电路基础
(Fundamentals of Electric Circuits, INF0120002.07)

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- 对称三相电路
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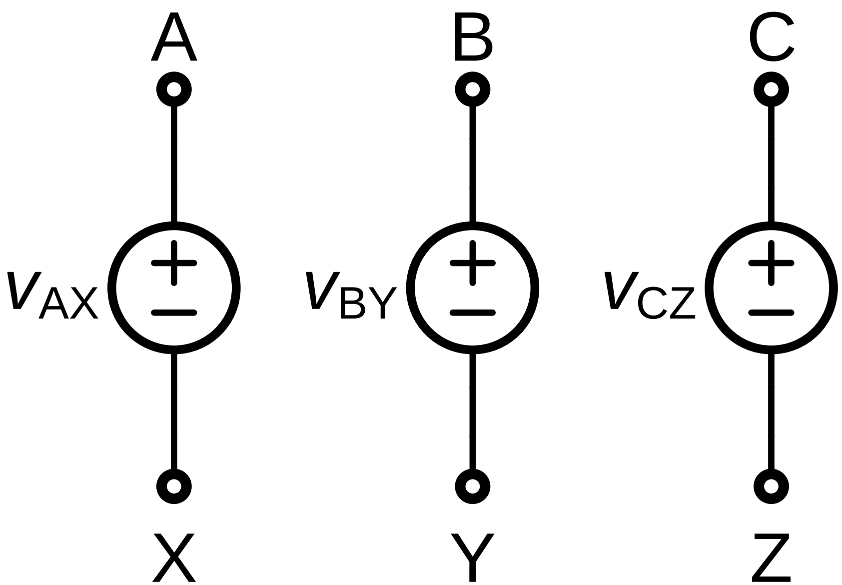
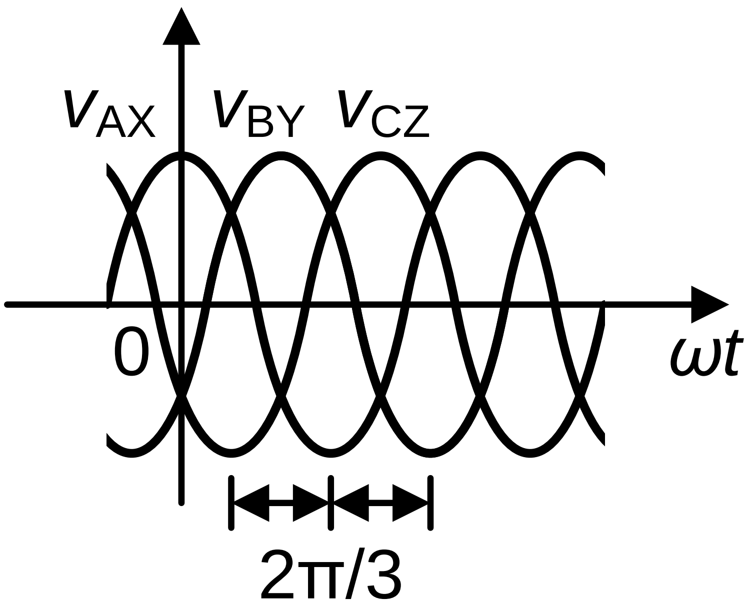
三相电路

三相制：A相、B相、C相，三相正弦交流电源。

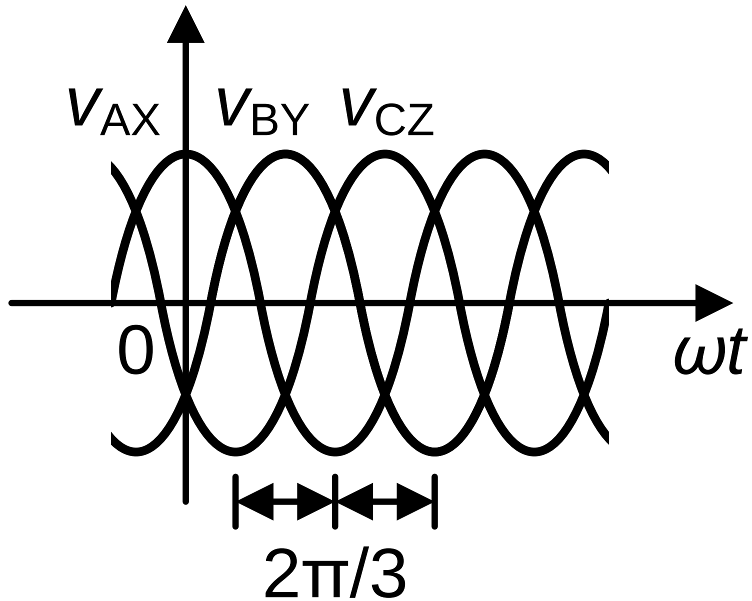


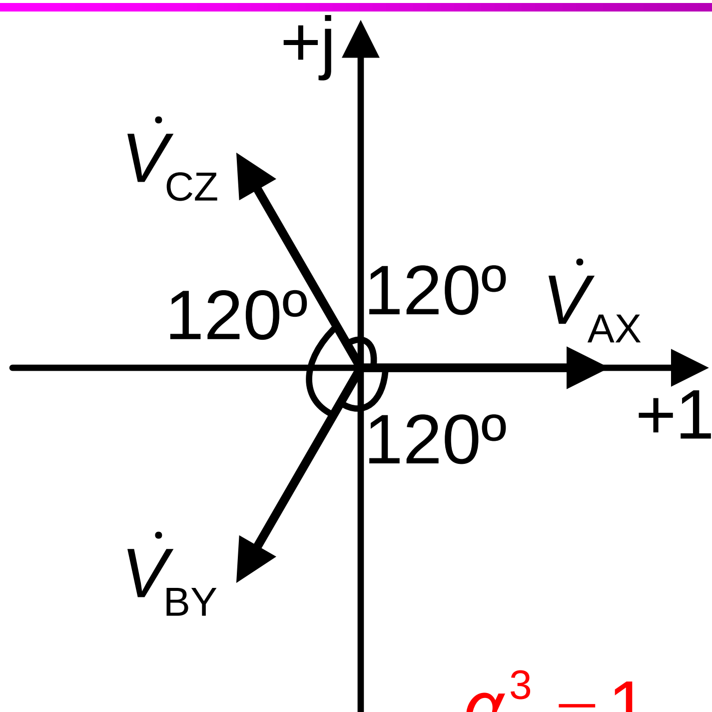


$$v_{AX} = V_m \cos(\omega t),\quad v_{BY} = V_m \cos(\omega t - 2\pi/3)$$

$$v_{CZ} = V_m \cos(\omega t + 2\pi/3)$$

正序：A→B→C





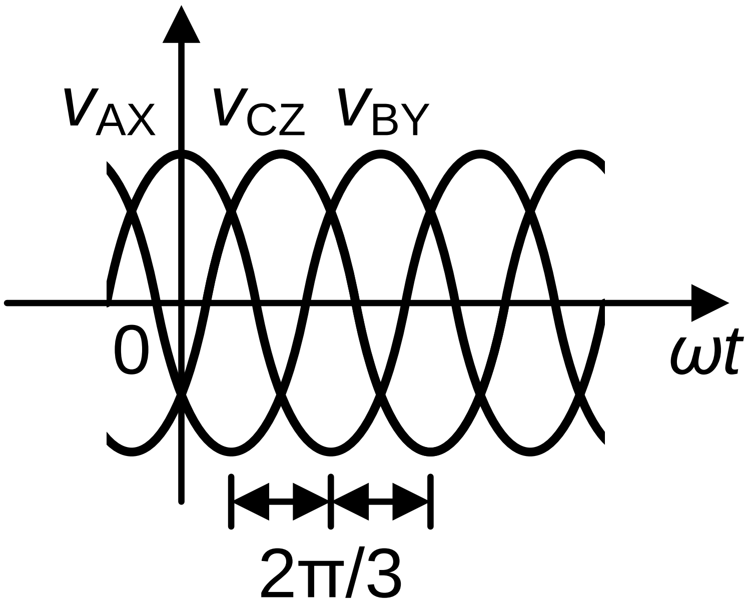
$$\dot{V}_{AX} = V_m \angle 0^\circ$$

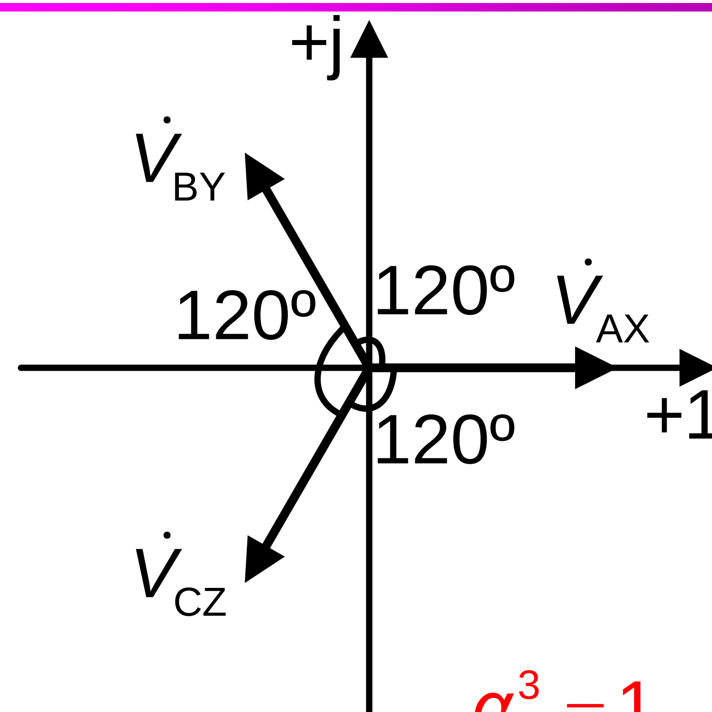
$$\alpha^3 = 1$$

$$\dot{V}_{BY} = \dot{V}_{AX} \angle -120^\circ = \frac{\dot{V}_{AX}}{\angle 120^\circ} = \frac{\dot{V}_{AX}}{\alpha}, \quad \alpha = \angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\dot{V}_{CZ} = \dot{V}_{BY} \angle -120^\circ = \dot{V}_{AX} \angle -240^\circ = \dot{V}_{AX} \angle +120^\circ = \frac{\dot{V}_{AX}}{\alpha^2}$$

反序：A→C→B





$$\alpha^3 = 1$$

$$\dot{V}_{AX} = V_m \angle 0^\circ$$

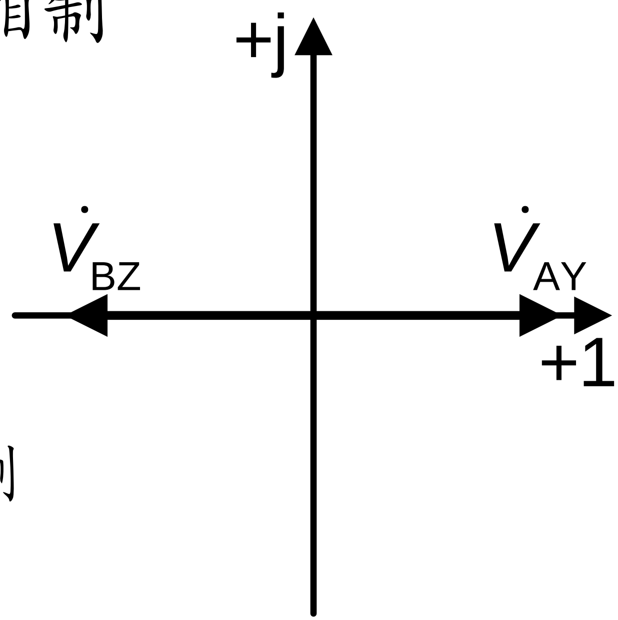
$$\dot{V}_{CZ} = \dot{V}_{AX}\angle -120^\circ = \frac{\dot{V}_{AX}}{\angle 120^\circ} = \frac{\dot{V}_{AX}}{\alpha}, \quad \alpha = \angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\dot{V}_{BY} = \dot{V}_{CZ}\angle -120^\circ = \dot{V}_{AX}\angle -240^\circ = \dot{V}_{AX}\angle +120^\circ = \frac{\dot{V}_{AX}}{\alpha^2}$$

