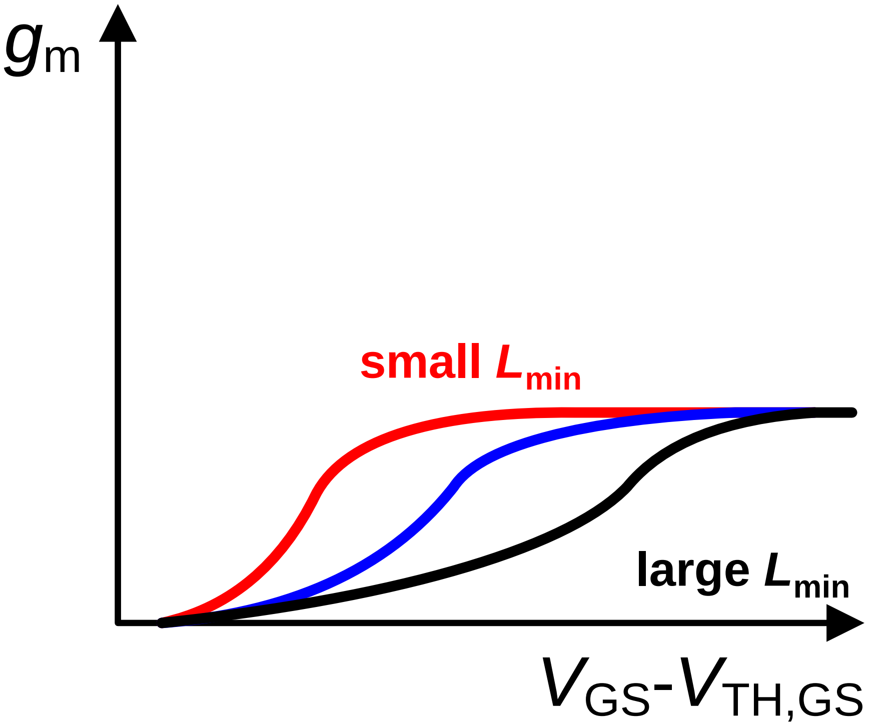
练习2：不同工艺下，L_{min}值，

不同t_{ox}值！



$$\boxed{W = \text{ct}}$$



$$\boxed{\frac{W}{L} = \text{ct}}$$

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MOST的I_D、g_m和g_m/I_D

摘要：关于I_D、g_m和g_m/I_D的公式

	I_D	g_m	$\frac{g_m}{I_D}=f(V_{GS}-V_{TH,GS})$	$\frac{g_m}{I_D}=f(I_D)$
wi	$I_t\frac{W}{L}e^{\frac{V_{GS}-V_{TH,GS}}{nV_T}}$ (1-25a)	$\frac{I_t}{nV_T}\frac{W}{L}e^{\frac{V_{GS}-V_{TH,GS}}{nV_T}}$ (1-25b)	$\frac{1}{nV_T}$ (1-26b)	$\frac{1}{nV_T}$ (1-26b)
ws			$(V_{GS}-V_{TH,GS})_{ws}=2nV_T$	$I_{D,ws}=\frac{KP}{2n}\frac{W}{L}(2nV_T)^2$
si	$\frac{KP}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS})^2$ (1-18c)	$2\frac{KP}{2n}\frac{W}{L}(V_{GS}-V_{TH,GS})$ (1-22a)	$\frac{2}{V_{GS}-V_{TH,GS}}$ (1-26a)	$2\sqrt{\frac{KP}{2n}\frac{W}{L}\frac{1}{I_{DS}}}$ (1-26a)
sv	$I_{D,si}=I_{D,vs}$ $\longrightarrow$		$(V_{GS}-V_{TH,GS})_{sv}=\frac{2nLC_{ox}v_{scl}}{KP}$	$I_{D,sv}=\frac{2nWLC_{ox}^2v_{scl}^2}{KP}$
	$\frac{g_{m,si}}{I_{D,si}}=\frac{g_{m,vs}}{I_{D,vs}}$ $\longrightarrow$		$(V_{GS}-V_{TH,GS})_{sv}=\frac{nLC_{ox}v_{scl}}{2KP}$	$I_{D,sv}=\frac{nWLC_{ox}^2v_{scl}^2}{2KP}$
		$g_{msi}=g_{mvs}$ $\longrightarrow$	$(V_{GS}-V_{TH,GS})_{sv}=\frac{2nLC_{ox}v_{scl}}{2KP}$	$I_{D,sv}=\frac{2nWLC_{ox}^2v_{scl}^2}{2KP}$
vs	$WC_{ox}v_{sat}(V_{GS}-V_{TH,GS})$ (1-38b)	$WC_{ox}v_{scl}$ (1-39)	$\frac{1}{V_{GS}-V_{TH,GS}}$	$\frac{WC_{ox}v_{scl}}{I_{D,vs}}$