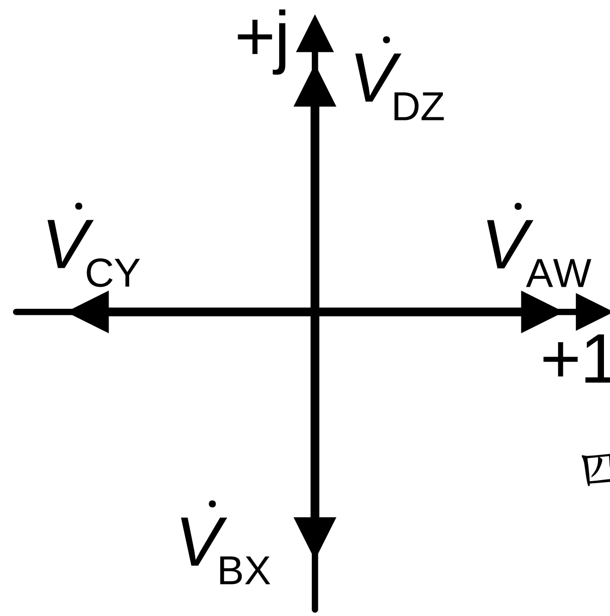
多相制

由多相电源供电的体系称为多相制。

对称N相正弦电压源包含N个振幅相等、频率相同的正弦电压，相邻的两个电压之间具有$2\pi/N$的相位差。例如：两相制、三相制、四相制、六相制、十二相制



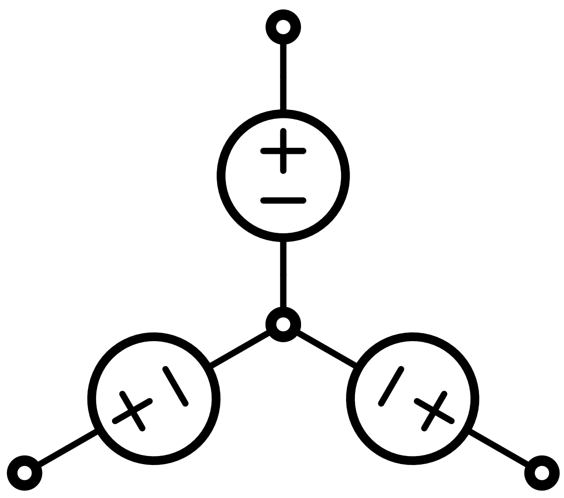
两相制

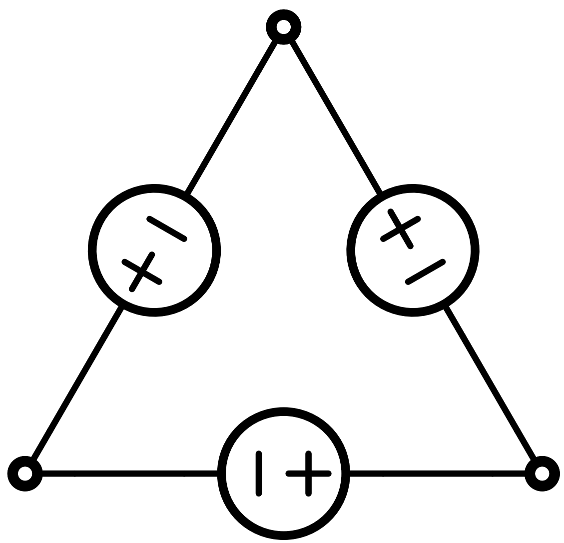


四相制

星形联结与三角形联结

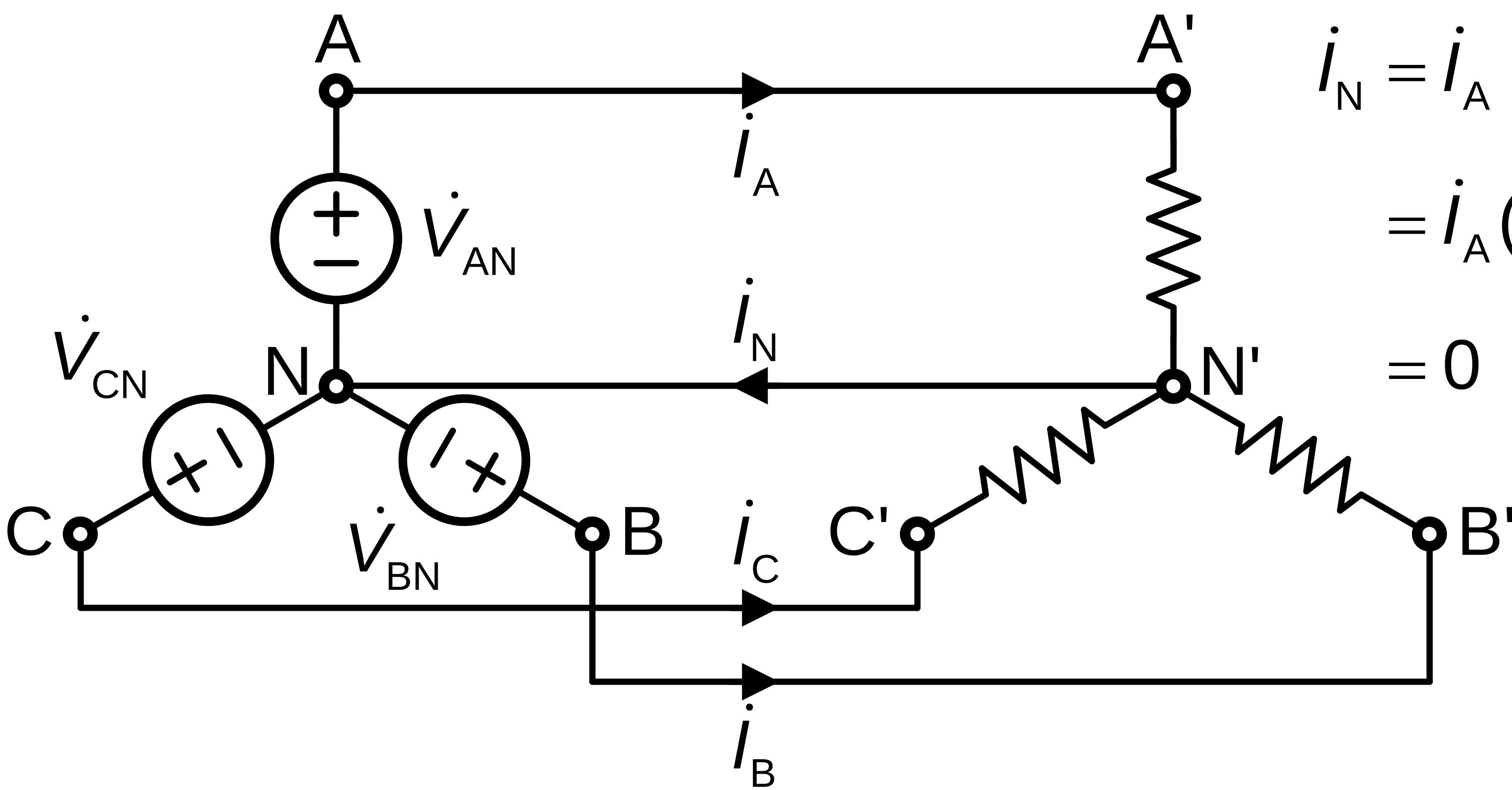
三相电源和三相负载都有两种基本连接方式：星形(Y)联结和三角形(Δ)联结。





三相四线制

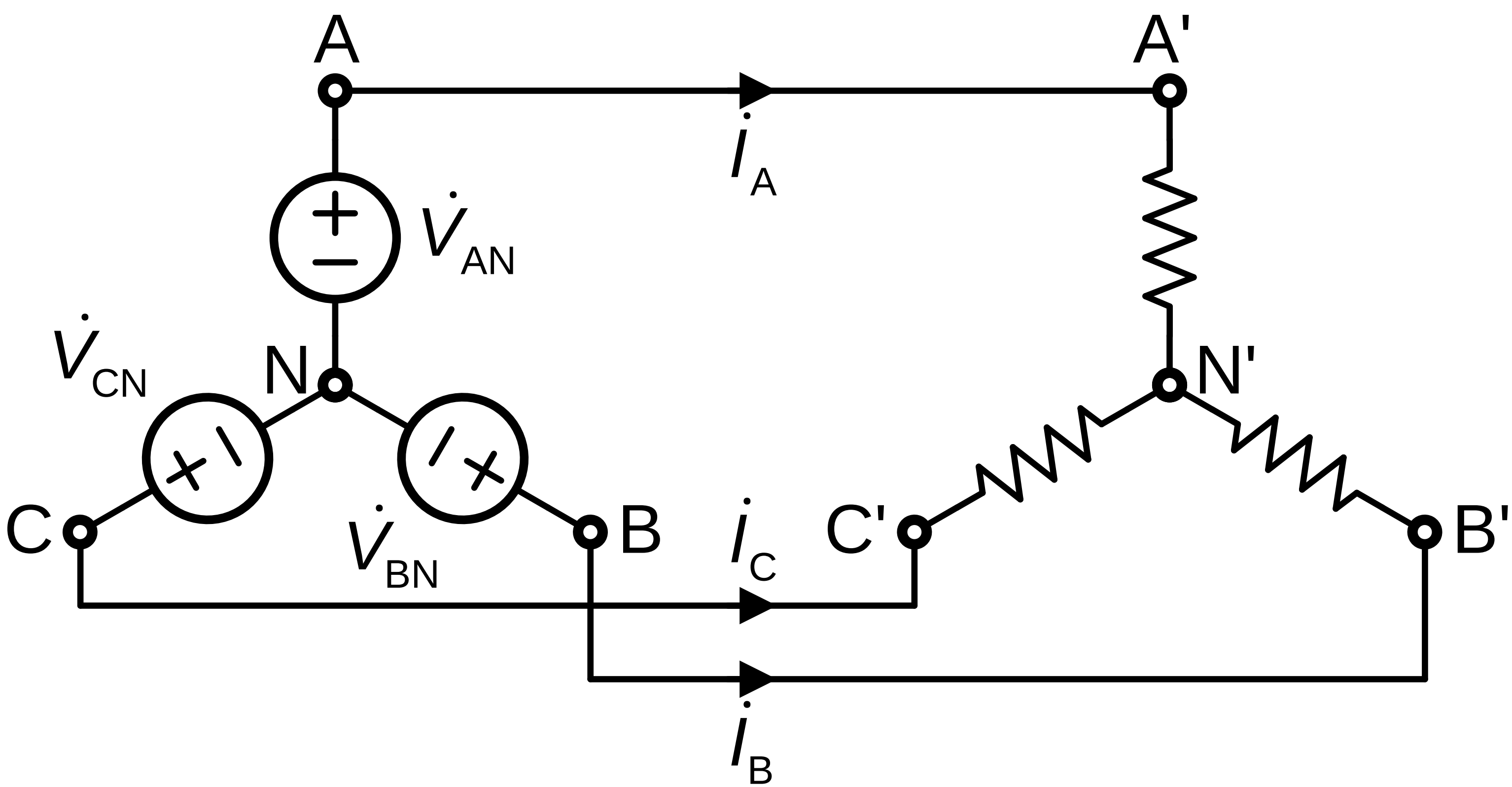
中性点，中线，端线



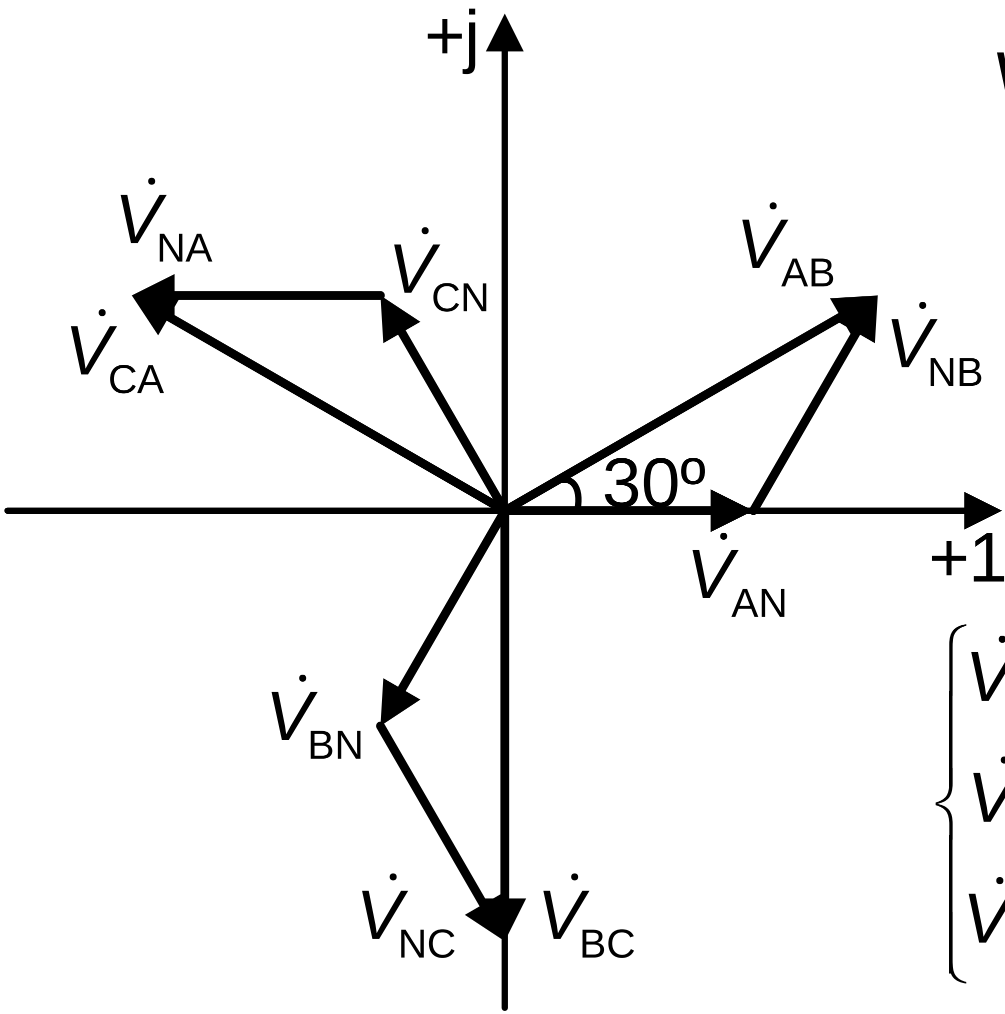
$$\dot{I}_N = \dot{I}_A + \dot{I}_B + \dot{I}_C$$
$$= \dot{I}_A\left(1 + \frac{1}{\alpha} + \frac{1}{\alpha^2}\right)$$
$$= 0$$

三相三线制：Y-Y形联结

线电流，线电压，相电流，相电压　　$I_L = I_P$



Y-Y形联结的相电压与线电压



$$\dot{V}_{AB}=\dot{V}_{AN}-\dot{V}_{BN}=\dot{V}_{AN}-\frac{\dot{V}_{AN}}{\alpha}$$

$$=\dot{V}_{AN}\left(1-\frac{1}{-1/2+j\sqrt{3}/2}\right)$$

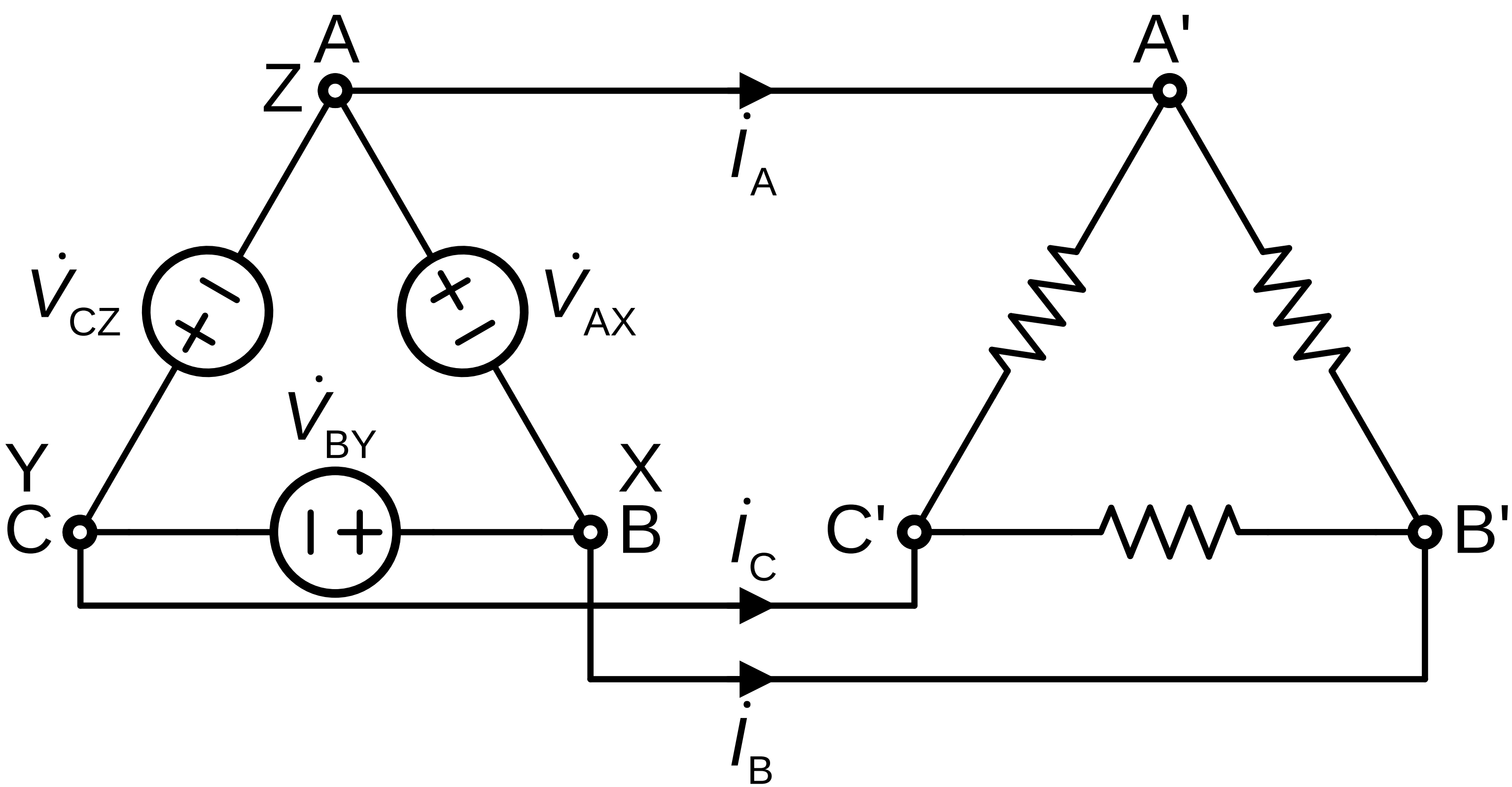
$$=\sqrt{3}\dot{V}_{AN}\angle 30^\circ$$

$$\begin{cases}\dot{V}_{AB}=\dot{V}_{AN}-\dot{V}_{BN}=\sqrt{3}\dot{V}_{AN}\angle 30^\circ\\ \dot{V}_{BC}=\dot{V}_{BN}-\dot{V}_{CN}=\sqrt{3}\dot{V}_{BN}\angle 30^\circ\\ \dot{V}_{CA}=\dot{V}_{CN}-\dot{V}_{AN}=\sqrt{3}\dot{V}_{CN}\angle 30^\circ\end{cases}$$

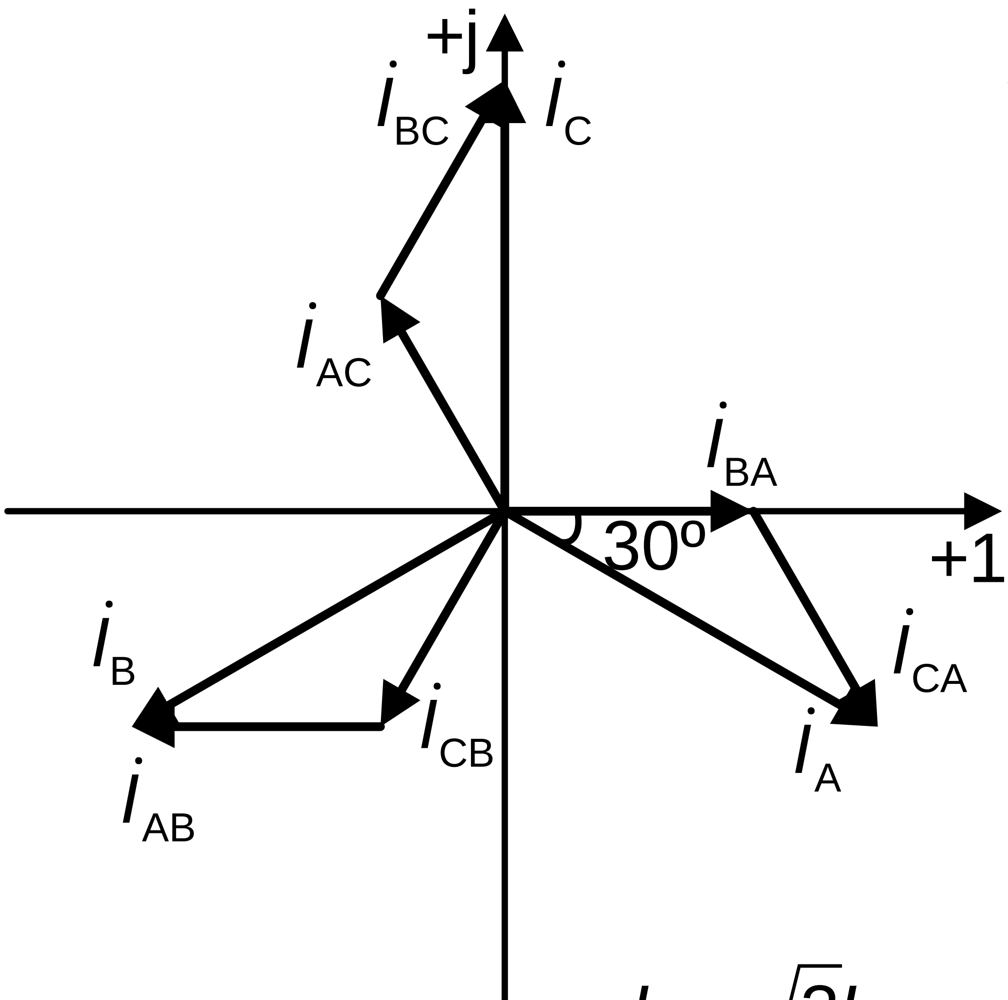
$$V_L=\sqrt{3}V_P$$

Δ-Δ形联结

线电流，线电压，相电流，相电压　　$V_L = V_P$



Δ-Δ形联结的相电流与线电流



$$I_L = \sqrt{3} I_P$$

$$\dot{i}_A = \dot{i}_{BA} - \dot{i}_{AC} = \dot{i}_{BA} - \frac{\dot{i}_{BA}}{\alpha^2}$$

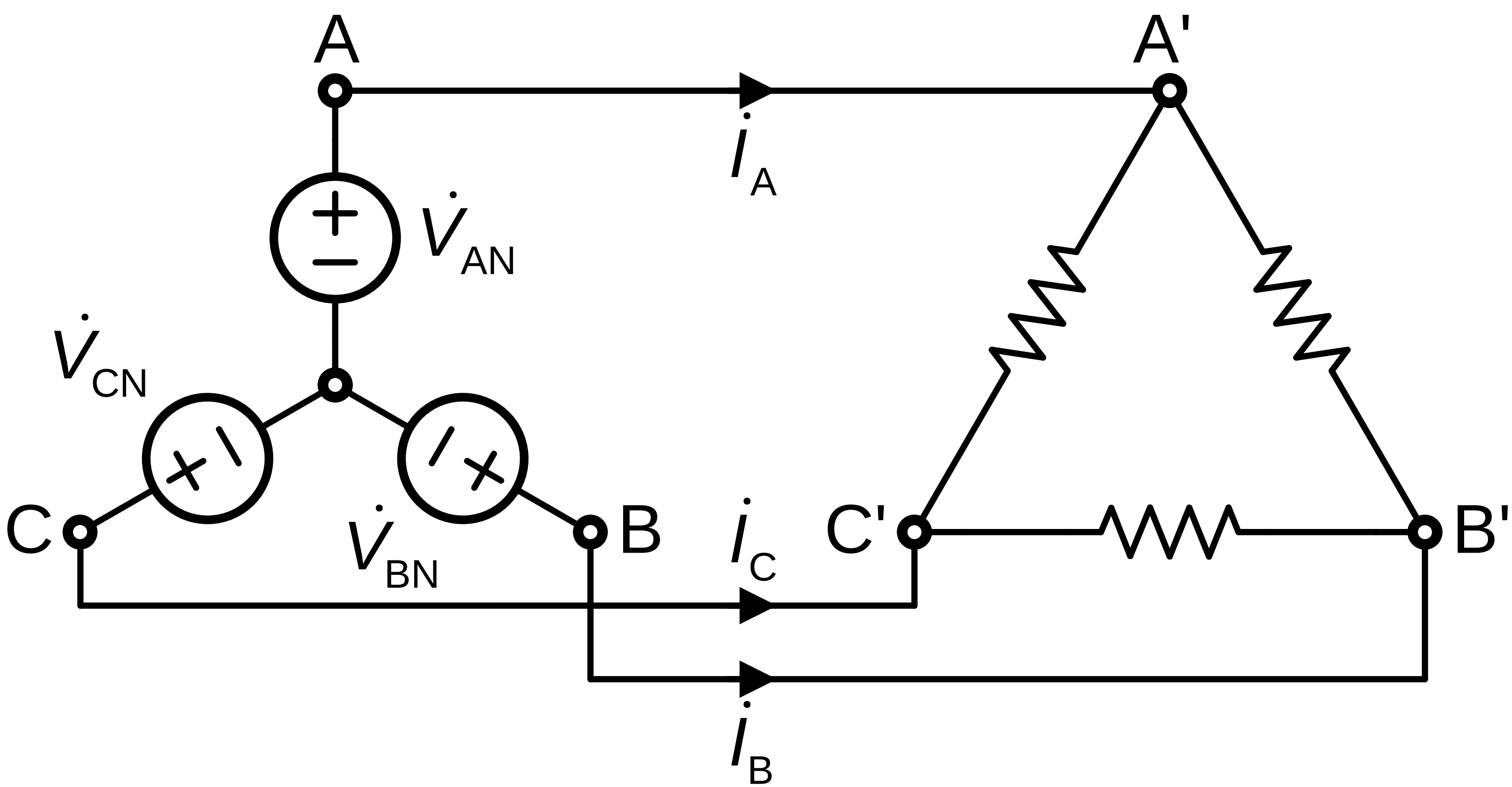
$$= \dot{i}_{BA}\left[1 - \left(-1/2 + j\sqrt{3}/2\right)\right]$$