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994; Table 1-4

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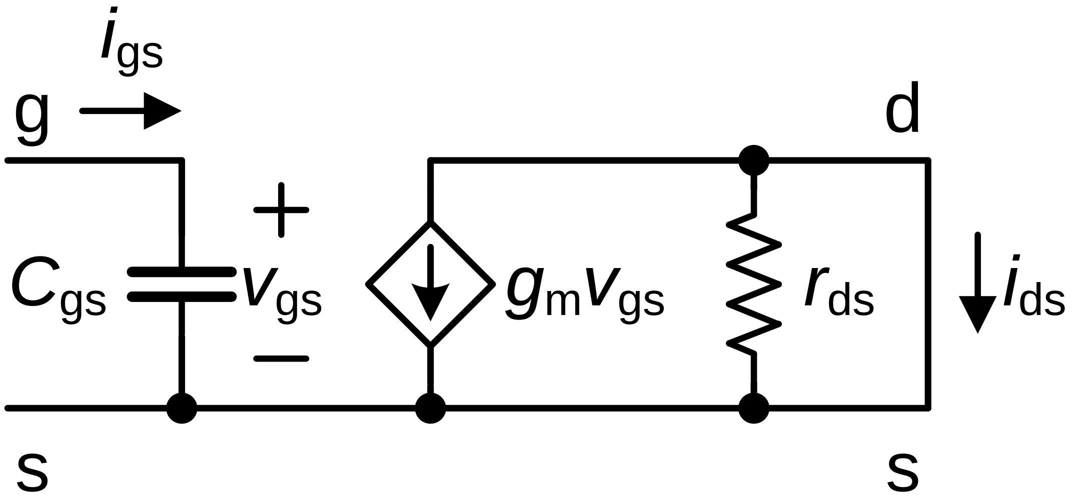
当$i_{ds}=i_{gs}$时，MOST的特征频率f_T



$$\begin{cases} i_{gs}=v_{gs}C_{gs}s \\ i_{ds}=g_m v_{gs} \end{cases}$$

$$C_{gs}=\frac{2}{3}WLC_{ox} \qquad g_m=2K'\frac{W}{L}(V_{GS}-V_{TH,GS}) \qquad K'=\frac{\mu C_{ox}}{2n}$$

$$\boxed{f_T=\frac{g_m}{2\pi C_{gs}}=\frac{1}{2\pi}\frac{3}{2n}\frac{\mu}{L^2}(V_{GS}-V_{TH,GS})}$$

$$\frac{1}{2\pi\cdot L/v_{scl}}=\boxed{\frac{v_{scl}}{2\pi L}}\times\frac{3}{2}?$$

$$g_m=WC_{ox}v_{scl}$$

高速设计

	高增益	高速
$V_{GS}-V_{TH,GS}$	低(0.2 V)	高(0.5 V)
L	大	小

$V_{GS}-V_{TH,GS}$ 决定 g_m/I_D 值，能效比!