$$= \sqrt{3}\,\frac{\dot{i}_{BA}}{\angle 30^\circ}$$

$$\begin{cases} \dot{i}_A = \dot{i}_{BA} - \dot{i}_{AC} = \sqrt{3}\,\dfrac{\dot{i}_{BA}}{\angle 30^\circ} \\ \dot{i}_B = \dot{i}_{CB} - \dot{i}_{BA} = \sqrt{3}\,\dfrac{\dot{i}_{CB}}{\angle 30^\circ} \\ \dot{i}_C = \dot{i}_{AC} - \dot{i}_{CB} = \sqrt{3}\,\dfrac{\dot{i}_{AC}}{\angle 30^\circ} \end{cases}$$

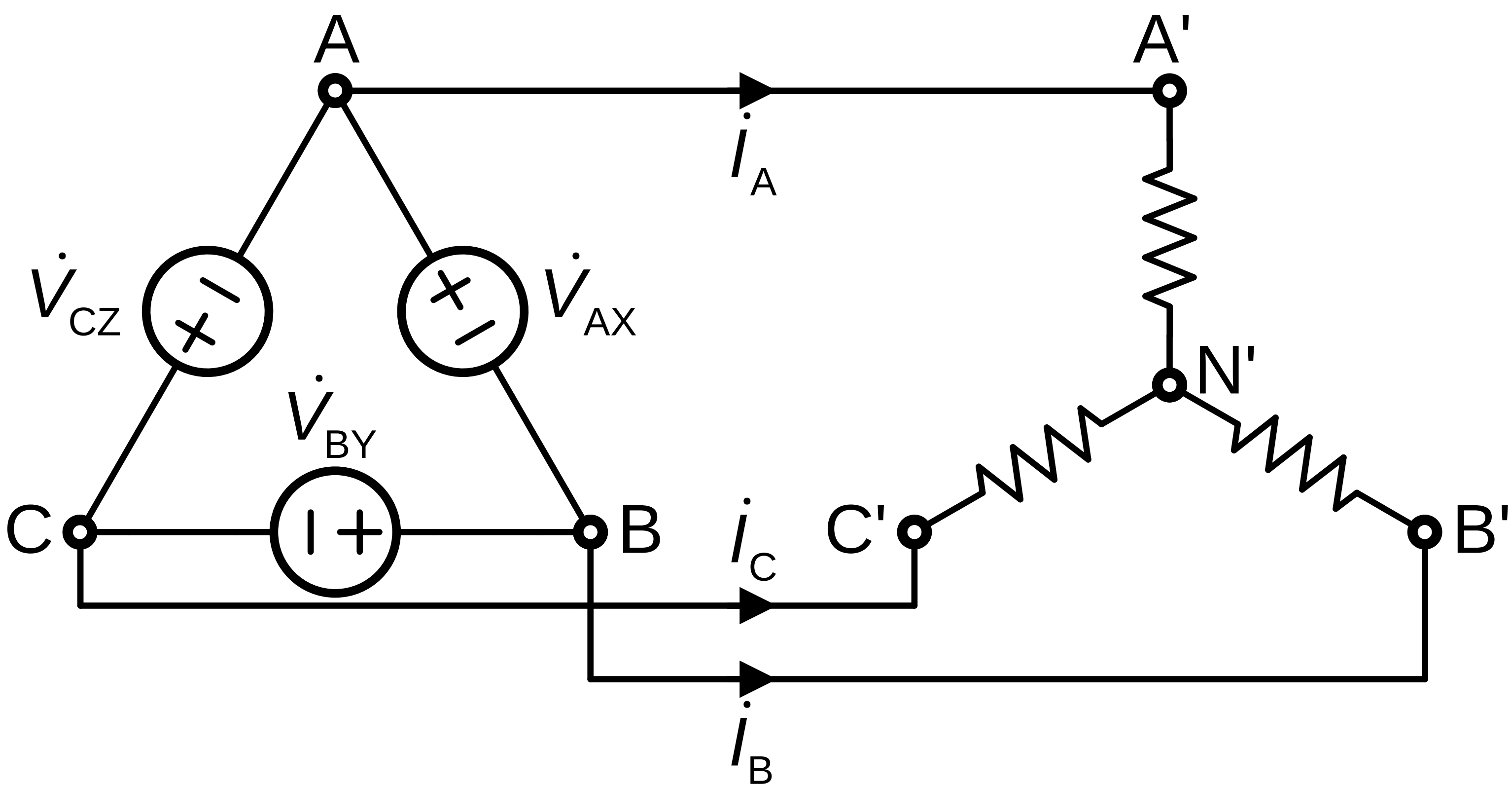
Y-Δ形联结

Y-Δ形联结等效为Y-Y形联结或者Δ-Δ形联结求解。



Δ-Y形联结

Δ-Y形联结等效为Y-Y形联结或者Δ-Δ形联结求解。



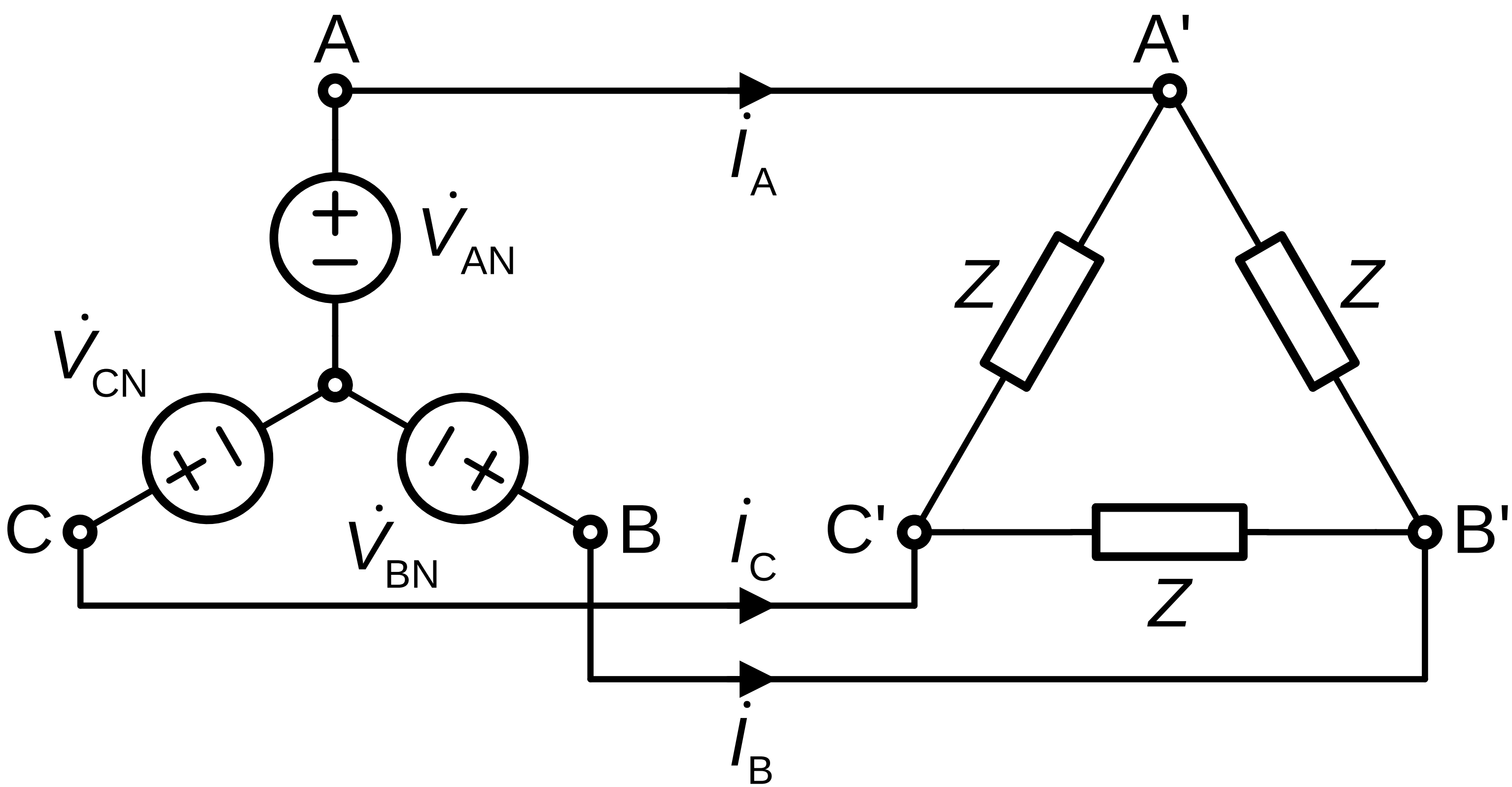
思考

Y-Y形联结有一相反接，会发生什么情况？

Δ-Δ形联结有一相反接，会发生什么情况？

例题1

如图所示，已知$\dot{V}_{AN} = 220\angle 0°$ V，负载 $Z = 1/2 + j\sqrt{3}/2$ Ω，求负载相电压、相电流及线电流的相量值。



$$\dot{V}_{A'B'}=\dot{V}_{AB}=\dot{V}_{AN}-\dot{V}_{BN}=\sqrt{3}\dot{V}_{AN}\angle 30^\circ=380\angle 30^\circ\ \text{V}$$

$$\dot{V}_{B'C'}=\frac{\dot{V}_{A'B'}}{\alpha}=\frac{380\angle 30^\circ}{\angle 120^\circ}=380\angle -90^\circ\ \text{V}$$

$$\dot{V}_{C'A'}=\frac{\dot{V}_{A'B'}}{\alpha^2}=\frac{380\angle 30^\circ}{\angle 240^\circ}=380\angle 150^\circ\ \text{V}$$

$$\dot{I}_{A'B'}=\frac{\dot{V}_{A'B'}}{Z}=\frac{380\angle 30^\circ}{1/2+\mathrm{j}\sqrt{3}/2}=380\angle -30^\circ\ \text{A}$$

$$\dot{I}_{B'C'}=\frac{\dot{I}_{A'B'}}{\alpha}=\frac{380\angle -30^\circ}{\angle 120^\circ}=380\angle -150^\circ\ \text{A}$$

$$\dot{I}_{C'A'}=\frac{\dot{I}_{A'B'}}{\alpha^2}=\frac{380\angle -30^\circ}{\angle 240^\circ}=380\angle 90^\circ\ \text{A}$$

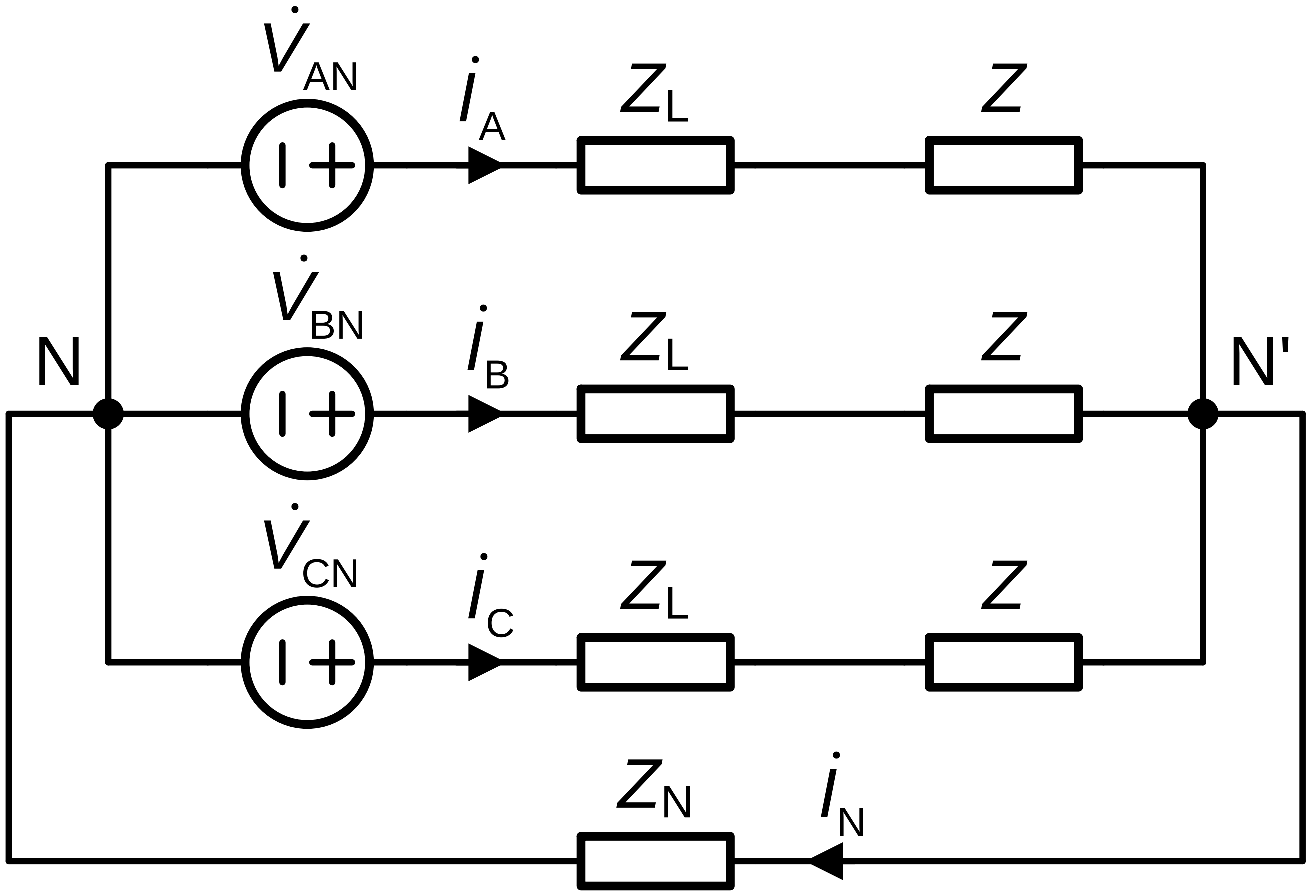
$$\dot{I}_{A}=\dot{I}_{A'B'}-\dot{I}_{C'A'}=\dot{I}_{A'B'}-\frac{\dot{I}_{A'B'}}{\alpha^2}=\sqrt{3}\dot{I}_{A'B'}\angle -30^\circ=660\angle -60^\circ\ \text{A}$$

$$\dot{I}_{B}=\frac{\dot{I}_{A}}{\alpha}=\frac{660\angle -60^\circ}{\angle 120^\circ}=660\angle 180^\circ\ \text{A}$$

$$\dot{I}_{C}=\frac{\dot{I}_{A}}{\alpha^2}=\frac{660\angle -60^\circ}{\angle 240^\circ}=660\angle 60^\circ\ \text{A}$$

对称三相电路

电压源对称，负载相同。



以电压源的中性点N为参考点，对负载中性点N'列节点电压方程

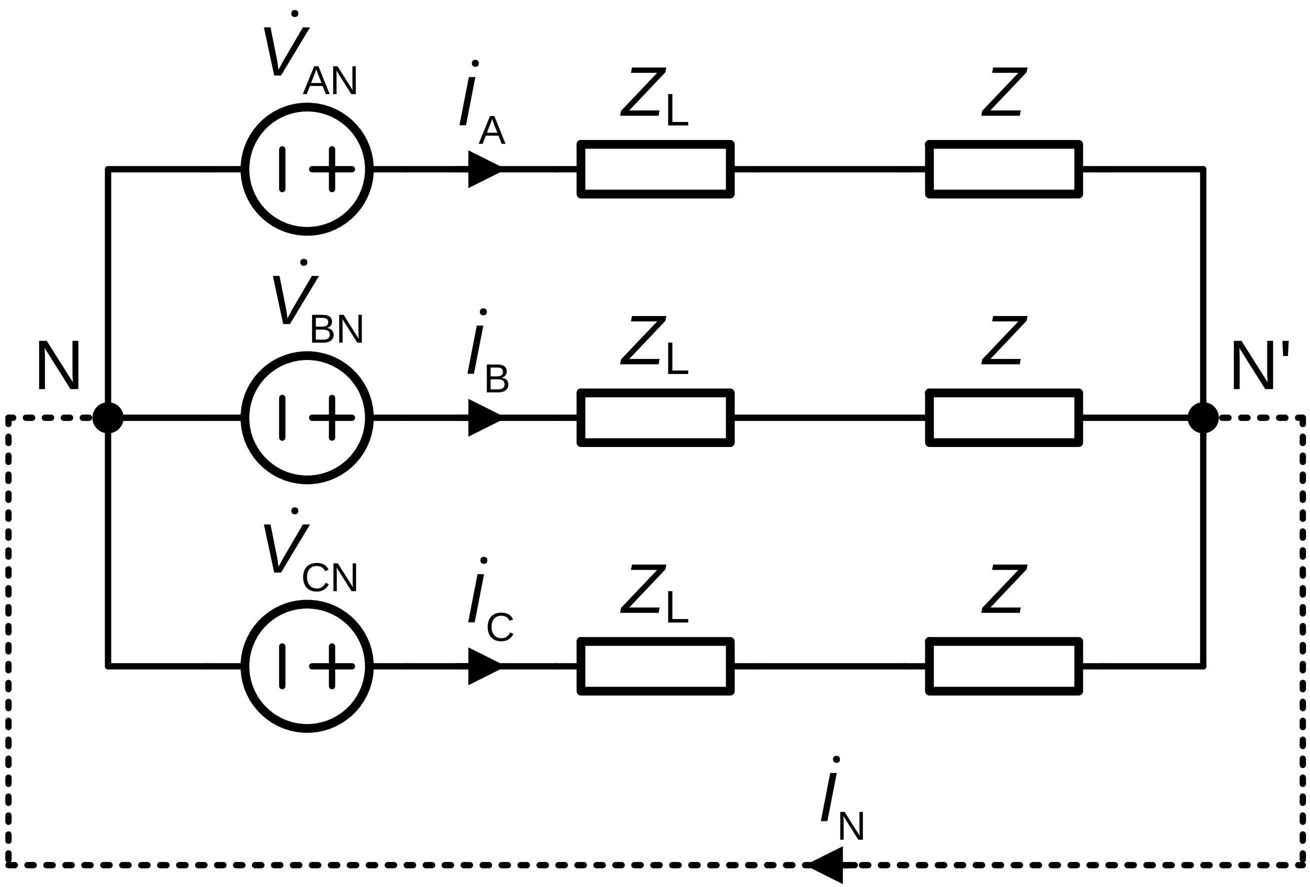
$$\left(\frac{3}{Z_L+Z}+\frac{1}{Z_N}\right)\dot{V}_{N'N}=\frac{\dot{V}_{AN}}{Z_L+Z}+\frac{\dot{V}_{BN}}{Z_L+Z}+\frac{\dot{V}_{CN}}{Z_L+Z}$$

$$\left(\frac{3}{Z_L+Z}+\frac{1}{Z_N}\right)\dot{V}_{N'N}=\frac{\dot{V}_{AN}+\dot{V}_{BN}+\dot{V}_{CN}}{Z_L+Z}=0$$