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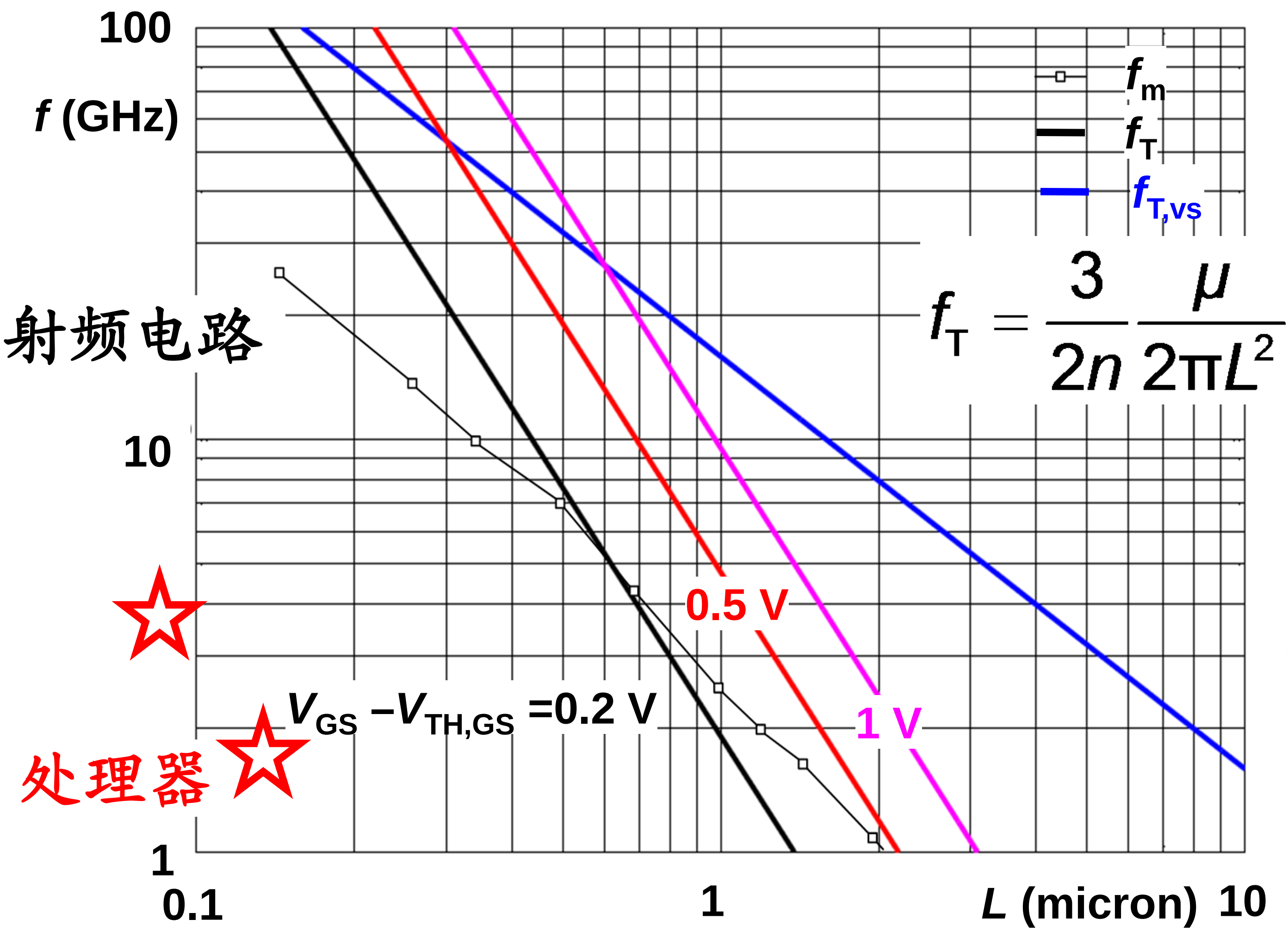
最大特征频率f_T与沟道长度L的关系



$$f_T = \frac{3}{2n}\frac{\mu}{2\pi L^2}(V_{GS} - V_{TH,GS})$$

$$f_{T,vs} = \frac{v_{scl}}{2\pi L}$$

$$f_m = \frac{f_T}{1+\alpha_{BD}}$$

$$\alpha_{BD} \approx \frac{C_{BD}}{C_{ox}}$$

饱和区和速度饱和区的特征频率f_T

$$f_\mathrm{T}=\frac{g_\mathrm{m}}{2\pi C_\mathrm{gs}} \qquad C_\mathrm{gs}=kW \qquad\qquad k=2\ \mathrm{fF/\mu m}=2\times10^{-11}\ \mathrm{F/cm}$$

$$g_\mathrm{m}=\frac{W}{L_\mathrm{min}}\frac{17\times10^{-5}}{1+2.8\times10^{4}L_\mathrm{min}/V_\mathrm{GST}} \quad L\text{的单位为cm}$$

$$\boxed{f_\mathrm{T}=\frac{1}{L_\mathrm{min}}\frac{13.5}{1+2.8L_\mathrm{min}/V_\mathrm{GST}}\ \mathrm{GHz}} \qquad L\text{的单位为}\mu\mathrm{m}$$

当V_GST=0.2 V，L_min<65 nm;

或V_GST=0.5 V，L_min<0.18 μm时，晶体管进入速度饱和区

饱和区和弱反型区的特征频率f_T

$$\left.\begin{array}{l} GM=\dfrac{1-\mathrm{e}^{-\sqrt{i}}}{\sqrt{i}} \\ I_D=i\times I_{D,ws} \end{array}\right\} \qquad \left.\begin{array}{l} f_T=\dfrac{g_m}{2\pi C_{GS}} \\ g_m=\dfrac{I_{D,ws}}{nV_T}\sqrt{i}\left(1-\mathrm{e}^{-\sqrt{i}}\right) \\ \dfrac{f_T}{f_{TH}}=\sqrt{i}\left(1-\mathrm{e}^{-\sqrt{i}}\right) \end{array}\right\}$$

$$f_{TH}=\frac{I_{D,ws}}{2\pi C_{gs} nV_T}=\frac{K'\dfrac{W}{L}V_{GST,ws}^2}{2\pi\dfrac{2}{3}WLC_{ox}nV_T}=\frac{3}{2\pi}\frac{\mu V_T}{L^2}$$

当i较小时：$\dfrac{f_T}{f_{TH}}=\sqrt{i}\left(1-\mathrm{e}^{-\sqrt{i}}\right)\approx i$

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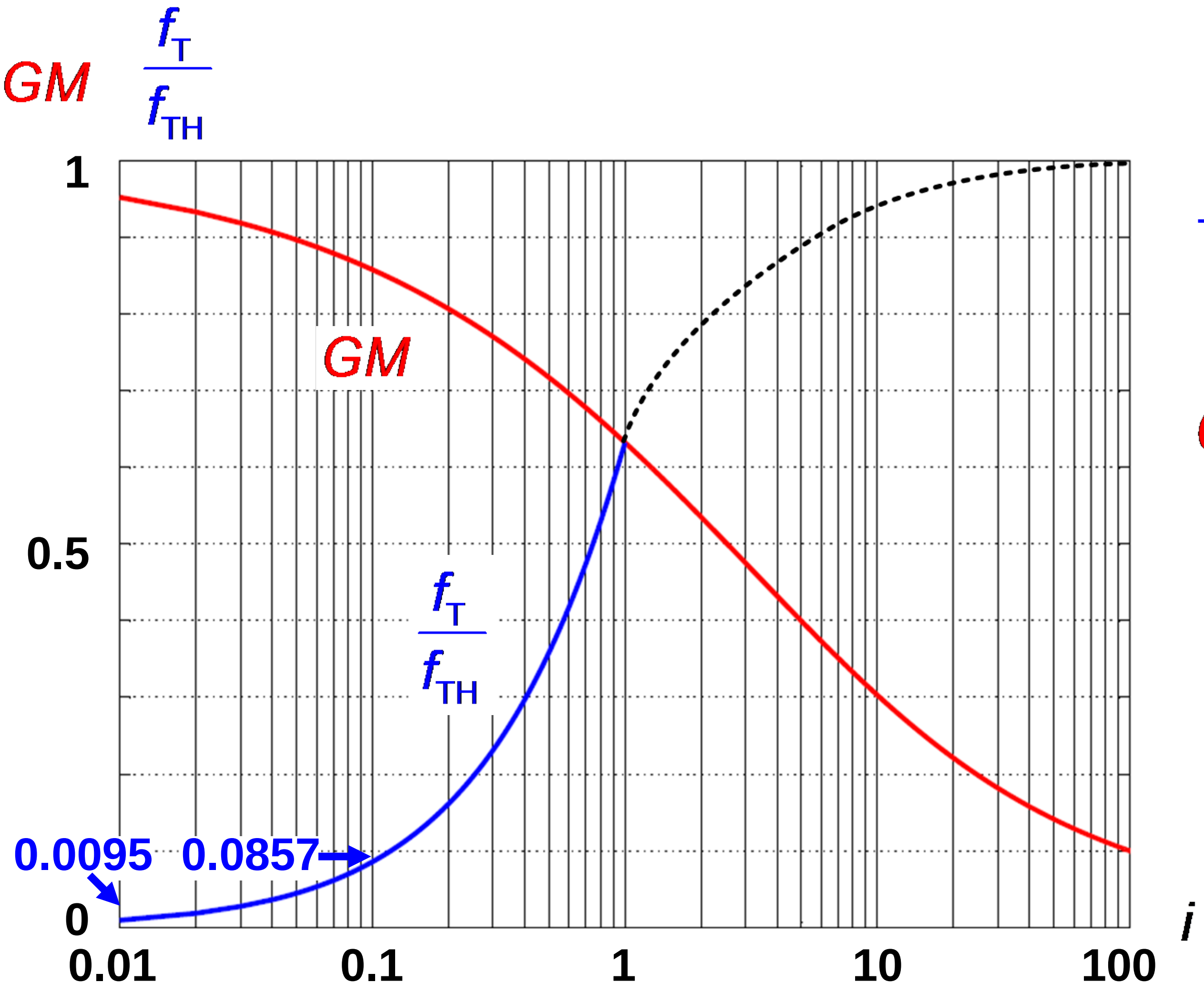
特征频率f_T与反型系数 i 的关系