$$\dot{V}_{N'N}=0$$

对称星形联结三相电路中，无论中性电阻为何值，负载中性点与电压源中性点之间的电压恒为零。

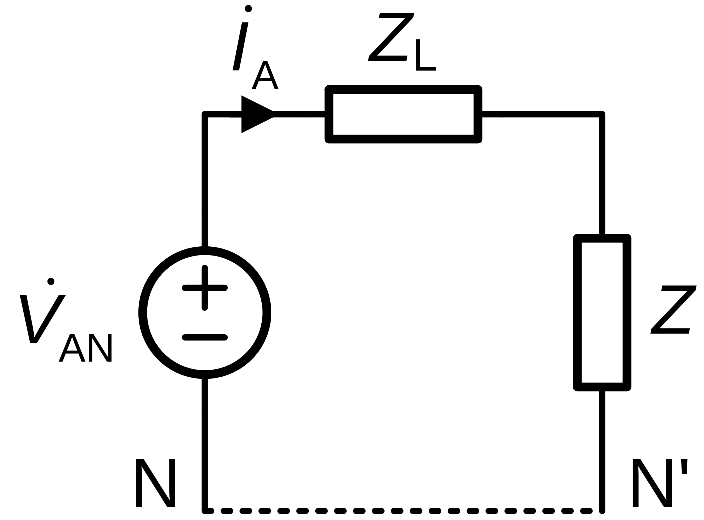
中性线短接



单相电路计算

对于对称三相星形联结电路，可取出一相按单相电路计算。

$$\dot{V}_{AN} = (Z_L + Z)\dot{I}_A$$



对称三相电路求解步骤

1.将Δ形联结电压源和负载都等效为Y形联结；

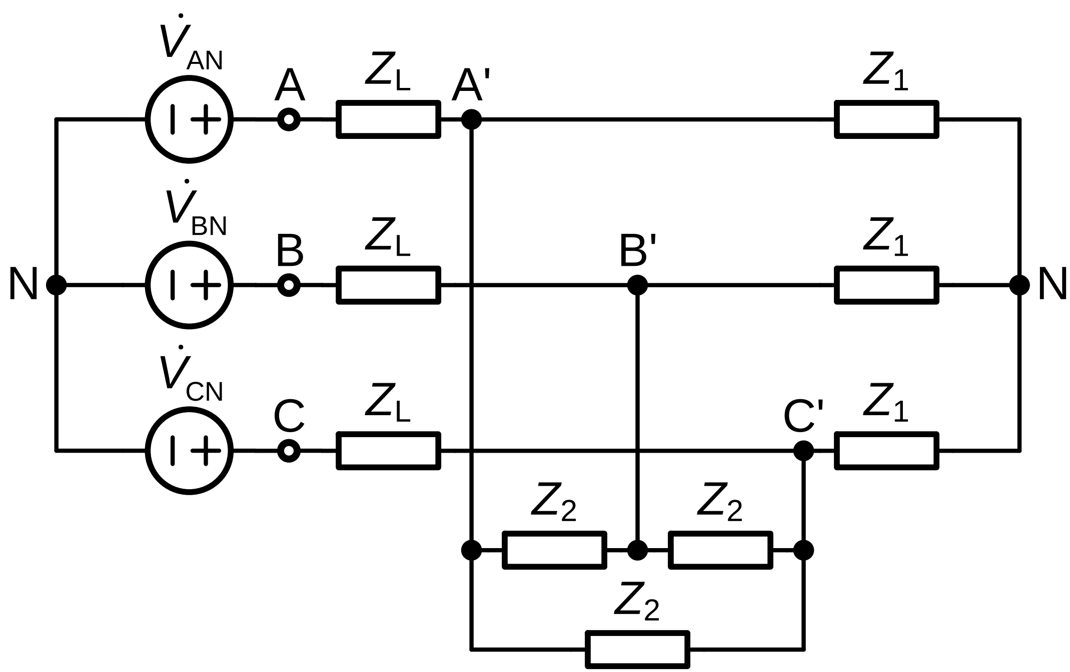
2. 将电压源中性点N与负载中性点N’短路；

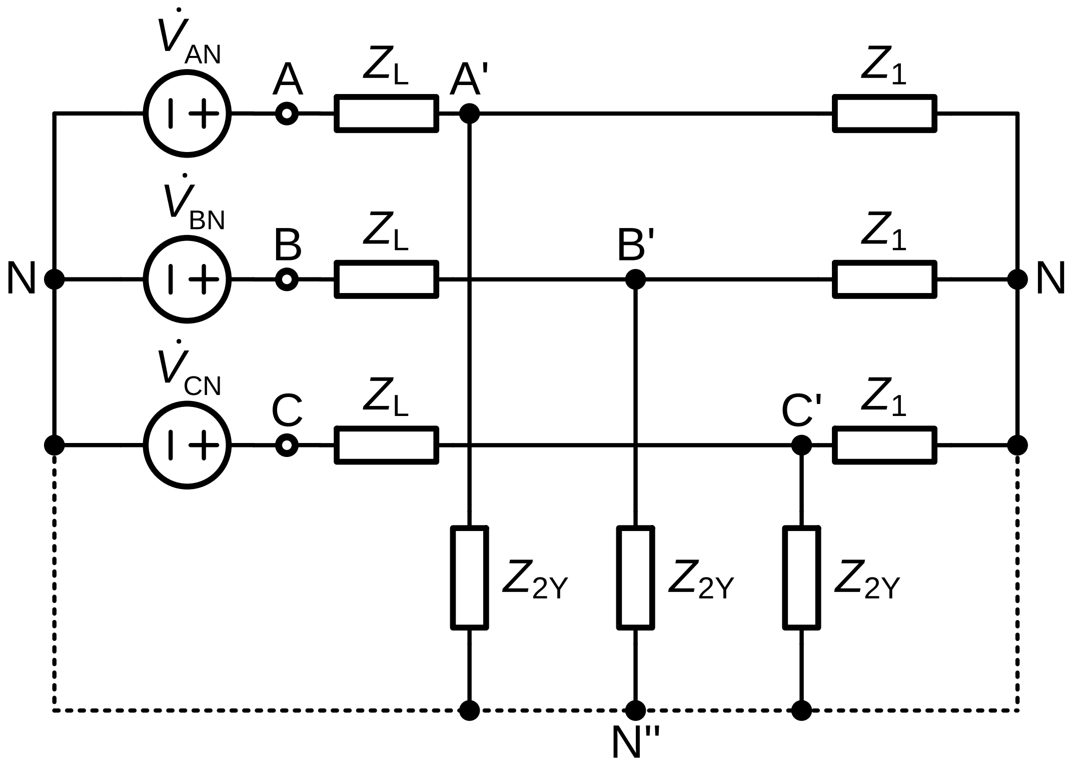
3. 取出一相电路进行计算；

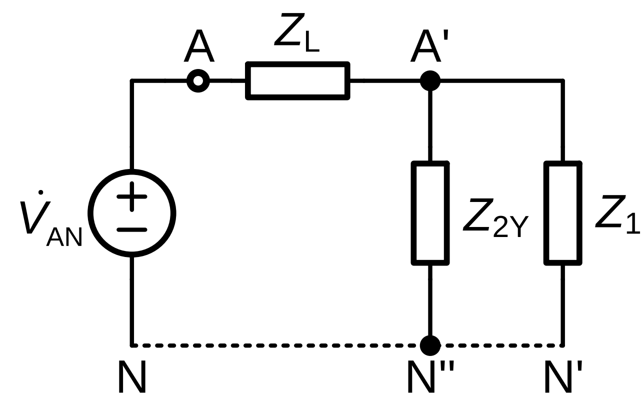
4. 根据对称关系推算其它两相的电压和电流。

例题2

如图所示，对称三相正弦交流电路，$Z_1 = 50\ \Omega$，$Z_2 = (150 + j150)\ \Omega$，$Z_L = j50\ \Omega$，$\dot{V}_{AB} = 150\sqrt{6}\angle 0^\circ\ V$，试求负载线电压和各负载的相电流。







$$\dot V_{AN} = \frac{\dot V_{AB}}{\sqrt{3}\angle 30^\circ} = 150\sqrt{2}\angle -30^\circ\ \text{V}$$

$$Z_{2Y} = \frac{Z_2}{3} = \frac{150 + j150}{3} = (50 + j50)\ \Omega$$

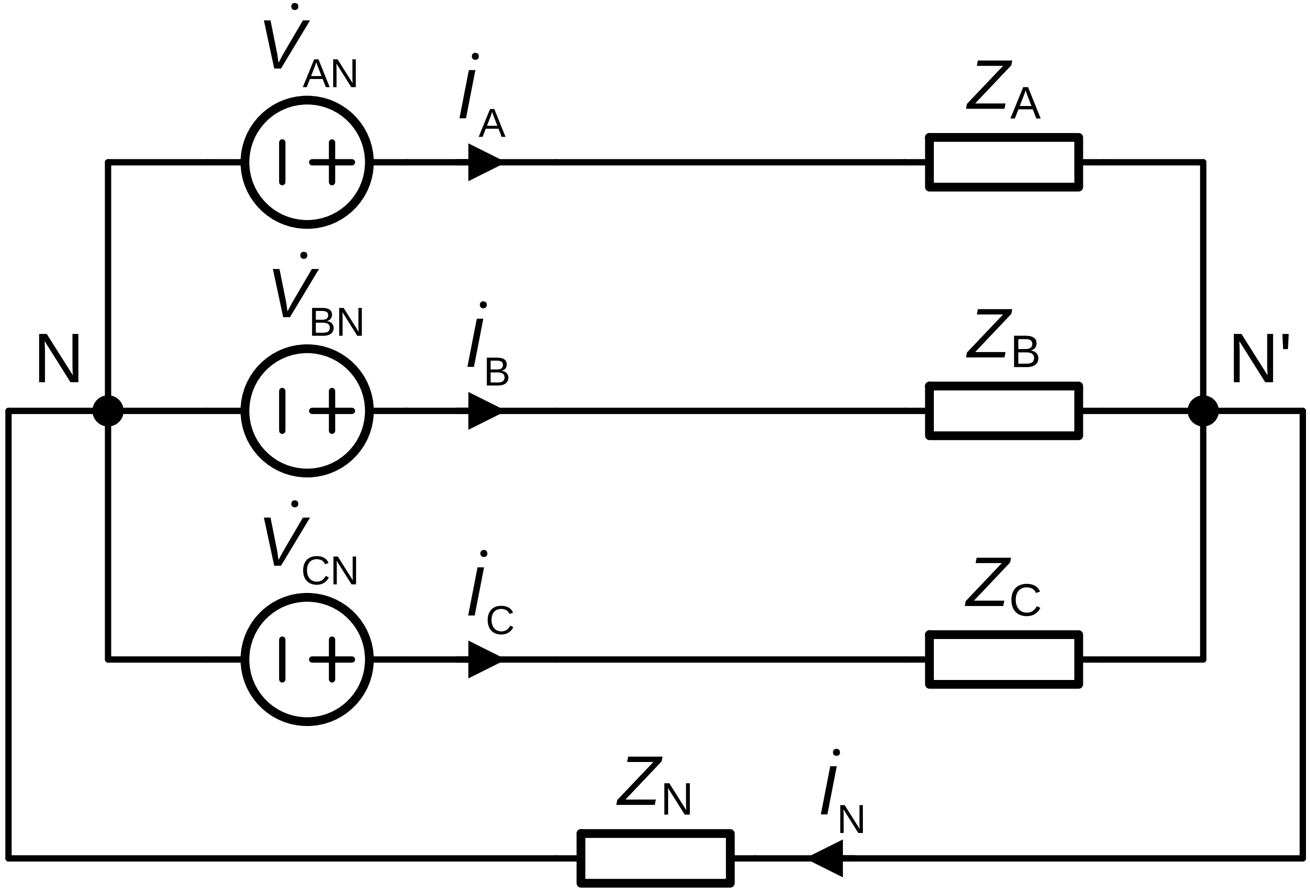
$$\dot V_{A'N'} = \dot V_{AN}\frac{Z_{2Y} \parallel Z_1}{Z_L + Z_{2Y} \parallel Z_1} = 150\sqrt{2}\angle -30^\circ \ \frac{(50 + j50)\ \parallel 50}{j50 + (50 + j50)\ \parallel 50} = 100\angle -75^\circ$$

$$\dot V_{A'B'} = \sqrt{3}\dot V_{A'N'}\angle 30^\circ = 100\sqrt{3}\angle -45^\circ$$

$$\dot I_{A'N'} = \frac{\dot V_{A'N'}}{Z_1} = \frac{100\angle -75^\circ}{50} = 2\angle -75^\circ,\ \dot I_{A'B'} = \frac{\dot V_{A'B'}}{Z_2} = \frac{100\sqrt{3}\angle -45^\circ}{150 + j150} = \frac{\sqrt{6}}{3}\angle -90^\circ$$

非对称三相电路

电压源不对称，或者负载不等。



电压源对称，负载不等。以电压源的中性点N为参考点，对负载中性点N'列节点电压方程

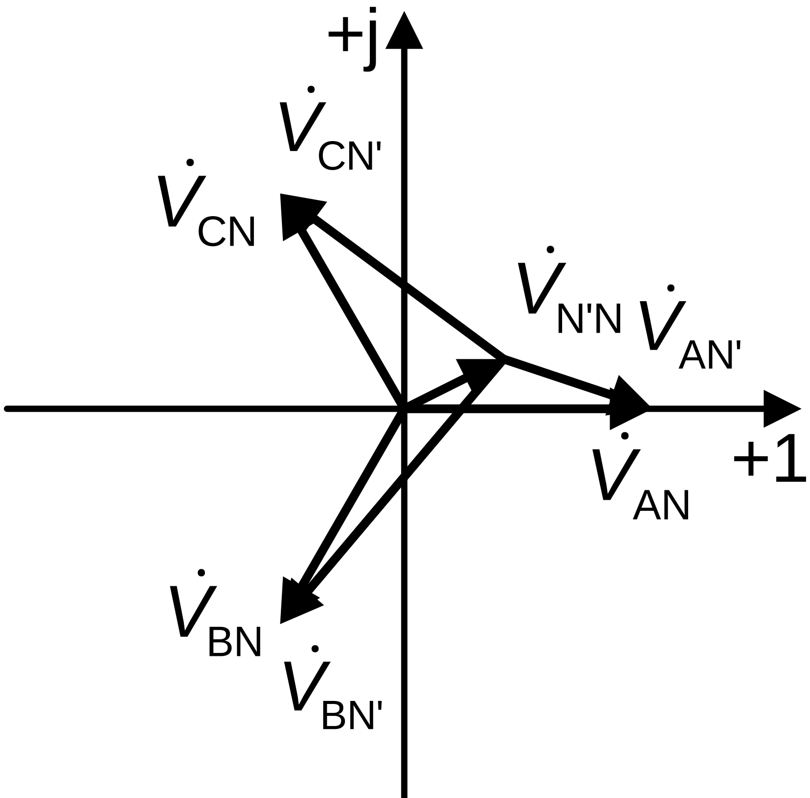
$$\left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}+\frac{1}{Z_N}\right)\dot{V}_{N'N}=\frac{\dot{V}_{AN}}{Z_A}+\frac{\dot{V}_{BN}}{Z_B}+\frac{\dot{V}_{CN}}{Z_C}$$

$$\dot{V}_{N'N}=\frac{\dot{V}_{AN}/Z_A+\dot{V}_{BN}/Z_B+\dot{V}_{CN}/Z_C}{1/Z_A+1/Z_B+1/Z_C+1/Z_N}\neq 0$$

由于负载不对称，使得电压源中性点和负载中性点之间的电压不为零。

负载中性点的位移

负载中性点与电压源中性点的电位不等的现象称为负载中性点的位移



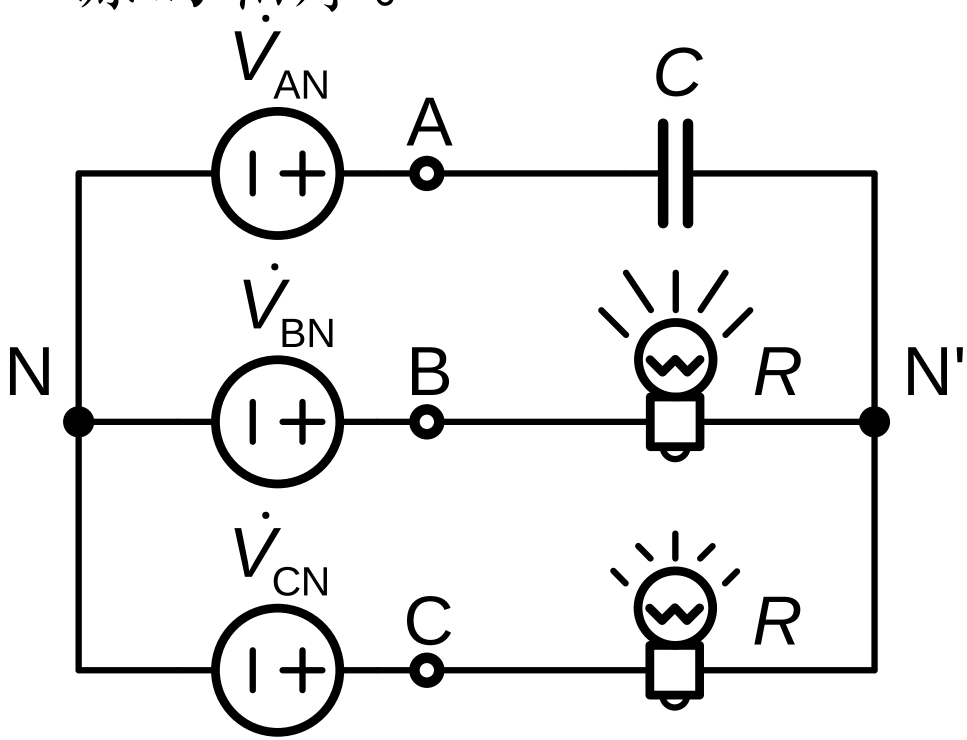
$$\begin{cases} \dot{V}_{AN'} = \dot{V}_{AN} - \dot{V}_{N'N} \\ \dot{V}_{BN'} = \dot{V}_{BN} - \dot{V}_{N'N} \\ \dot{V}_{CN'} = \dot{V}_{CN} - \dot{V}_{N'N} \end{cases}$$