$$\frac{f_T}{f_{TH}} = \sqrt{i}\left(1-e^{-\sqrt{i}}\right)$$

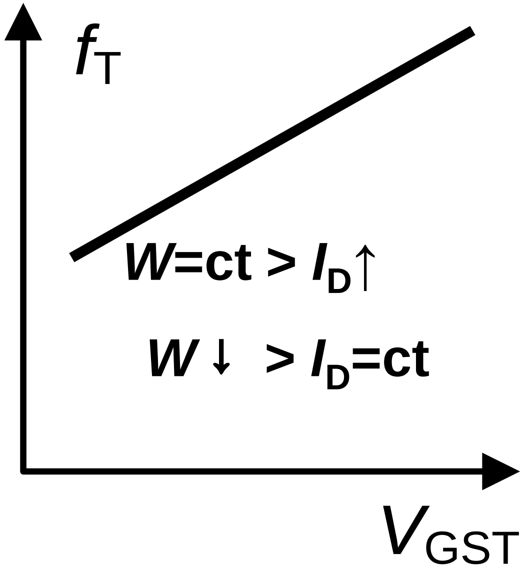
$$GM = \frac{1-e^{-\sqrt{i}}}{\sqrt{i}}$$

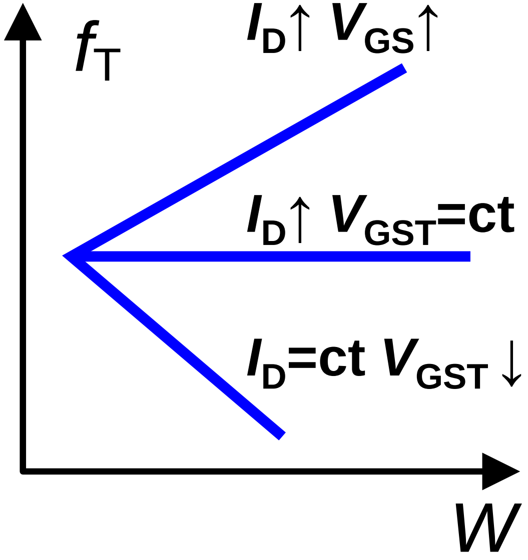
$$i = \frac{I_D}{I_{Dt}}$$

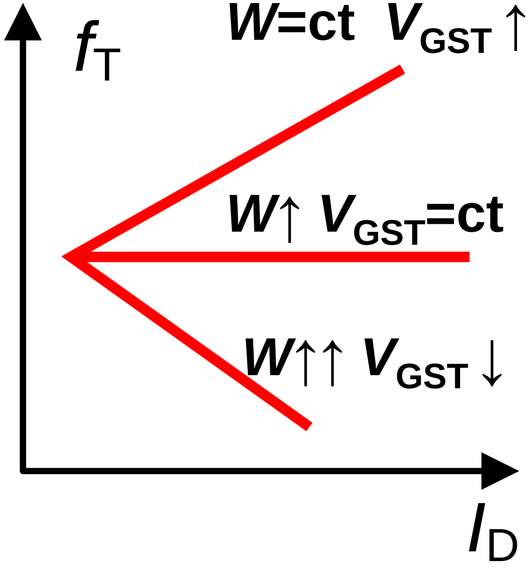
练习3: MOST的特征频率f_T?

所有$L=L_{min}$

$$f_T = \frac{1}{2\pi}\frac{3}{2n}\frac{\mu}{L_{min}^2}(V_{GS}-V_{TH,GS}) = \frac{3}{2}\frac{\sqrt{K' I_D}}{\pi C_{ox}\sqrt{W L_{min}^3}} = \frac{3}{2}\frac{I_D}{\pi W L_{min} C_{ox}(V_{GS}-V_{TH,GS})}$$







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“单页” MOST型晶体管模型

$$V_{\mathrm{GS}} - V_{\mathrm{T}} \approx 0.2\ \mathrm{V}$$

$$K' = \frac{\mu C_{\mathrm{ox}}}{2n}$$

$$K_{\mathrm{n}}' \approx 100\ \mu\mathrm{A/V}^2$$

$$K_{\mathrm{p}}' \approx 40\ \mu\mathrm{A/V}^2$$

$$I_{\mathrm{DS}} = K' \frac{W}{L} (V_{\mathrm{GS}} - V_{\mathrm{TH,GS}})^2$$

$$g_{\mathrm{m}} = 2K' \frac{W}{L} (V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}) = 2\sqrt{K' \frac{W}{L} I_{\mathrm{D}}} = \frac{2I_{\mathrm{D}}}{V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}}$$

$$r_{\mathrm{ds}} = \frac{V_{\mathrm{E}} L}{I_{\mathrm{D}}}$$

$$V_{\mathrm{En}} \approx 5\ \mathrm{V}/\mu\mathrm{m}$$

$$V_{\mathrm{Ep}} \approx 8\ \mathrm{V}/\mu\mathrm{m}$$

$$f_{\mathrm{T}} = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{\mathrm{GS}} - V_{\mathrm{TH,GS}}) \approx \frac{v_{\mathrm{scl}}}{2\pi L}$$

$$v_{\mathrm{scl}} \approx 10^7\ \mathrm{cm/s}$$

MOS型与BJT型晶体管的比较

表2-8 MOS型与BJT型晶体管的比较

Specification		MOST	Bipolar transistor	
1. I_{IN}		0	I_{C}/β	β ?
R_{IN}		∞	$r_{\pi}+r_{\mathrm{B}}$	
2. V_{DSsat}		$V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}=\sqrt{\frac{I_{\mathrm{D}}}{K'W/L}}$	few V_{T}	
3. $\frac{g_{\mathrm{m}}}{I}$	wi	$\frac{1}{nV_{\mathrm{T}}}$	$\frac{1}{V_{\mathrm{T}}}$	$n=1+\frac{C_{\mathrm{j}}}{C_{\mathrm{ox}}}$
	si	$\frac{2}{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}$	$\frac{1}{V_{\mathrm{T}}}$	4…6 X
	vs	$\frac{1}{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}$	$\frac{1}{V_{\mathrm{T}}}$	