后果：有的相电压过高，有的相电压过低。

措施：中性线阻抗为零。

例题3：相序指示器

如图所示电路为相序指示器，设$R = 1/(\omega C)$，试说明如何根据两个白炽灯亮度差异确定对称三相电压源的相序。



$$\dot{V}_{N'N} = \frac{\dot{V}_{AN}/Z_A + \dot{V}_{BN}/Z_B + \dot{V}_{CN}/Z_C}{1/Z_A + 1/Z_B + 1/Z_C}$$

$$= \frac{j\omega C\dot{V}_{AN} + \dot{V}_{AN}/(\alpha R) + \dot{V}_{AN}/(\alpha^2 R)}{j\omega C + 1/R + 1/R}$$

$$= \frac{-1 + j3}{5}\dot{V}_{AN}$$

$$\dot{V}_{BN'} = \dot{V}_{BN} - \dot{V}_{N'N} = \dot{V}_{AN}\left[1/\alpha - (-1 + j3)/5\right]$$

$$= (1.5\angle -101.5°)\dot{V}_{AN}$$

$$\dot{V}_{CN'} = \dot{V}_{CN} - \dot{V}_{N'N} = \dot{V}_{AN}\left[1/\alpha^2 - (-1 + j3)/5\right]$$

$$= (0.4\angle 138.4°)\dot{V}_{AN}$$

$V_{BN'} > V_{CN'}$ 亮的为B相，暗的为C相。

三相电路的功率

对称三相电压源的瞬时功率为常数。这种性质称为瞬时功率平衡。

$$v_{AN}=\sqrt{2}V_P\cos(\omega t+\varphi_v),\ i_{NA}=\sqrt{2}I_P\cos(\omega t+\varphi_i)$$

$$v_{BN}=\sqrt{2}V_P\cos(\omega t+\varphi_v-120°),\ i_{NB}=\sqrt{2}I_P\cos(\omega t+\varphi_i-120°)$$

$$v_{CN}=\sqrt{2}V_P\cos(\omega t+\varphi_v+120°),\ i_{NC}=\sqrt{2}I_P\cos(\omega t+\varphi_i+120°)$$

$$p_A=v_{AN}i_{NA}=V_PI_P\cos(\varphi_v-\varphi_i)+V_PI_P\cos(2\omega t+\varphi_v+\varphi_i)$$

$$p_B=v_{BN}i_{NB}=V_PI_P\cos(\varphi_v-\varphi_i)+V_PI_P\cos(2\omega t+\varphi_v+\varphi_i-240°)$$

$$p_C=v_{CN}i_{NC}=V_PI_P\cos(\varphi_v-\varphi_i)+V_PI_P\cos(2\omega t+\varphi_v+\varphi_i+240°)$$

$$p=p_A+p_B+p_C=3V_PI_P\cos(\varphi_v-\varphi_i)$$

对称三相电压源的功率

有功功率：

$$P = P_A + P_B + P_C = 3V_P I_P \cos\varphi = 3V_P I_P \lambda = \sqrt{3} V_L I_L \lambda$$

无功功率：

$$Q = Q_A + Q_B + Q_C = 3V_P I_P \sin\varphi = 3V_P I_P \sqrt{1-\lambda^2} = \sqrt{3} V_L I_L \sqrt{1-\lambda^2}$$

视在功率：

$$S = S_A + S_B + S_C = 3V_P I_P = \sqrt{3} V_L I_L = \sqrt{P^2 + Q^2}$$

非对称三相电压源的功率

有功功率：

$$P = P_{\mathrm{A}} + P_{\mathrm{B}} + P_{\mathrm{C}} = V_{\mathrm{AN}} I_{\mathrm{NA}} \cos\varphi_{\mathrm{A}} + V_{\mathrm{BN}} I_{\mathrm{NB}} \cos\varphi_{\mathrm{B}} + V_{\mathrm{CN}} I_{\mathrm{NC}} \cos\varphi_{\mathrm{C}}$$

无功功率：

$$Q = Q_{\mathrm{A}} + Q_{\mathrm{B}} + Q_{\mathrm{C}} = V_{\mathrm{AN}} I_{\mathrm{NA}} \sin\varphi_{\mathrm{A}} + V_{\mathrm{BN}} I_{\mathrm{NB}} \sin\varphi_{\mathrm{B}} + V_{\mathrm{CN}} I_{\mathrm{NC}} \sin\varphi_{\mathrm{C}}$$

视在功率：

$$S = \sqrt{P^2 + Q^2}$$

例题4

已知对称三相星形感性负载的线电压为380 V，线电流为10 A，平均功率为5.7 kW。

1.求三相负载的功率因数及等效阻抗；

2.设C相负载短路，再求各相电流、线电流和平均功率。

$$\lambda = \frac{P}{\sqrt{3}V_{\mathrm{L}}I_{\mathrm{L}}} = \frac{5.7\ \mathrm{kW}}{\sqrt{3}\times 380\ \mathrm{V}\times 10\ \mathrm{A}} \approx 0.866 \qquad \varphi = \arccos\lambda = 30^\circ$$

$$Z = \frac{V_{\mathrm{P}}}{I_{\mathrm{P}}}\angle\varphi = \frac{V_{\mathrm{L}}/\sqrt{3}}{I_{\mathrm{L}}}\angle\varphi = \frac{220\ \mathrm{V}}{10\ \mathrm{A}}\angle 30^\circ = 22\angle 30^\circ \approx (19.05 + \mathrm{j}11)\ \Omega$$

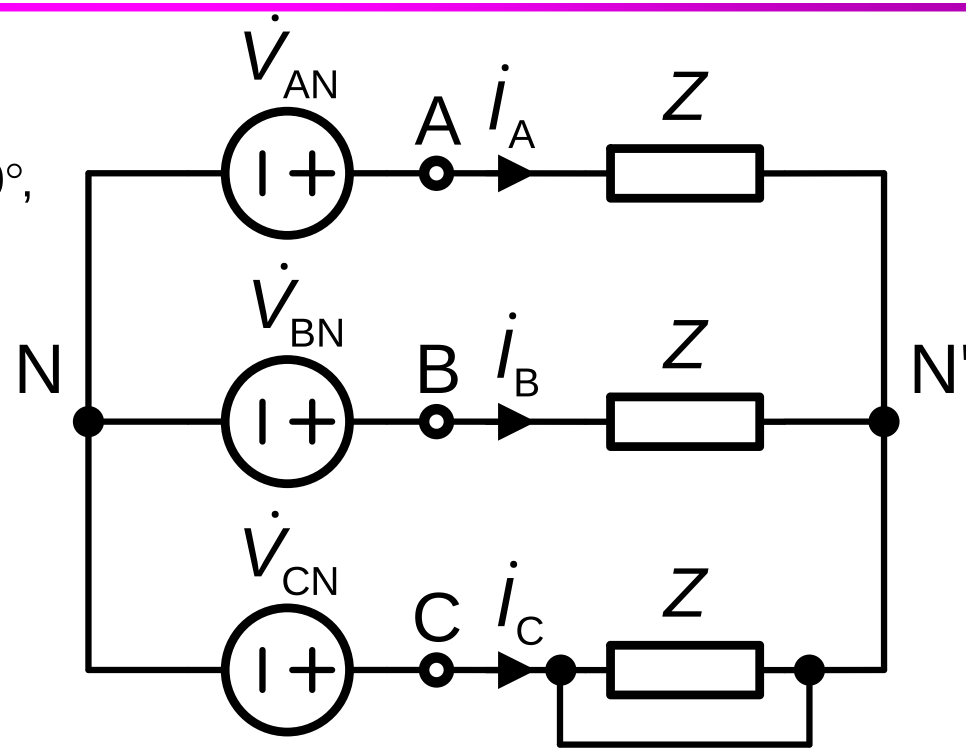
$\dot{V}_{AN} = 220\angle 0°,$

$\dot{V}_{BN} = 220\angle -120°,\ \dot{V}_{CN} = 220\angle 120°,$

$$\dot{I}_A = \frac{\dot{V}_{AN'}}{Z} = \frac{\dot{V}_{AN} - \dot{V}_{CN}}{Z} = \frac{220\angle 0° - 220\angle 120°}{22\angle 30°} = 10\sqrt{3}\angle -60°$$

$$\dot{I}_B = \frac{\dot{V}_{BN'}}{Z} = \frac{\dot{V}_{BN} - \dot{V}_{CN}}{Z} = \frac{220\angle -120° - 220\angle 120°}{22\angle 30°} = 10\sqrt{3}\angle -120°$$

$$\dot{I}_C = -\dot{I}_A - \dot{I}_B = 30\angle 90°$$



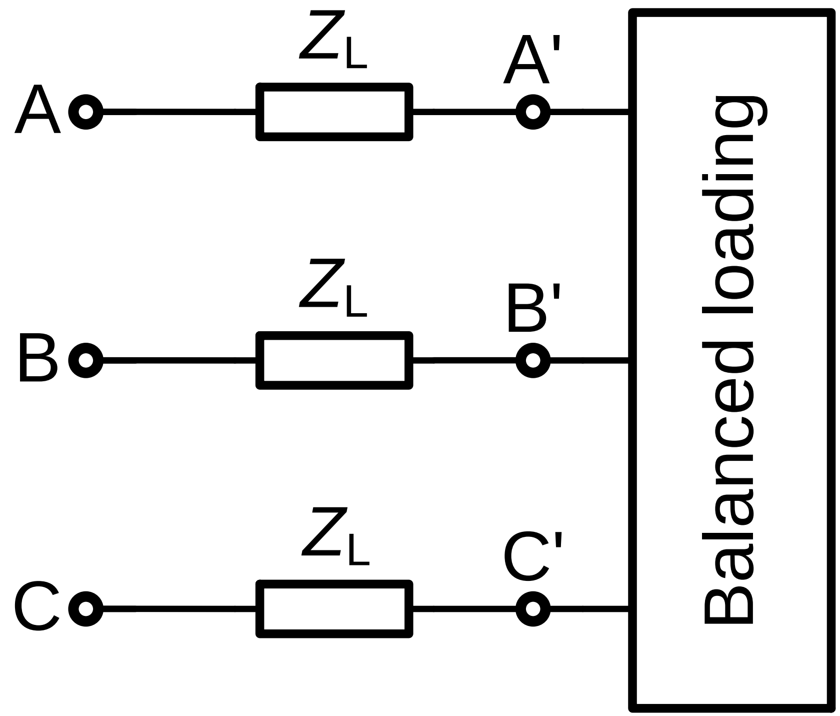
$$P = V_{AN}I_A\cos 60° + V_{BN}I_B\cos 0° + V_{CN}I_C\cos 30° = 220\times 10\sqrt{3}\times 0.5 + 220\times 10\sqrt{3} + 220\times 30\times \sqrt{3}/2 \approx 11.4\ \text{kW}$$

$$P = I_A^2R + I_B^2R = (10\sqrt{3})^2\times 11\sqrt{3} + (10\sqrt{3})^2\times 11\sqrt{3} \approx 11.4\ \text{kW}$$