Ref.: Laker Sansen Table 2-8

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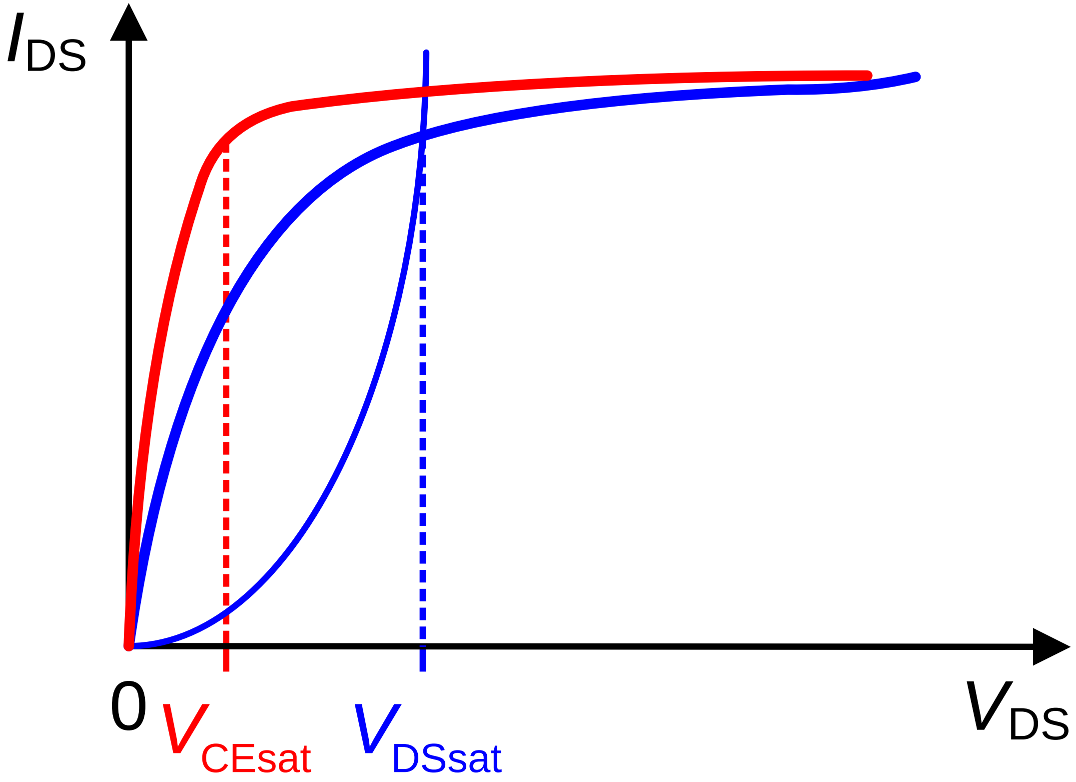
MOST-BJT的比较：最小V_{DS}



$$V_{DSsat} \approx V_{GS} - V_{TH,GS}$$

$$V_{GS} - V_{TH,GS} \approx \sqrt{\frac{I_D}{K'\frac{W}{L}}}$$

$$V_{CEsat} \approx V_T\text{'s}$$

Ref.: Laker Sansen Table 2-8

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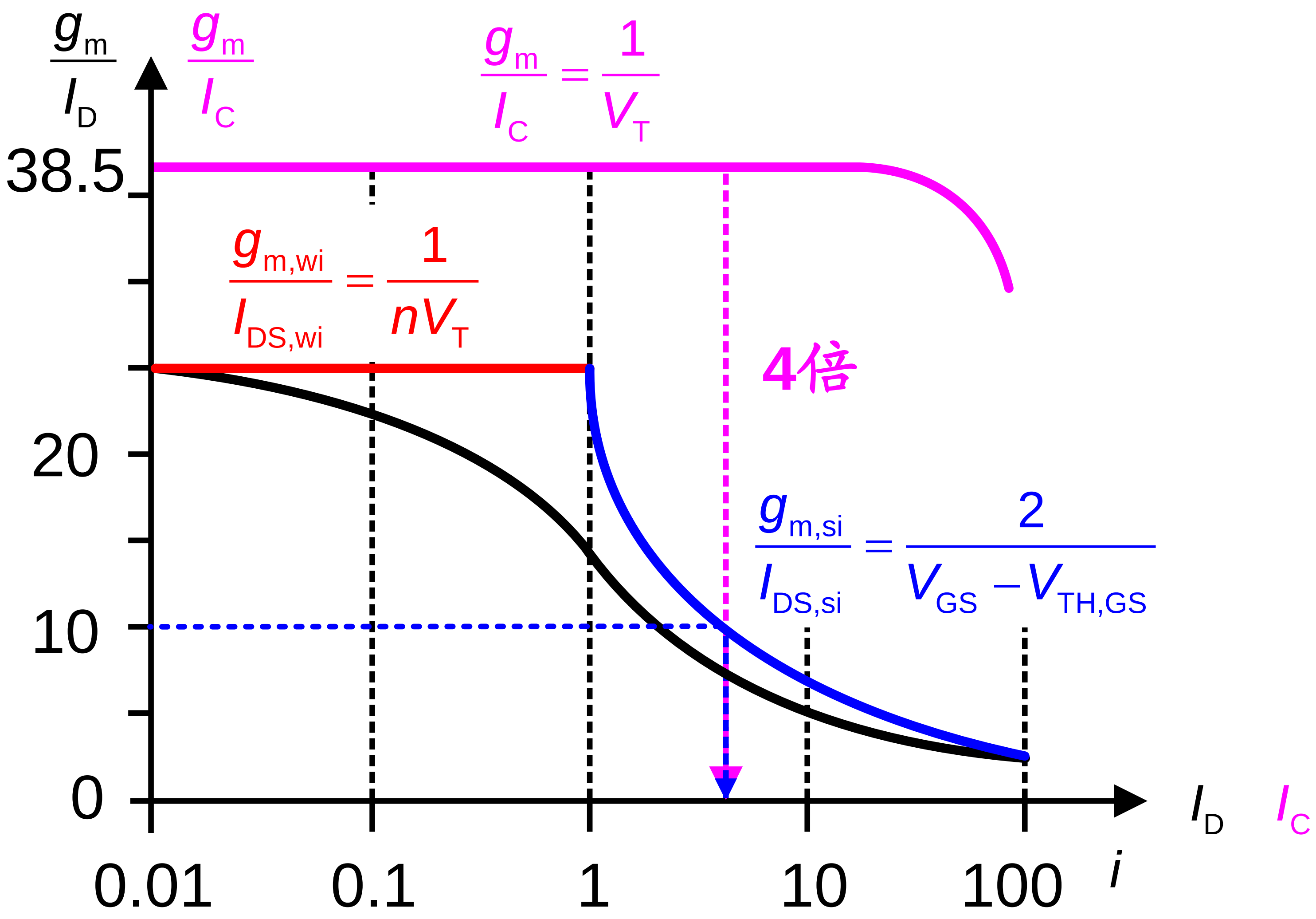
MOST-BJT的比较：能效比g_m/I_D



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g_{m}的设计流程

$$I_{\mathrm{D}}=K'\frac{W}{L}(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})^2$$

$$g_{\mathrm{m}}=2K'\frac{W}{L}(V_{\mathrm{GS}}-V_{\mathrm{TH,GS}})=2\sqrt{K'\frac{W}{L}I_{\mathrm{D}}}=\frac{2I_{\mathrm{D}}}{V_{\mathrm{GS}}-V_{\mathrm{TH,GS}}}$$

2个方程，4个变量 >> 2个变量不受约束

选择V_{GS}-$V_{\mathrm{TH,GS}}$和L！

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MOS型与BJT型晶体管的比较

表2-8 MOS型与BJT型晶体管的比较

Specification		MOST	Bipolar transistor
4.Design planning		$\frac{W}{L},\ V_{GS}-V_{TH,GS}$	V_T
5. *I*-range		1 decade	7 decade
6. Max f_T	low *I*	C_{GS}, C_{GD}	C_j, C_μ
	high *I*	v_{scl}/L	v_{scl}/W_B
7. Noise $\overline{dv_i^2}$	Therm.	$4kT(\frac{2/3}{g_m}+R_G)$	$4kT(\frac{1/2}{g_m}+R_B)$
	1/f	10X	
8. Offset		10X	$v_{scl}\approx 10^7$ cm/s

Ref.: Laker Sansen Table 2-8

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关于晶体管模型的参考书目

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