例题5

如图所示对称三相电路，已知感性负载额定电压为380 V,额定功率为3.3 kW，功率因数为0.5，线路阻抗为(1 + 4j) Ω。

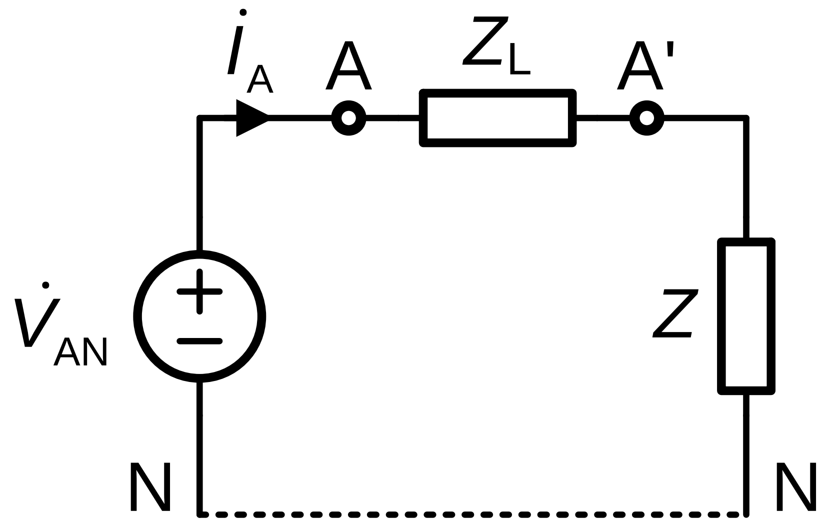
1. 若要求负载的线电压为额定电压，问电源线电压为多少？
2. 若电源线电压为380 V,求负载的线电压和负载实际消耗的平均功率。



$$\dot{V}_{\text{A'N'}} = \frac{V_{\text{A'B'}}}{\sqrt{3}}\angle 0^\circ = \frac{380}{\sqrt{3}}\angle 0^\circ = 220\angle 0^\circ,$$

$$I_{\text{A}} = \frac{P}{3V_{\text{A'N'}}\lambda} = \frac{3.3\ \text{kW}}{3\times 220\times 0.5} = 10\ \text{A}$$

$$\varphi = \arccos\lambda = 60^\circ$$



$$\dot{I}_{\text{A}} = 10\angle -60^\circ\ \text{A}$$

$$\dot{V}_{\text{AN}} = \dot{V}_{\text{A'N'}} + Z_{\text{L}}\dot{I}_{\text{A}}$$
$$= 220\angle 0^\circ + (1 + \text{j}4)\times 10\angle -60^\circ \approx 260\angle 2.5^\circ$$

$$V_{\text{AB}} = \sqrt{3}V_{\text{AN}} = 450\ \text{V}$$

$$V_{\text{L}} = \frac{380}{450}\times 380 \approx 321\ \text{V}$$

$$P = 3.3\ \text{kW}\times\left(\frac{380}{450}\right)^2 = 2.355\ \text{kW}$$

三相电路功率的测量

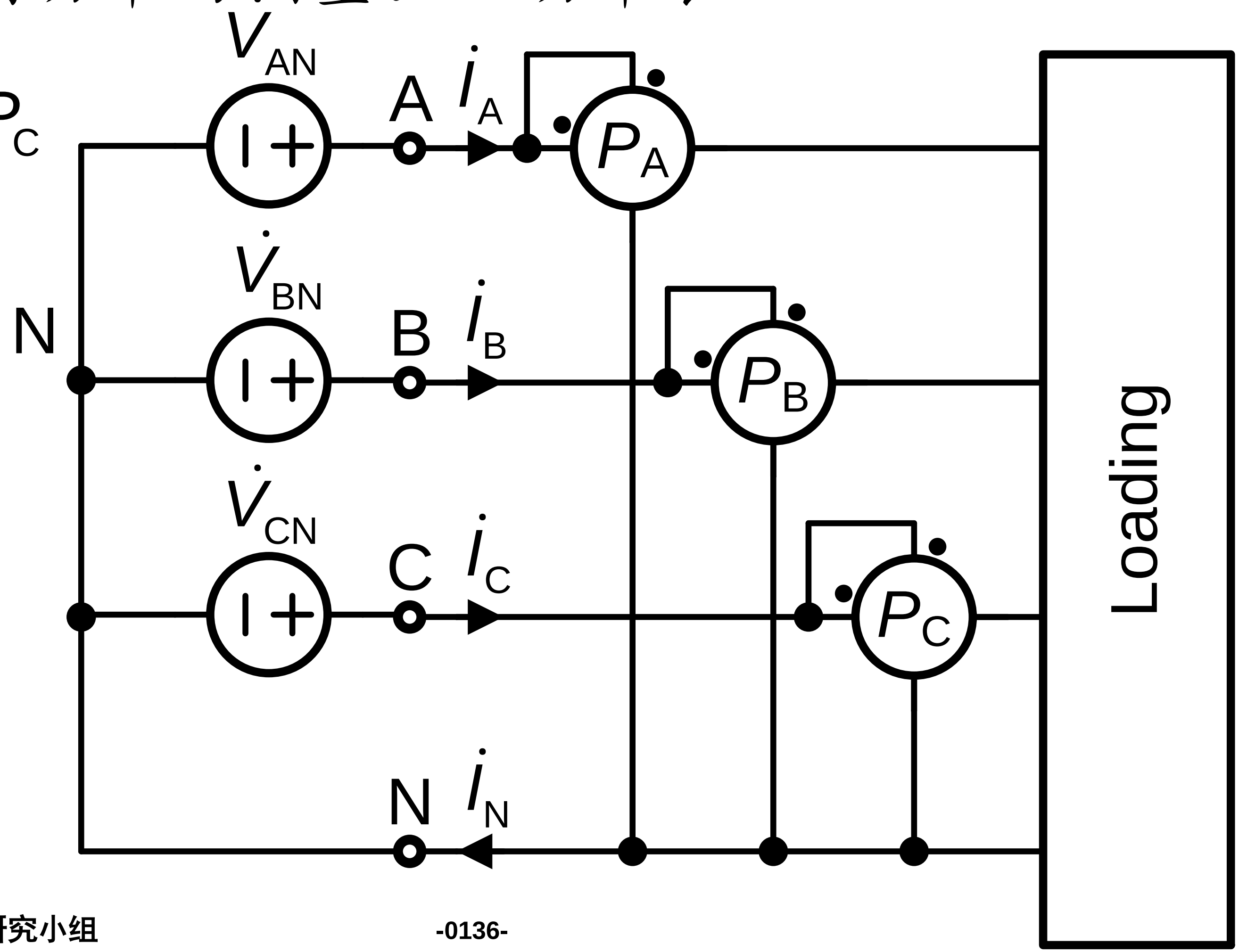
三相四线制功率的测量：三功率表法

$$P = P_A + P_B + P_C$$

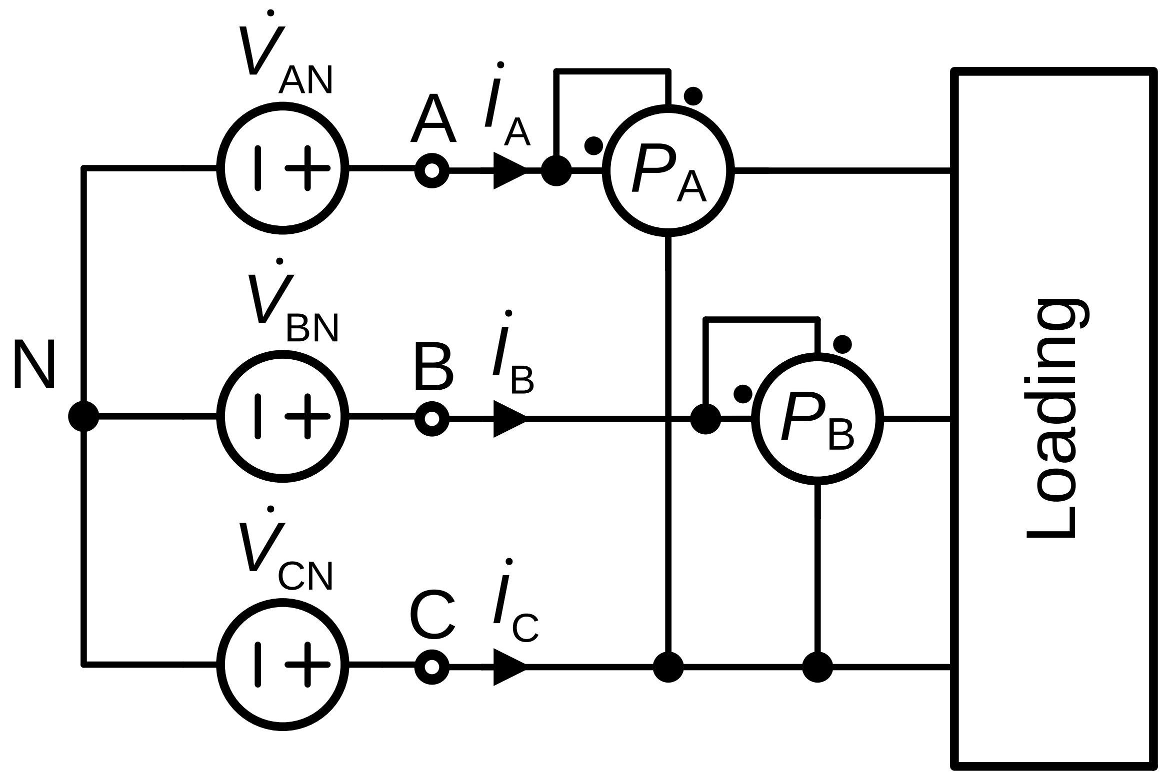
$$P_A = \mathrm{Re}[\dot{V}_{AN}\dot{I}_A^*]$$

$$P_B = \mathrm{Re}[\dot{V}_{BN}\dot{I}_B^*]$$

$$P_C = \mathrm{Re}[\dot{V}_{CN}\dot{I}_C^*]$$



三相三线制功率的测量：两功率表法



论证

$$P_{\mathrm{A}} = \mathrm{Re}[\dot{V}_{\mathrm{AC}}\dot{I}_{\mathrm{A}}^{*}], \quad P_{\mathrm{B}} = \mathrm{Re}[\dot{V}_{\mathrm{BC}}\dot{I}_{\mathrm{B}}^{*}]$$

$$\begin{aligned} P &= P_{\mathrm{A}} + P_{\mathrm{B}} = \mathrm{Re}[\dot{V}_{\mathrm{AC}}\dot{I}_{\mathrm{A}}^{*} + \dot{V}_{\mathrm{BC}}\dot{I}_{\mathrm{B}}^{*}] \\ &= \mathrm{Re}[(\dot{V}_{\mathrm{AN}} - \dot{V}_{\mathrm{CN}})\dot{I}_{\mathrm{A}}^{*} + (\dot{V}_{\mathrm{BN}} - \dot{V}_{\mathrm{CN}})\dot{I}_{\mathrm{B}}^{*}] \\ &= \mathrm{Re}[\dot{V}_{\mathrm{AN}}\dot{I}_{\mathrm{A}}^{*} + \dot{V}_{\mathrm{BN}}\dot{I}_{\mathrm{B}}^{*} + \dot{V}_{\mathrm{CN}}(-\dot{I}_{\mathrm{A}}^{*} - \dot{I}_{\mathrm{B}}^{*})] = \mathrm{Re}[\dot{V}_{\mathrm{AN}}\dot{I}_{\mathrm{A}}^{*} + \dot{V}_{\mathrm{BN}}\dot{I}_{\mathrm{B}}^{*} + \dot{V}_{\mathrm{CN}}\dot{I}_{\mathrm{C}}^{*}] \\ &= \mathrm{Re}[\tilde{S}_{\mathrm{A}} + \tilde{S}_{\mathrm{B}} + \tilde{S}_{\mathrm{C}}] = \mathrm{Re}[\tilde{S}] \end{aligned}$$

对称三相三线制的两功率表法

令 $\dot{V}_{\text{AN}} = V_{\text{AN}}\angle 0°,\ \dot{I}_{\text{A}} = I_{\text{A}}\angle -\varphi$

$$P_{\text{A}} = \text{Re}[\dot{V}_{\text{AC}}\dot{I}_{\text{A}}^{*}] = V_{\text{AC}}I_{\text{A}}\cos(\varphi - 30°) = V_{\text{L}}I_{\text{L}}\cos(\varphi - 30°)$$

$$P_{\text{B}} = \text{Re}[\dot{V}_{\text{BC}}\dot{I}_{\text{B}}^{*}] = V_{\text{BC}}I_{\text{B}}\cos(\varphi + 30°) = V_{\text{L}}I_{\text{L}}\cos(\varphi + 30°)$$

无功功率：

$$Q = \sqrt{3}(P_{\text{A}} - P_{\text{B}}) = \sqrt{3}V_{\text{L}}I_{\text{L}}\sin\varphi$$

功率因数：

$$\varphi = \arctan\frac{Q}{P} = \sqrt{3}\,\frac{P_{\text{A}} - P_{\text{B}}}{P_{\text{A}} + P_{\text{B}}}$$